A TEXTBOOK OF

ELECTRICAL TECHNOLOGY

IN S.I. UNITS

VOLUME III

TRANSMISSION, DISTRIBUTION AND UTILIZATION

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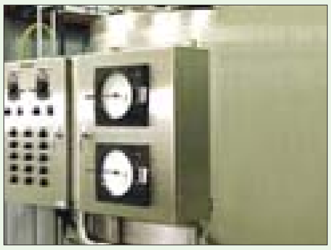

Factors Influencing Costs and Tariffs of Electric Supply — Demand — Average Demand — Maximum Demand — Demand Factor—Diversity of Demand—Diversity Factor — Load Factor—Significance of Load Factor — Plant Factor or Capacity Factor— Utilization Factor (or Plant use Factor)— Connected Load Factor—Load Curves of a Generating Station — Tariffs-Flat Rate— Sliding Scale — Two-part Tariff — Kelvin's Law-Effect of Cable Insulation —Note on Power Factor—Disadvantages of Low Power Factor—Economics of Power Factor—Economical Limit of Power Factor Correction — Objective Tests.

CHAPTER 40

Learning Objectives

- Transmission and Distribution of D.C. Power
- Two-wire and Three-wire System
- Voltage Drop and Transmission Efficiency
- Methods of Feeding Distributor
- D.C.Distributor Fed at One End
- Uniformaly Loaded Distributor
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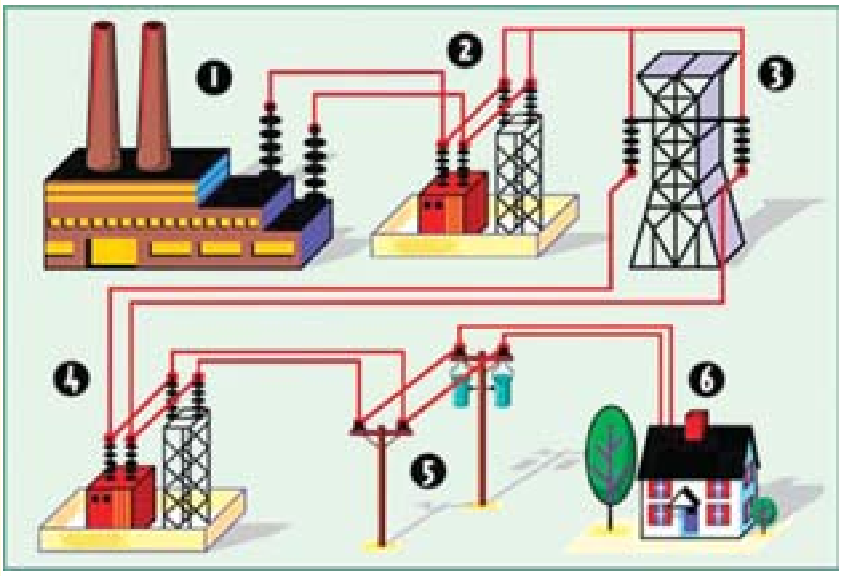
D.C. TRANSMISSION AND DISTRIBUTION



(1) Electricity leaves the power plant, (2) Its voltage is increased at a step-up transformer, (3) The electricity travels along a transmission line to the area where power is needed, (4) There, in the substation, voltage is decreased with the help of step-down transformer, (5) Again the transmission lines carry the electricity, (6) Electricity reaches the final consumption points

40.1. Transmission and Distribution of D.C. Power

By transmission and distribution of electric power is meant its conveyance from the central station where it is generated to places, where it is demanded by the consumers like mills, factories, residential and commercial buildings, pumping stations etc. Electric power may be transmitted by two methods.

(i) By overhead system or *(ii)* By underground system—this being especially suited for densely-populated areas though it is somewhat costlier than the first method. In over-head system, power is conveyed by bare conductors of copper or aluminium which are strung between wooden or steel poles erected at convenient distances along a route. The bare copper or aluminium wire is fixed to an insulator which is itself fixed onto a cross-arm on the pole. The number of cross-arms carried by a pole depends on the number of wires it has to carry. Line supports consist of *(i)* pole structures and *(ii)* tower. Poles which are made of wood, reinforced concrete or steel are used up to 66 kV whereas steel towers are used for higher voltages.

The underground system employs insulated cables which may be single, double or triple-core etc.

A good system whether overhead or underground should fulfil the following requirements :

1. The voltage at the consumer's premises must be maintained within ± 4 or ± 6% of the declared voltage, the actual value depending on the type of load*.
2. The loss of power in the system itself should be a small percentage (about 10%) of the power transmitted.
3. The transmission cost should not be unduly excessive.
4. The maximum current passing through the conductor should be limited to such a value as not to overheat the conductor or damage its insulation.
5. The insulation resistance of the whole system should be very high so that there is no undue leakage or danger to human life.

It may, however, be mentioned here that these days all production of power is as a.c. power and nearly all d.c. power is obtained from large a.c. power systems by using converting machinery like synchronous or rotary converters, solid-state converters and motor-generator sets etc. There are many sound reasons for producing power in the form of alternating current rather than direct current.

(i) It is possible, in practice, to construct large high-speed a.c. generators of capacities up to 500 MW. Such generators are economical both in the matter of cost per kWh of electric energy produced as well as in operation. Unfortunately, d.c. generators cannot be built of ratings higher than 5 MW because of commutation trouble. Moreover, since they must operate at low speeds, it necessitates large and heavy machines.

(ii) A.C. voltage can be efficiently and conveniently raised or lowered for economic transmission and distribution of electric power respectively. On the other hand, d.c. power has to be generated at comparatively low voltages by units of relatively low power ratings. As yet, there is no economical method of raising the d.c. voltage for transmission and lowering it for distribution.

Fig. 40.1 shows a typical power system for obtaining d.c. power from a.c. power. Other details such as instruments, switches and circuit breakers etc. have been omitted.

Two 13.8 kV alternators run in parallel and supply power to the station bus-bars. The voltage is stepped up by 3-phase transformers to 66 kV for transmission purposes** and is again stepped down to 13.8 kV at the sub-station for distribution purposes. Fig. 40.1 shows only three methods commonly used for converting a.c. power to d.c. power at the sub-station.

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* According to Indian Electricity Rules, voltage fluctuations should not exceed ± 5% of normal voltage for L.T. supply and $\pm 12\frac{1}{2}$ % for H.T. supply.

** Transmission voltages of upto 400 kV are also used.

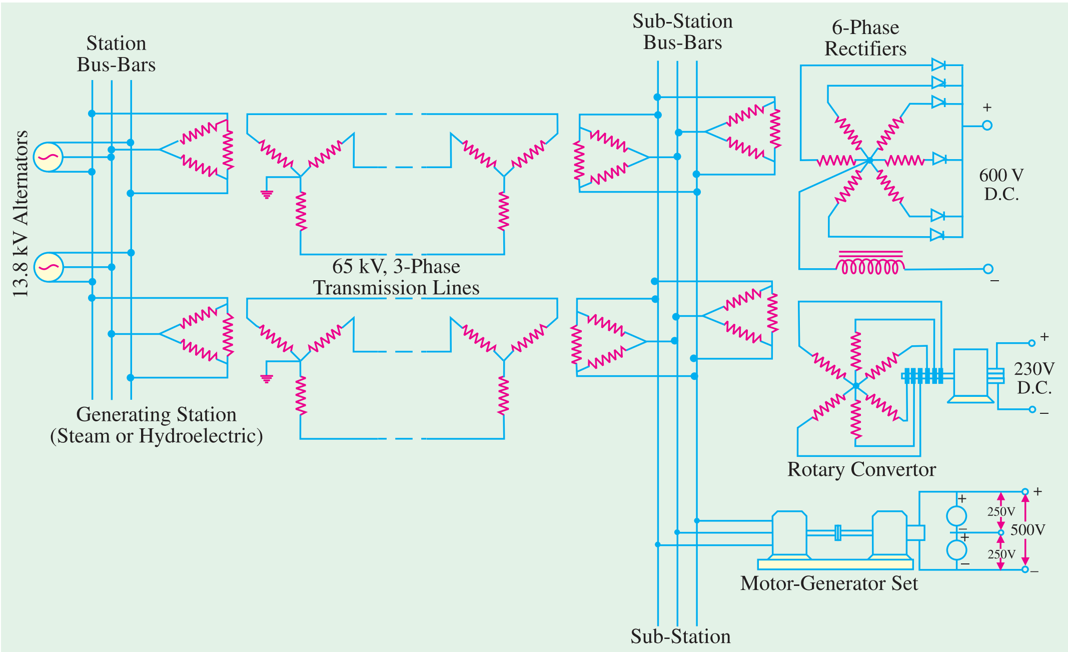


Fig. 40.1

(*a*) a 6-phase mercury-arc rectifier gives 600 V d.c. power after the voltage has been stepped down to a proper value by the transformers. This 600-V d.c. power is generally used by electric railways and for electrolytic processes.

(*b*) a rotary converter gives 230 V d.c. power.

(*c*) a motor-generator set converts a.c. power to 500/250 d.c. power for 3-wire distribution systems.

The above figure shows a motor-generator set. Nowadays, we use solid-state devices, called rectifiers, to convert standard AC to DC current. Back in the olden days, they needed a “motor dynamo” set to make the conversion as shown above. An AC motor would turn a DC Generator, as pictured above

In Fig. 40.2 is shown a schematic diagram of ***low tension*** distribution system for d.c. power. The whole system consists of a network of cables or conductors which convey power from central station to the consumer’s premises. The station bus-bars are fed by a number of generators (only two shown in the figure) running in parallel. From the bus-bars, the power is carried by many ***feeders*** which radiate to various parts of a city or locality. These feeders deliver power at certain points to a distributor which runs along the various streets. The points *FF*, as shown in the figure, are known as ***feeding points***. Power connections to the various consumers are given ***from this distributor and not directly from the feeder***. The wires which convey power from the distributor to the consumer’s premises are known as ***service mains (S)***. Sometimes when there is only one distributor in a locality, several sub-distributors (*SD*) branching off from the distributor are employed and service mains are now connected to them instead of distributor as shown in the figure.

Obviously, a feeder is designed on the basis of its current-carrying capacity whereas the design of distributor is based on the voltage drop occurring in it.

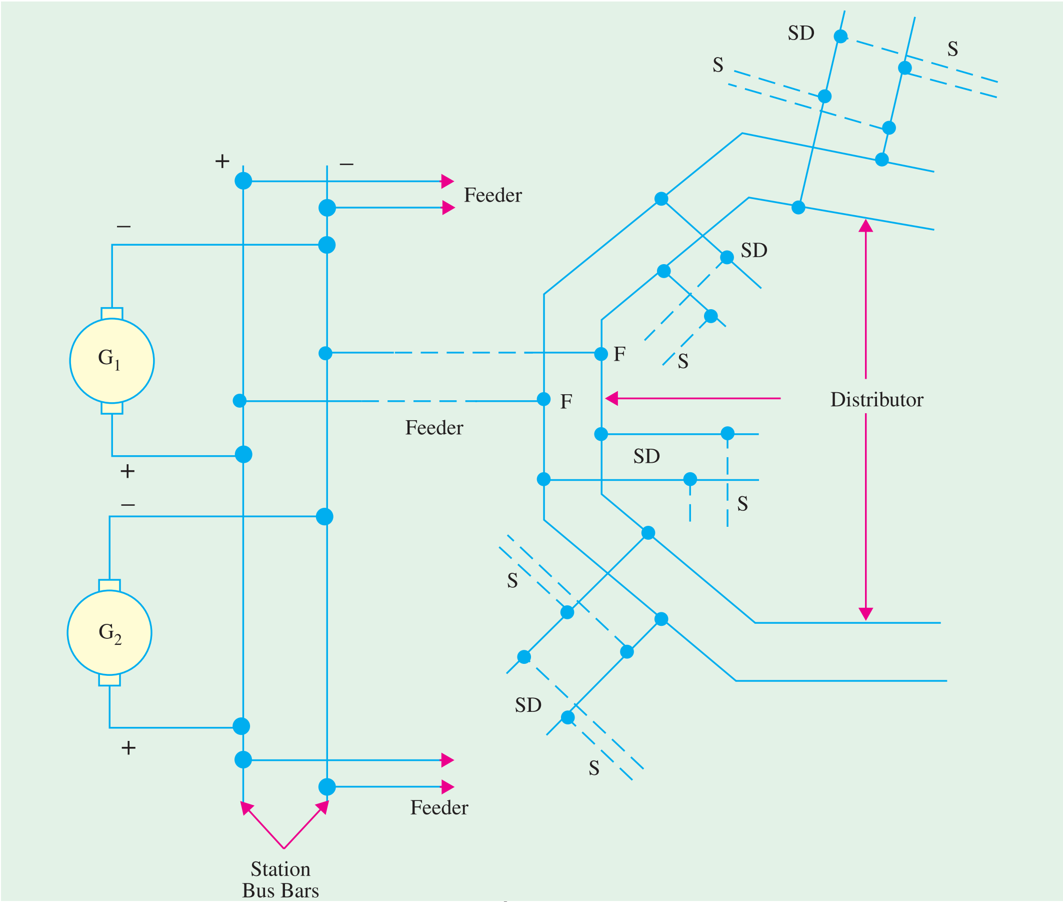


Fig. 40.2

40.2. Two-wire and Three-wire Systems

In d.c. systems, power may be fed and distributed either by ***(i)* 2-wire system** or ***(ii)* 3-wire system**.

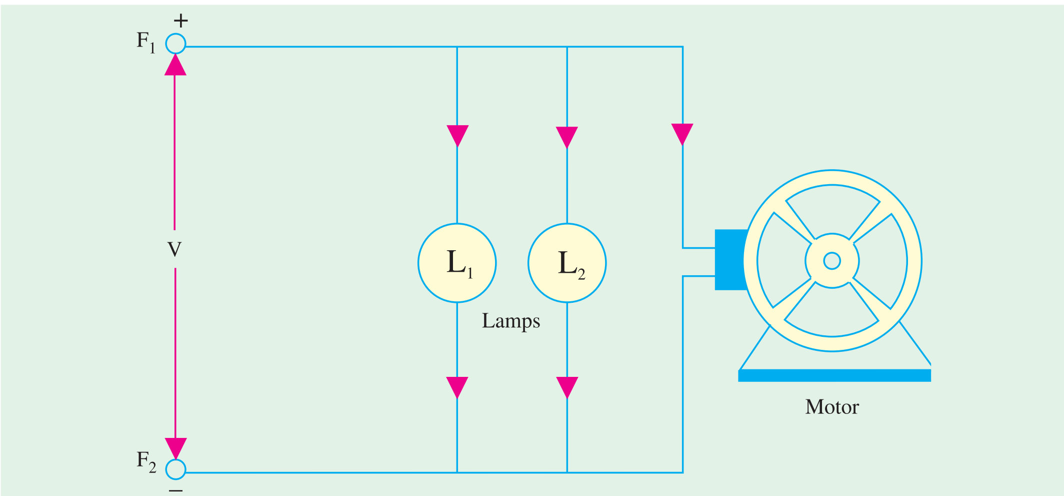


Fig. 40.3

In the 2-wire system, one is the outgoing or positive wire and the other is the return or negative wire. In the case of a 2-wire distributor, lamps, motors and other electrical apparatus are connected in parallel between the two wires as shown in Fig. 40.3. As seen, the potential difference and current have their maximum values at feeding points F_1 and F_2. The standard voltage between the conductors is 220 V.

The 2-wire system when used for transmission purposes, has much lower efficiency and economy as compared to the 3-wire system as shown later.

A 3-wire has not only a higher efficiency of transmission (Fig. 40.4) but when used for distribution purposes, it makes available two voltages at the consumer's end (Fig. 40.5). This 3-wire system consists of two '***outers***' and a ***middle*** or ***neutral*** wire which is earthed at the generator end. Its potential is midway between that of the outers *i.e.* if the p.d. between the outers is 460 V, then the p.d. of positive outer is 230 V ***above*** the neutral and that of negative outer is 230 V ***below*** the neutral. Motors requiring higher voltage are connected across the outers whereas lighting and heating circuits requiring less voltage are connected between any one of the outers and the neutral.

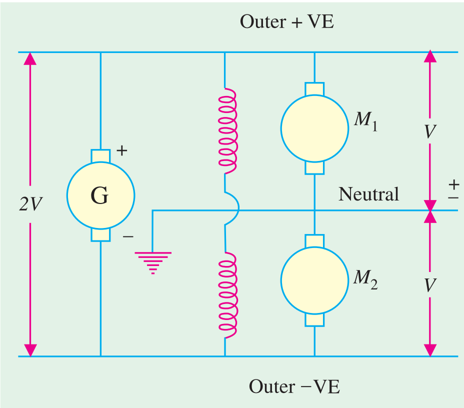


Fig. 40.4

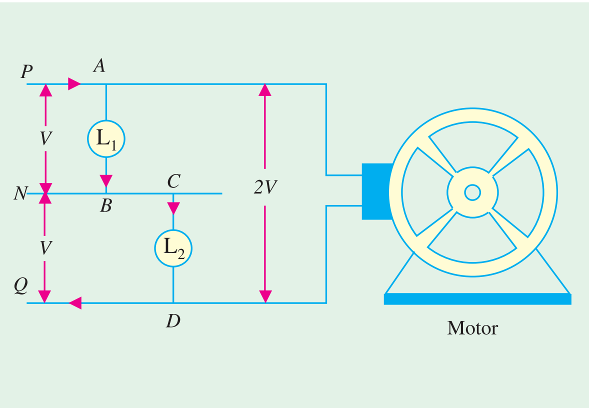


Fig. 40.5

The addition of the middle wire is made possible by the connection diagram shown in Fig. 40.4. G is the main generator which supplies power to the whole system. M_1 and M_2 are two identical shunt machines coupled mechanically with their armatures and shunt field winding joined in series across the outers. The junction of their armatures is earthed and the neutral wire is taken out from there.

40.3 Voltage Drop and Transmission Efficiency

Consider the case of a ***2-wire feeder*** (Fig. 40.6). AB is the sending end and CD the receiving end. Obviously, the p.d. at AB is higher than at CD. The difference in potential at the two ends is the potential drop or 'drop' in the cable. Suppose the transmitting voltage is 250 V, current in AC is 10 amperes, and resistance of each feeder conductor is 0.5 Ω, then drop in each feeder conductor is $10 \times 0.5 = 5$ volt and drop in both feeder conductor is $5 \times 2 = 10$ V.

$\therefore$ *P.d.* at Receiving end CD is $= 250 - 10 = 240$ V

Input power at AB $= 250 \times 10 = 2{,}500$ W

Output power at CD $= 240 \times 10 = 2{,}400$ W

$\therefore$ power lost in two feeders $= 2{,}500 - 2{,}400 = 100$W

The above power loss could also be found by using the formula

$$\text{Power loss} = 2I^2R = 2 \times 10^2 \times 0.5 = 100 \text{ W}$$

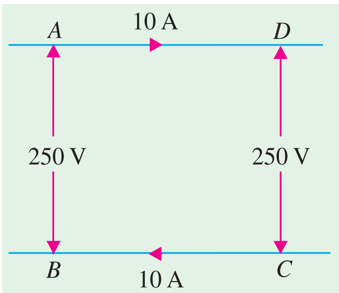


Fig. 40.6

The efficiency of transmission is defined, like any other efficiency, as the ratio of the output to input

$\therefore$ efficiency of transmission $= \dfrac{\text{power delivered by the line}}{\text{power received by the line}}$

In the present case, power delivered by the ***feeder*** is = 2500 W and power received by it as 2400 W.

$\therefore \quad \eta = 2400 \times 100/2500 = 96\%$

In general, if V_1 is the voltage at the sending end and V_2 at the receiving end and I the current delivered, then

$$\text{Input} = V_1 I, \qquad \text{output} = V_2 I \qquad \therefore \quad \eta = \frac{V_2 I}{V_1 I} = \frac{V_2}{V_1}$$

Now $\quad V_2 = V_1 - \text{drop in both conductors}$

$= V_1 - IR,$ where R is the resistance of both conductors

$$\therefore \quad \eta = \frac{V_1 - IR}{V_1} = 1 - \frac{IR}{V_1} \qquad \text{...(i)}$$

or $$\% \text{ efficiency} = 100 \times \left(1 - \frac{IR}{V_1}\right) = 100 - \left(\frac{IR}{V_1} \times 100\right)$$

Now, $(IR/V_1) \times 100$ represents the voltage drop in both conductors expressed as a percentage of the voltage at the sending end. It is known as percentage drop.

$\therefore \quad \% \eta = 100 - \%$ drop

In the present case total drop is 10 volt which expressed as percentage becomes $(10/250) \times 100 = 4\%$. Hence $\% \eta = 100 - 4 = 96\%$ ...**as before**

It is seen from equation ***(i)*** above that for a given drop, transmission efficiency can be considerably increased by increasing the voltage at the transmitting end. *i.e.*, V_1. Moreover, the cross-section of copper in the cables is decreased in proportion to the increase in voltage which results in a proportionate reduction of the cost of copper in the cables.

The calculation of drop in a ***feeder*** is, as seen from above, quite easy because of the fact that current is constant throughout its length. But it is not so in the case of ***distributors*** which are tapped off at various places along their entire lengths. Hence, their different sections carry different currents over different lengths. For calculating the total voltage drop along the entire length of a distributor, following information is necessary.

A DC power distribution system consists of a network of cables or conductors which conveys power from central station to the consumer's premises.

(i) value of current tapped at each load point.

(ii) the resistance of each section of the distributor between tapped points.

Example 40.1. *A DC 2-wire feeder supplies a constant load with a sending-end voltage of 220 V. Calculate the saving in copper if this voltage is doubled with power transmitted remaining the same.*

Solution. Let $\quad l =$ length of each conductor in metre

$\sigma =$ current density in A/m^2

$P =$ power supplied in watts

***(i)* 220 V Supply**

Current per feeder conductor $I_1 = P/220$
Area of conductor required $A_1 = I_1/\sigma = P/220\,\sigma$
Volume of Cu required for both conductors is

$$V_1 = 2A_1 l = \frac{2Pl}{220\sigma} = \frac{Pl}{110\sigma}$$

***(ii)* 440 V Supply**

$$V_2 = \frac{Pl}{220\sigma}$$

$$\text{\%age saving in Cu} = \frac{V_1 - V_2}{V_1} \times 100 = \frac{\dfrac{Pl}{110\,\sigma} - \dfrac{Pl}{220\,\sigma}}{Pl/110\,\sigma} \times 100 = \mathbf{50\%}$$

40.4. Methods of Feeding a Distributor

Different methods of feeding a distributor are given below :

1. feeding at one end
2. feeding at both ends with equal voltages
3. feeding at both ends with unequal voltages
4. feeding at some intermediate point

In adding, the nature of loading also varies such as

(*a*) concentrated loading (*b*) uniform loading (*c*) combination of (*a*) and (*b*).

Now, we will discuss some of the important cases separately.

40.5. D.C. Distributor Fed at One End

In Fig. 40.7 is shown one conductor AB of a distributor with concentrated loads and fed at one end.

Let i_1, i_2, i_3 etc. be the currents tapped off at points C, D, E and F and I_1, I_2, I_3 etc. the currents in the various sections of the distributor. Let r_1, r_2, r_3 etc. be the ohmic resistances of these various sections and ***R_1, R_2, R_3 etc. the total resistance from the feeding end A to the successive tapping points.***

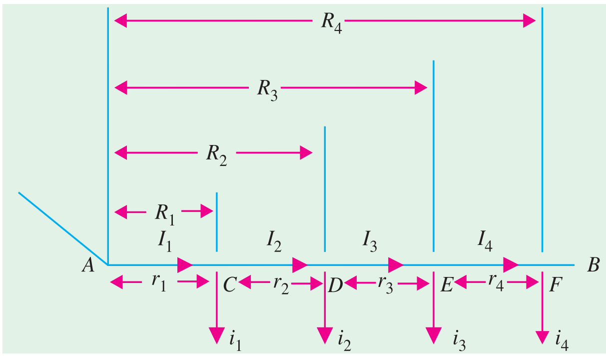


Fig. 40.7

Then, total drop in the distributor is

$$= r_1 I_1 + r_2 I_2 + r_3 I_3 + \ldots\ldots\ldots.$$

Now $\quad I_1 = i_1 + i_2 + i_3 + i_4 + \ldots\ldots;\ I_2 = i_2 + i_3 + i_4 + \ldots\ldots.;\ I_3 = i_3 + i_4 + \ldots\ldots$

$\therefore \quad = r_1(i_1 + i_2 + i_3 + \ldots\ldots.) + r_2(i_2 + i_3 + i_4 + \ldots\ldots..) + r_3(i_3 + i_4 + \ldots\ldots)$

$= i_1 r_1 + i_2(r_1 + r_2) + i_3(r_1 + r_2 + r_3) + \ldots\ldots = i_1 R_1 + i_2 R_2 + i_3 R_3 + \ldots\ldots.$

$=$ ***sum of the moments of each load current about feeding point A.***

1. Hence, the drop at ***the far end*** of a distributor fed at one end is given by the sum of the moments of various tapped currents about the feeding point *i.e.* $v = \Sigma\, i\, R$.

2. It follows from this that the total voltage drop is the same as that produced by a single load equal to the sum of the various concentrated loads, acting at the centre of gravity of the load system.

3. Let us find the drop at any intermediate point like E. The value of this drop is

$$= i_1 R_1 + i_2 R_2 + i_3 R_3 + R_3(i_4 + i_5 + i_6 + \ldots..)^*$$

* The reader is advised to derive this expression himself.

$$= \begin{pmatrix} \text{sum of moments} \\ \text{upto } E \end{pmatrix} + \begin{pmatrix} \text{moment of load beyond} \\ E \text{ assumed acting at } E \end{pmatrix}$$

In general, the drop at any intermediate point is ***equal to the sum of moments of various tapped currents upto that point plus the moment of all the load currents beyond that point assumed to be acting at that point.***

The total drop over both conductors would, obviously, be twice the value calculated above.

40.6. Uniformly Loaded Distributor

In Fig. 40.8 is shown one conductor AB of a distributor fed at one end A and uniformly loaded with i amperes per unit length. Any convenient unit of length may be chosen *i.e.* a metre or 10 metres but at every such unit length, the load tapped is the same. Hence, let

i = current tapped off per unit length $\qquad$ l = total length of the distributor

r = resistance per unit length of the distributor

Now, let us find the voltage drop at a point C (Fig. 40.9) which is at a distance of x units from feeding end A. The current at point C is $(il - ix) = i\,(l - x)$.

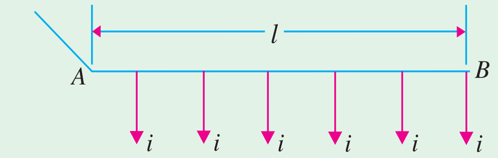


Fig. 40.8

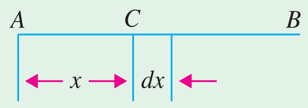


Fig. 40.9

Consider a small section of length dx near point C. Its resistance is (rdx). Hence, drop over length dx is

$$dv = i\,(l - x)\,(rdx) = (ilr - ixr)dx$$

The total drop up to point x is given by integrating the above quantity between proper limits.

$$\therefore \qquad \int_0^{v} dx = \int_0^{x} (ilx - ixr)\,dx \qquad \therefore \quad v = ilrx - \frac{1}{2}irx^2 = ir\left(lx - \frac{x^2}{2}\right)$$

The drop at point B can be obtained by putting $x = l$ in the above expression.

$$\therefore \qquad \text{drop at point } B = ir\left(l^2 - \frac{l^2}{2}\right) = \frac{irl^2}{2} = \frac{(i \times l) \times (r \times l)}{2} = \frac{1}{2}IR = I \times \frac{1}{2}R$$

where $i \times l = I$ – total current entering at point A; $r \times l = R$–the total resistance of distributor AB.

Total drop in the distributor $AB = \dfrac{1}{2}IR$

(*i*) It follows that in a uniformly loaded distributor, total drop is equal to that produced by the whole of the load assumed concentrated at the middle point.

(*ii*) Suppose that such a distributor is fed at both ends A and B with ***equal voltages***. In that case, the point of minimum potential is obviously the middle point. We can thus imagine as if the distributor were cut into two at the middle point, giving us two uniformly-loaded distributors each fed at one end with equal voltages. The resistance of each is $R/2$ and total current fed into each distributor is $I/2$. Hence, drop at the middle point is

$$= \frac{1}{2} \times \left(\frac{I}{2}\right) \times \left(\frac{R}{2}\right) = \frac{1}{8}IR$$

It is 1/4th of that of a distributor fed at one end only. The advantage of feeding a distributor at both ends, instead of at one end, is obvious.

The equation of drop at point C distant x units from feeding point $A = irlx - \dfrac{1}{2}irx^2$ shows that the diagram of drop of a uniformly-loaded distributor fed at one end is a parabola.

Example 40.2. *A uniform 2-wire d.c. distributor 200 metres long is loaded with 2 amperes/metre. Resistance of single wire is 0.3 ohm/kilometre. Calculate the maximum voltage drop if the distributor is fed (a) from one end (b) from both ends with equal voltages.*

(Elect. Technology ; Bombay Univ.)

Solution. **(*a*)** Total resistance of the distributor is $R = 0.3 \times (200/1000) = 0.06$ W

Total current entering the distributor is $I = 2 \times 200 = 400$ A

Total drop on the whole length of the distributor is (Art. 40.6)

$$= \frac{1}{2} IR = \frac{1}{2} \times 400 \times 0.06 = \mathbf{12\ V}$$

(*b*) Since distributor is fed from both ends, total voltage drop is

$$= \frac{1}{8} IR = \frac{1}{8} \times 400 \times 0.06 = \mathbf{3\ V}$$

As explained in Art. 40.6 the voltage drop in this case is one-fourth of that in case **(*a*)** above.

Example 40.3. *A 2-wire d.c. distributor AB is 300 metres long. It is fed at point A. The various loads and their positions are given below :*

At point	*distance from A in metres*	*concentrated load in A*
C	*40*	*30*
D	*100*	*40*
E	*150*	*100*
F	*250*	*50*

If the maximum permissible voltage drop is not to exceed 10 V, find the cross-sectional area of the distributor. Take $\rho = (1.78 \times 10^{-8})\ \Omega$-m. **(Electrical Power-I ; Bombay Univ.)**

Solution. The distributor along with its tapped currents is shown in Fig. 40.10.

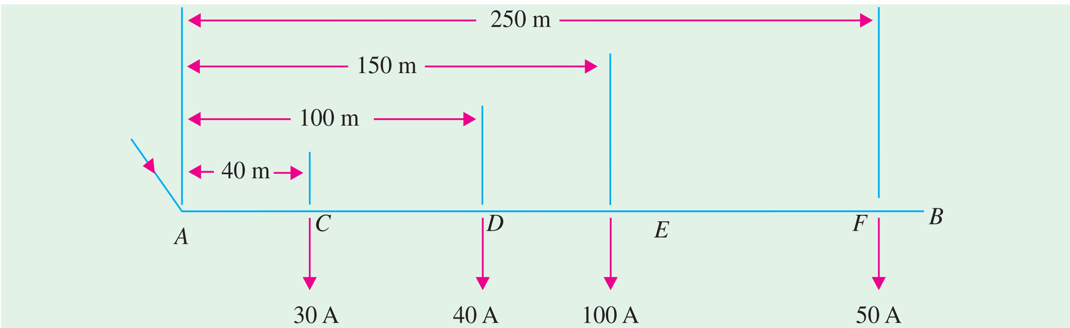


Fig. 40.10

Let 'A' be cross sectional area of distributor

As proved in Art. 40.5 total drop over the distributor is

$$v = i_1R_1 + i_2R_2 + i_3R_3 + i_4R_4 \qquad \text{—for one conductor}$$

$$= 2(i_1R_1 + i_2R_2 + i_3R_3 + i_4R_4) \qquad \text{—for two conductors}$$

where R_1 = resistance of $AC = 1.78 \times 10^{-8} \times 40/A$

R_2 = resistance of $AD = 1.78 \times 10^{-8} \times 100/A$

R_3 = resistance of $AE = 1.78 \times 10^{-8} \times 150/A$

R_4 = resistance of $AF = 1.78 \times 10^{-8} \times 250/A$

Since v = 10 V, we get

$$10 = \frac{2 \times 1.78 \times 10^{-8}}{A}(30 \times 40 + 40 \times 100 + 100 \times 150 + 50 \times 250)$$

$$= \frac{2 \times 1.78 \times 10^{-8}}{A} \times 32,700$$

$$\therefore \quad A = 3.56 \times 327 \times 10^{-7} \text{ m}^2 = \mathbf{1,163\ cm^2}$$

40.7. Distributor Fed at Both Ends with Equal Voltages

It should be noted that in such cases

(i) the maximum voltage drop must always occur at one of the load points and

(ii) if both feeding ends are at the same potential, then the voltage drop between each end and this point must be the same, which in other words, means that the sum of the moments about ends must be equal.

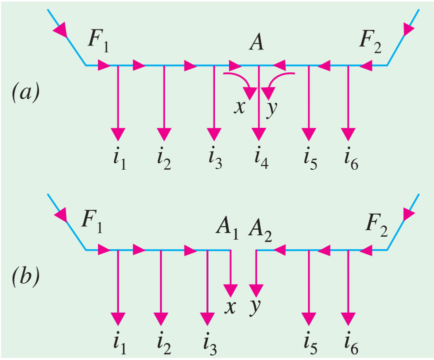


Fig. 40.11

In Fig. 40.11 (*a*) is shown a distributor fed at two points F_1 and F_2 with equal voltages. The potential of the conductor will gradually fall from F_1 onwards, reach a minimum value at one of the tapping, say, A and then rise again as the other feeding point F_2 is approached. All the currents tapped off between points F_1 and A will be supplied from F_1 while those tapped off between F_2 and A will be supplied from F_2. The current tapped at point A itself will, in general, be partly supplied by F_1 and partly by F_2. Let the values of these currents be x and y respectively. If the distributor were actually cut off into two at A—the point of minimum voltage, with x amperes tapped off from the left and y amperes tapped off from the right, then potential distribution would remain unchanged, showing that we can regard the distributor as consisting of two separate distributors each fed from one end only, as shown in Fig. 40.11 (*b*). The drop can be calculated by locating point A and then values of x and y can be calculated.

This can be done with the help of the following pair of equations :

$$x + y = i_4 \quad \text{and} \quad \text{drop from } F_1 \text{ to } A_1 = \text{drop from } F_2 \text{ to } A_2.$$

Example 40.4. *A 2-wire d.c. distributor $F_1 F_2$ 1000 metres long is loaded as under :*

Distance from F_1 (in metres) :	*100*	*250*	*500*	*600*	*700*	*800*	*850*	*920*
Load in amperes :	*20*	*80*	*50*	*70*	*40*	*30*	*10*	*15*

The feeding points F_1 and F_2 are maintained at the same potential. Find which point will have the minimum potential and what will be the drop at this point ? Take the cross-section of the conductors as 0.35 cm^2 and specific resistance of copper as (1.764×10^{-6}) Ω-cm.

Solution. The distributor along with its tapped currents is shown in Fig. 40.12. The numbers along the distributor indicate length in metres.

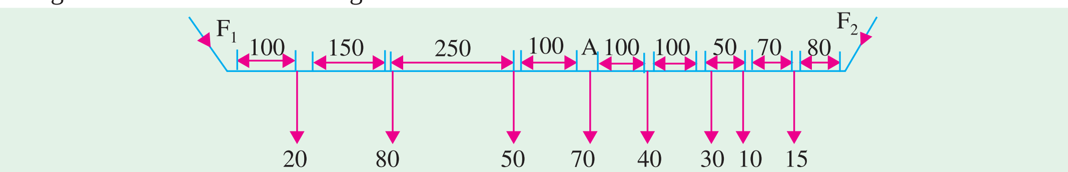


Fig. 40.12

The easiest method of locating the point of minimum potential is to take the moments about the two ends and then by comparing the two sums make a guess at the possible point. The way it is done is as follows :

It is found that 4th point from F_1 is the required point *i.e.* point of minimum potential. Using the previous two equations, we have

$$x + y = 70 \quad \text{and} \quad 47{,}000 + 600x = 20{,}700 + 400\,y$$

Solving the two equations, we get $x = 1.7$ A, $y = 70 - 1.7 = 68.3$ A

Drop at A per conductor = 47,000 + 600 × 1.7 = 48,020 ampere-metre.

$$\text{Resistance/metre} = \frac{\rho l}{A} = \frac{1.764 \times 10^{-8} \times 1}{0.35 \times 10^{-4}} = 50.4 \times 10^{-5}\ \Omega/\text{m}$$

$$\text{Hence, drop per conductor} = 48{,}020 \times 50.4 \times 10^{-5} = 24.2\ \text{V}$$

Reckoning both conductors, the drop at A is = **48.4 V**

Moments about F_1 in ampere-metres	*Sum*	*Moments about F_2 in ampere-metres*	*Sum*
20 × 100 = 2,000	2,000	15 × 80 = 1,200	1,200
80 × 250 = 20,000	22,000	10 × 150 = 1,500	2,700
50 × 500 = 25,000	47,000	30 × 200 = 6,000	8,700
		40 × 300 = 12,000	20,700
		70 × 400 = 28,000	48,700

Alternative Solution

The alternative method is to take the total current fed at one end, say, F_1 as x and then to find the current distribution as shown in Fig. 40.13 (*a*). The drop over the whole distributor is equal to the sum of the products of currents in the various sections and their resistances. For a distributor fed at both ends with ***equal voltages***, this drop equals zero. In this way, value of x can be found.

Resistance per metre single = $5.04 \times 10^{-4}\ \Omega$

Resistance per metre double = $10.08 \times 10^{-4}\ \Omega$

$\therefore \quad 10.08 \times 10^{-4}\,[100x + 150(x - 20) + 250\,(x - 100) + 100\,(x - 150) + 100\,(x - 200) + 100(x - 260) + 50(x - 290) + 70(x - 300) + 80\,(x - 315)] = 0 \quad \text{or} \quad 1000x = 151{,}700$

$\therefore \quad x = 151.7$ A

This gives a current distribution as shown in Fig. 40.13 (*b*). Obviously, point A is the point of minimum potential.

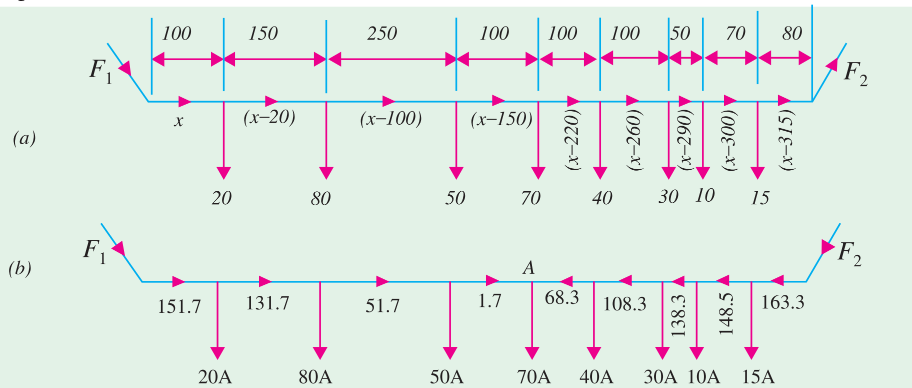


Fig. 40.13

Drop at A (considering both conductors) is

$$= 10.08 \times 10^{-4}\,(100 \times 151.7 + 150 \times 131.7 + 250 \times 51.7 + 100 \times 1.7)$$

$$= 10.08 \times 10^{-4} \times 48{,}020 = \mathbf{48.4\ V}$$ **—as before.**

Example 40.5. *The resistance of a cable is 0.1 Ω per 1000 metre for both conductors. It is loaded as shown in Fig. 40.14 (a). Find the current supplied at A and at B. If both the ends are supplied at 200 V.* **(Electrical Technology-II, Gwalior Univ.)**

Solution. Let the current distribution be as shown in Fig. 40.14 (*b*). Resistance for both conductors = 0.1/1000=10^{-4} Ω/m. The total drop over the whole cable is zero because it is fed at both ends by equal voltages.

$\therefore$ 10^{-4} [500 *i* + 700 (*i* – 50) + 300 (*i* – 150) + 250 (*i* – 300)] = 0

i = **88.6 A**

This gives the current distribution as shown in Fig. 40.14 (*c*).

Current in *AC* = 88.6 A

Current in *CD* = (88.6 – 50) = 38.6 A

Current in *DE* = (38.6 – 100) = – 61.4 A

Current in *EB* = (–61.4 – 150) = –211.4 A

Hence, current entering the cable at point *B* is = **211.4 A.**

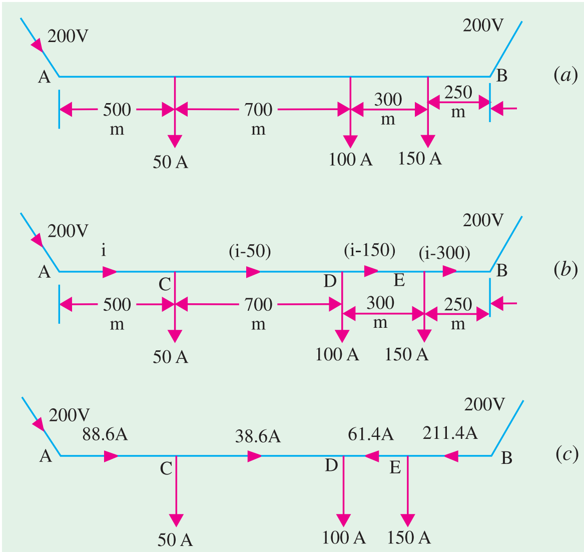


Fig. 40.14

Example 40.6. *The resistance of two conductors of a 2-conductor distributor shown in Fig. 39.15 is 0.1 Ω per 1000 m for both conductors. Find* ***(a)*** *the current supplied at A* ***(b)*** *the current supplied at B* ***(c)*** *the current in each section* ***(d)*** *the voltages at C, D and E. Both A and B are maintained at 200 V.* **(Electrical Engg. Grad. I.E.T.E.)**

Solution. The distributor along with its tapped currents is shown in Fig. 40.15. Let the current distribution be as shown. Resistance per metre double is = 0.1/1000 = 10^{-4} Ω.

The total drop over the whole distributor is zero because it is fed at both ends by equal voltages.

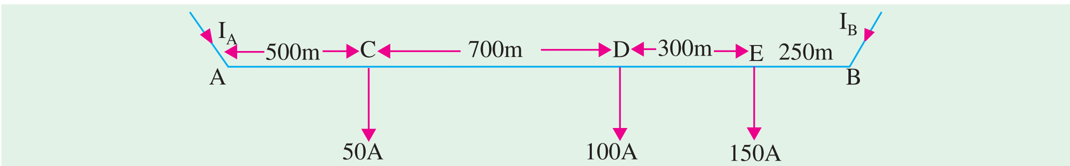


Fig. 40.15

$\therefore$ 10^{-4} [500*i* + 700 (*i* – 50) + 300 (*i* – 150) + 250 (*i* – 300)] = 0

or 1750*i* = 155,000 or *i* = 88.6 A

This gives the current distribution shown in Fig. 40.16.

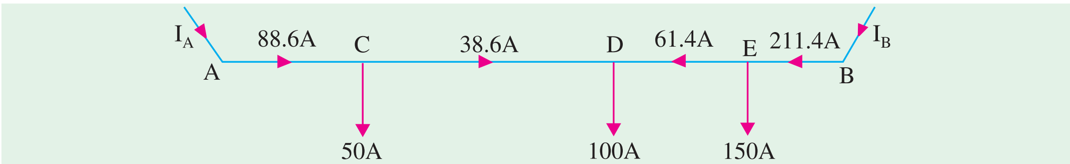


Fig. 40.16

(a) I_A = **88.6 A** ***(b)*** I_B = **–211.4 A**

(c) Current in section *AC* = **88.6 A**; Current in *CD* = (88.6 – 50) = **38.6 A**

Current in section *DE* = (31.6 – 100) = **– 61.4 A**

Current in *EB* = (– 61.4 – 150) = **– 211.4 A**

(d) Drop over AC = 10^{-4} × 500 × 88.6 = 4.43 V; Drop over *CD* = 10^{-4} × 700 × 38.6 = **2.7 V**

Drop over $DE = 10^{-4} \times 300 \times -61.4 = -1.84$ V $\therefore$ Voltage at $C = 200 - 4.43 =$ **195.57 V**

Voltage at $D = (195.57 - 2.7) =$ **192.87 V**, Voltage at $E = 192.87 - (-1.84) =$ **194.71 V**

Example 40.7. *A 200 m long distributor is fed from both ends A and B at the same voltage of 250 V. The concentrated loads of 50, 40, 30 and 25 A are coming on the distributor at distances of 50, 75, 100 and 150 m respectively from end A. Determine the minimum potential and locate its position. Also, determine the current in each section of the distributor. It is given that the resistance of the distributor is 0.08 Ω per 100 metres for go and return.*

(Electric Power-I (Trans & Dist) Punjab Univ. 1993)

Solution. As shown in Fig. 40.17, let current fed at point A be i. The currents in various sections are as shown. Resistance per metre of the distributor (go and return) is $0.08/100 = 0.0008\ \Omega$.

Since the distributor is fed at both ends with equal voltages, total drop over it is zero.

Hence, $\quad 0.0008\,[50i + 25(i - 50) + 25(i - 90) + 50(i - 120) + 50(i - 145)] = 0$

or $\quad 200i = 16{,}750 \quad$ or $\quad i = 83.75$ A

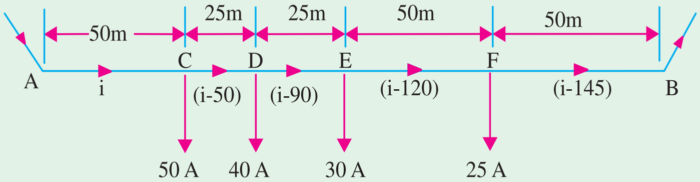


Fig. 40.17

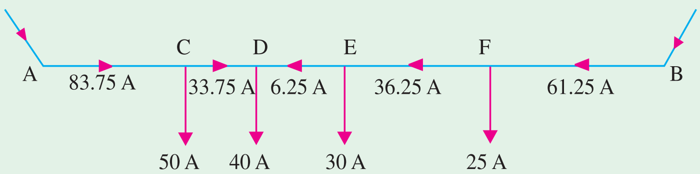


Fig 40.18

The actual current distribution is shown in Fig. 40.18. Obviously, point D is the point of minimum potential.

Drop at point D is $\quad = 0.0008(50 \times 83.75 + 25 \times 33.75) = 4.025$ V

$\therefore$ minimum potential $\quad = 250 - 4.025 =$ **245.98 V**

40.8. Distributor Fed at Both Ends with Unequal Voltages

This case can be dealt with either by taking moments (in amp-m) about the two ends and then making a guess about the point of minimum potential or by assuming a current x fed at one end and then finding the actual current distribution.

Consider the case already discussed in Ex. 40.4. Suppose, there is a difference of ν volts between ends F_1 and F_2 with F_1 being at higher potential. Convert ν volts into ampere-metres with the help of known value of resistance/metre. Since F_2 is at a lower potential, these ampere-metres appear in the coloumn for F_2 as initial drop.

If, for example, ν is 4 volts, then since resistance/metre is $5.04 \times 10^{-4}\ \Omega$, initial ampere-metres for F_2 are

$$= \frac{4 \times 10^4}{5.04} = 7{,}938 \text{ amp-metres}$$

The table of respective moments will be as follows :

Moments about F_1	Sum	Moments about F_2	Sum
$20 \times 100 = 2000$	2,000	initial = 7,938	
$80 \times 250 = 20{,}000$	22,000	$15 \times 80 = 1{,}200$	9,138
$50 \times 500 = 25{,}000$	47,000	$10 \times 150 = 1{,}500$	10,638
		$30 \times 200 = 6{,}000$	16,638
		$40 \times 300 = 12{,}000$	28,638
		$70 \times 400 = 28{,}000$	56,638

As seen, the dividing point is the same as before.

$\therefore \quad x + y = 70 \quad$ and $\quad 47{,}000 + 600x = 28{,}638 + 400y$

Solving for x and y, we get $\quad x = 9.64$ A $\quad$ and $\quad y = 60.36$ A

After knowing the value of x, the drop at A can be calculated as before.

The alternative method of solution is illustrated in Ex. 40.12.

40.9. Uniform Loading with Distributor Fed at Both Ends

Consider a distributor PQ of length l units of length, having resistance per unit length of r ohms and with loading per unit length of i amperes. Let the difference in potentials of the two feeding points be ν volts with Q at the lower potential. The procedure for finding the point of minimum potential is as follows:

Let us assume that point of minimum potential M is situated at a distance of x units from P. Then drop from P to M is $= irx^2/2$ volts (Art. 40.6)

Since the distance of M from Q is $(l - x)$, the drop from Q to

M is $= \dfrac{ir\,(l-x)^2}{2}$ volt.

Since potential of P is greater than that of Q by ν volts,

$$\therefore \quad \frac{irx^2}{2} = \frac{ir\,(l-x)^2}{2} + \nu \quad \text{or} \quad x = \frac{l}{2} + \frac{\nu}{irl}$$

A circuit-board as shown above uses DC current

40.10. Concentrated and Uniform Loading with Distributor Fed at One End

Such cases are solved in two stages. First, the drop at any point due to concentrated loading is found. To this add the voltage drop due to uniform loading as calculated from the relation.

$$ir\left(lx - \frac{x^2}{2}\right)$$

As an illustration of this method, please look up Ex. 40.9 and 40.13.

Example 40.8. *Each conductor of a 2-core distributor, 500 metres long has a cross-sectional area of 2 cm^2. The feeding point A is supplied at 255 V and the feeding point B at 250 V and load currents of 120 A and 160 A are taken at points C and D which are 150 metres and 350 metres respectively from the feeding point A. Calculate the voltage at each load. Specific resistance of copper is 1.7 × 10^{-6} Ω-cm.* **(Elect. Technology-I, Bombay Univ.)**

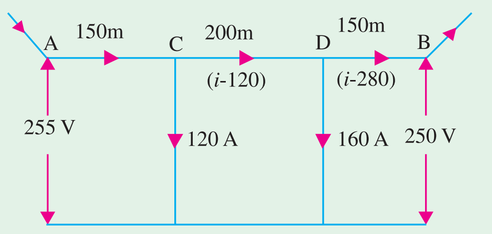


Fig. 40.19

Solution. $\rho = 1.7 \times 10^{-6}\ \Omega\text{-cm}$

$= 1.7 \times 10^{-8}\ \Omega.\text{m}$

Resistance per metre single $= \rho \dfrac{l}{A} = 1.7 \times 10^{-8}/2 \times 10^{-4} = 85 \times 10^{-6}$ m

Resistance per metre double $= 2 \times 85 \times 10^{-6} = 17 \times 10^{-5}\ \Omega$

The dorp over the whole distributor = 255 – 250 = 5 V

$\therefore\quad 17 \times 10^{-5}\,[150i + 200\,(i - 120) + 150\,(i - 280)] = 5$

$\therefore\quad i = 190.8$ A — **(Fig. 40.19)**

Current in section AC = 190.8 A; Current in section CD = (190.8 – 120) = 70.8 A

Current in section DB = (190.8 – 280) = – 89.2 A (from B to D)

Voltage at point C = 255 – drop over AC

$= 255 - (17 \times 10^{-5} \times 150 \times 190.8) =$ **250.13 V**

Voltage at point D = 250 – drop over BD

$= 250 - (17 \times 10^{-5} \times 150 \times 89.2) =$ **247.72 V**

Example 40.9. *A 2-wire distributor 500 metres long is fed at P at 250 V and loads of 40A, 20A, 60A, 30A are tapped off from points A, B, C and D which are at distances of 100 metres, 150 metres, 300 metres and 400 metres from P respectively. The distributor is also uniformly loaded at the rate of 0.1 A/m. If the resistance of the distributor per metre (go and return) is 0.0005 Ω, calculate the voltage at* ***(i)*** *point Q and* ***(ii)*** *point B.*

Solution. First, consider drop due to concentrated load only.

Drop in $PA = 150 \times (100 \times 0.0005) = 7.5$ V

Drop in $AB = 110 \times (50 \times 0.0005) = 2.75$ V

Drop in $BC = 90 \times (150 \times 0.0005) = 6.75$ V

Drop in $CD = 30 \times (100 \times 0.0005) = 1.5$ V

$\therefore$ total drop due to this load = 18.5 V

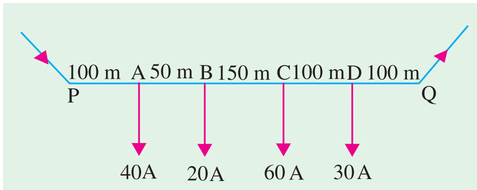


Fig. 40.20

Now, let us consider drop due to uniform load only.

Drop over length $l = ir\dfrac{l^2}{2} = 0.1 \times 0.0005 \times 500^2/2 = 6.25$ V

(i) $\therefore$ potential of point Q = 250 – (18.5 + 6.25) = **225.25 V**

(ii) Consider point B at the distance of 150 metres from P.

Drop due to concentrated loading = 7.5 + 2.75 = 10.25 V.

Drop due to uniform loading = $ir\,(lx - x^2/2)$; Here l = 500 m ; x = 150 m

$$\therefore \quad \text{drop} = 0.1 \times 0.0005\left(500 \times 150 - \frac{150^2}{2}\right) = 3.1875 \text{ V}$$

$\therefore$ potential of point P = 250 – (10.25 + 3.1875) = **13.44 V**

Example 40.10. *A distributor AB is fed from both ends. At feeding point A, the voltage is maintained at 236 V and at B at 237 V. The total length of the distributor is 200 metres and loads are tapped off as under :*

(i) *20 A at 50 metres from A* ***(ii)*** *40 A at 75 metres from A*

(iii) *25 A at 100 metres from A* ***(iv)*** *30 A at 150 metres from A*

The resistance per kilometre of one conductor is 0.4Ω. Calculate the currents in the various sections of the distributor, the minimum voltage and the point at which it occurs.

(Electrical Technology, Calcutta Univ.)

Solution. The distributor along with currents in its various sections is shown in Fig. 40.21.

Resistance/metre single $= 0.4/1000 = 4 \times 10^{-4}\ \Omega$

Resistance/metre double $= 8 \times 10^{-4}\ \Omega$

Voltage drop on both conductors of 200-metre long distributor is

$$= 8 \times 10^{-4} [50i + 26 (i - 20) + 25(i - 60) + 50(i - 85) + 50(i - 115)]$$
$$= 8 \times 10^{-4} (200i - 12{,}000) \text{ volt}$$

This drop must be equal to the potential difference between A and B.

$\therefore\quad 8 \times 10^{-4} (200i - 12{,}000) = 236 - 237 = -1$

$\therefore\quad i = 53.75$ A

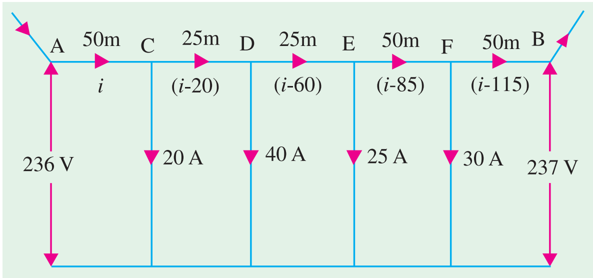


Fig. 40.21

Current in section AC = **53.75 A**

Current in section CD = 53.75 – 20 = **33.75 A**

Current in section DE = 53.75 – 60 = **– 6.25 A**

Current in section EF = 53.75 – 85 = **– 31.25 A**

Current in section FB = 53.75 – 115 = **– 61.25 A**

The actual current distribution is as shown in Fig. 40.22.

Obviously, minimum voltage occurs at point D *i.e.* 75 ***metre from point*** A (or 125 m from B)

Voltage drop across both conductors of the distributor over the length AD is

$$= 8 \times 10^{-4} (50 \times 53.75 + 25 \times 33.75) = 2.82 \text{ V}$$

$\therefore$ potential of point

$D = 236 - 2.82$
$= \mathbf{233.18\ V}$

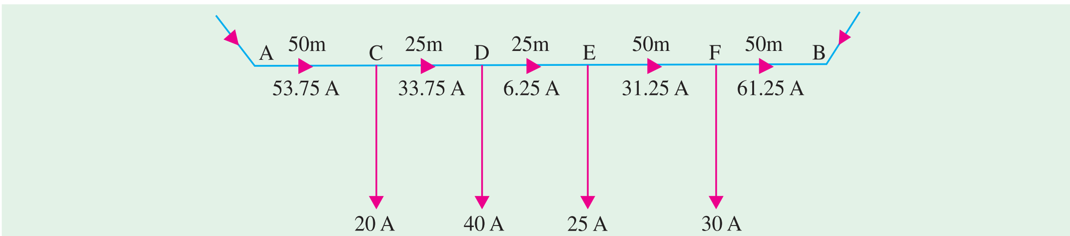


Fig. 40.22

Example 40.11. *A distributor cable AB is fed at its ends A and B. Loads of 12, 24, 72 and 48 A are taken from the cable at points C, D, E and F. The resistances of sections AC, CD, DE, EF and FB of the cable are 8, 6, 4, 10 and 5 milliohm respectively (for the go and return conductors together).*

The p.d. at point A is 240 V, the p.d. at the load F is also to be 240 V. Calculate the voltage at the feeding point B, the current supplied by each feeder and the p.d.s. at the loads C,D and E.

(Electrical Technology ; Utkal Univ.)

Solution. Let the current fed at the feeding point A be i. The current distribution in various sections becomes as shown in Fig. 40.23.

Voltage drop on both sides of the distributor over the section AF is

$$= [8i + 6(i - 12) + 4(i - 36) + 10(i - 108)] \times 10^{-3} = (28i - 1296) \times 10^{-3} \text{ V}$$

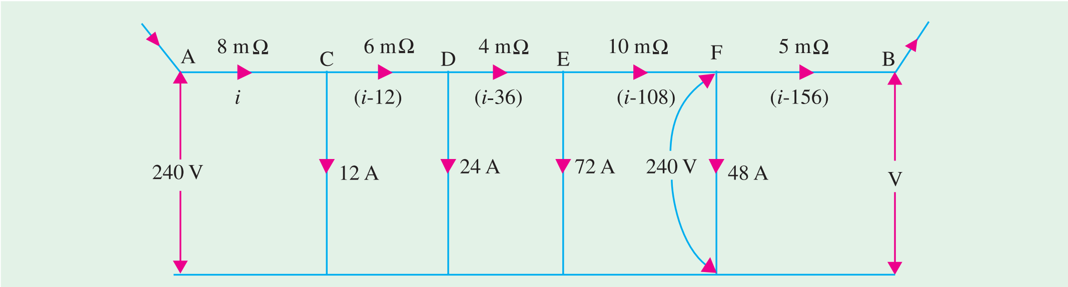


Fig. 40.23

Since points A and F are the same potential, the p.d. between A and F is zero.

$\therefore \qquad (28i - 1296) \times 10^{-3} = 0 \qquad \text{or} \qquad i = 46.29 \text{ A}$

Current in section AC = 46.29 A

Current in section CD = 46.29 – 12 = 34.29 A

Current in section DE = 46.29 – 36 = 10.29 A

Current in section EF = 46.29 – 108 = – 61.71 A

Current in section FB = 46.29 – 156 = –109.71 A

The actual current distribution is as shown in Fig. 40.24.

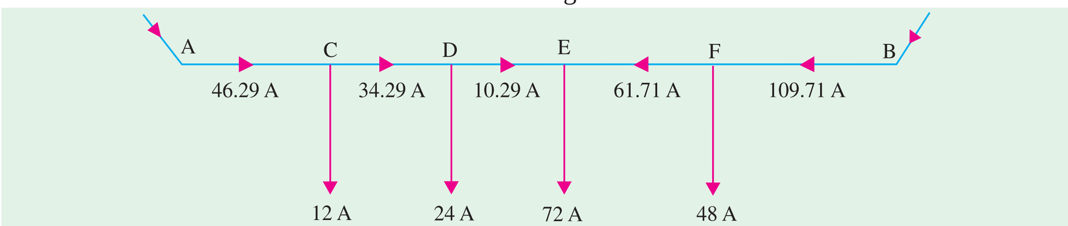


Fig. 40.24

Current applied by feeder at point A is 46.29 A and that supplied at point B is 109.71 A.

Voltage at feeding point B = 240 – drop over FB = $240 - (-5 \times 10^{-3} \times 109.71)$ = **240.55 V.**

Voltage at point C = 240 – drop over AC = $240 - (8 \times 10^{-3} \times 46.29)$ = **239.63 V**

Voltage at point D = 239.63 – drop over CD = $239.63 - (6 \times 10^{-3} \times 34.29)$ = **239.42 V**

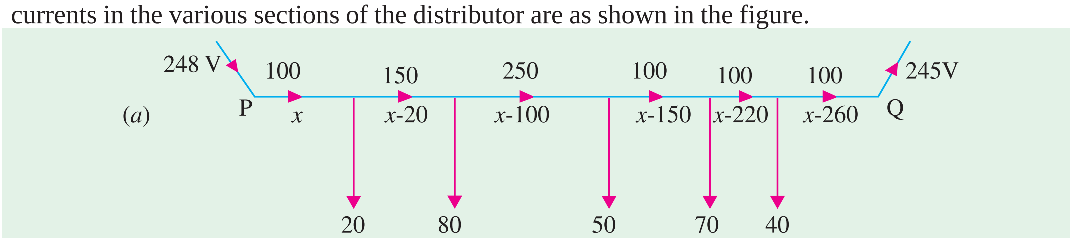
Voltage at point E = 239.42 – drop over DE = $239.42 - (4 \times 10^{-3} \times 10.29)$ = **239.38 V**

Example 40.12. *A two-wire, d.c. distributor PQ, 800 metre long is loaded as under :*

Distance from P (metres) :	*100*	*250*	*500*	*600*	*700*
Loads in amperes :	*20*	*80*	*50*	*70*	*40*

The feeding point at P is maintained at 248 V and that at Q at 245 V. The total resistance of the distributor (lead and return) is 0.1 Ω. Find ***(a)*** *the current supplied at P and Q and* ***(b)*** *the power dissipated in the distributor.*

Solution. As shown in Fig. 40.25 (a), let x be the current supplied from end P. The other currents in the various sections of the distributor are as shown in the figure.



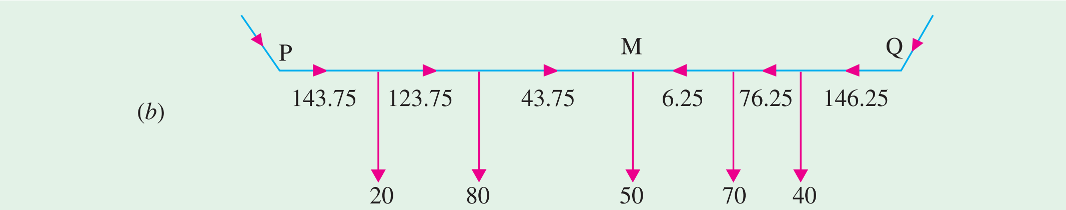


Fig. 40.25

Now, the total drop over *PQ* should equal the potential difference between ends *P* and *Q i.e.*, = 248 – 245 = 3.

Resistance/metre of both conductors = 0.1/800 = 1/8000 Ω

$$\therefore \quad \frac{1}{8000}[100x + 150(x-20) + 250(x-100) + 100(x-150) + 100(x-220) + 100(x-246)] = 3$$

or $\quad 800x = 115{,}000 \qquad \therefore \quad x = 143.75$ A

The actual distribution is shown in Fig. 40.25 (*b*) from where it is seen that point *M* has the minimum potential.

$$I_P = \textbf{143.75 A.} \qquad I_Q = \textbf{116.25 A}$$

$$\text{Power loss} = \Sigma I^2R = \frac{1}{8000}[143.75^2 \times 100 + 123.75^2 \times 150 + 43.75^2 \times 250 + 6.25^2 \times 100 + 76.25^2 \times 100 + 116.25^2 \times 100] = \textbf{847.3 W}$$

Example 40.13. *The two conductors of a d.c. distributor cable 1000 m long have a total resistance of 0.1 Ω. The ends A and B are fed at 240 V. The cable is uniformly loaded at 0.5 A per metre length and has concentrated loads of 120 A, 60 A, 100 A and 40 A at points distant 200, 400, 700 and 900 m respectively from the end A. Calculate* ***(i)*** *the point of minimum potential on the distributor* ***(ii)*** *the value of minimum potential and* ***(iii)*** *currents fed at the ends A and B.*

(Power System-I, AMIE, Sec. B, 1993)

Solution. Concentrated loads are shown in Fig. 40.26. Let us find out the point of minimum potential and currents at points *A* and *B*.

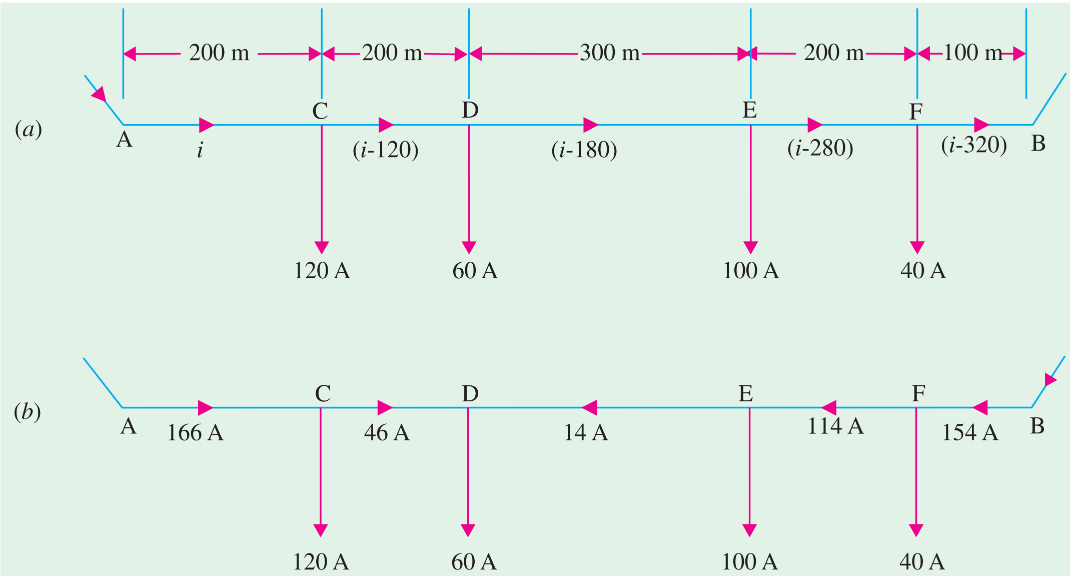


Fig. 40.26

It should be noted that location of point of minimum potential is not affected by the uniformly-spread load of 0.5 A/m. In fact, let us, for the time being, imagine that it is just not there. Then, assuming i to be the input current at A, the different currents in various sections are as shown. Since points A and B are fed at equal voltages, total drop over the distributor is zero. Distributor resistance per metre length (go and return) is $0.1/1000 = 10^{-4}\ \Omega$.

$\therefore \quad 10^{-4}\,[200i + 200(i-120) + 300(i-180) + 200(i-280) + 100(i-320)] = 0$

or $\quad 1000i = 166{,}000 \quad \therefore \quad i = 166$ A

Actual current distribution is shown in Fig. 40.26 (b) from where it is seen that point D is the point of minimum potential. The uniform load of 0.5 A/m upto point D will be supplied from A and the rest from point B.

Uniform load from A to $D = 400 \times 0.5 = 200$ A. Hence, $I_A = 166 + 200 =$ **366 A.**

Similarly, $\quad I_B = 154 + (600 \times 0.5) =$ **454 A.**

Drop at D due to concentrated load is $= 10^{-4}\,(166 \times 200 + 46 \times 200) =$ **4.24 V.**

Drop due to uniform load can be found by imagining that the distributor is cut into two at point D so that AD can be looked upon as a distributor fed at one end and loaded uniformly. In that case, D becomes the other end of the distributor.

$\therefore$ drop at D due to uniform load (Art. 40.6)

$$= irl^2/2 = 0.5 \times 10^{-4} \times 400^2/2 = 4 \text{ V}$$

$\therefore$ total drop at $D = 4.24 + 4 = 8.24$ V $\quad \therefore$ potential of $D = 240 - 8.24 =$ **231.76 V**

Example 40.14. *It is proposed to lay out a d.c. distribution system comprising three sections—the first section consists of a cable from the sub-station to a point distant 800 metres from which two cables are taken, one 350 metres long supplying a load of 22 kW and the other 1.5 kilometre long and supplying a load of 44 kW. Calculate the cross-sectional area of each cable so that the total weight of copper required shall be minimum if the maximum drop of voltage along the cable is not to exceed 5% of the normal voltage of 440 V at the consumer's premises. Take specific resistance of copper at working temperature equal to 2 μ Ω-cm.*

Solution. Current taken from 350-m section is $I_1 = 22{,}000/440 = 50$ A

Current taken from 1.5 km section, $I_2 = 44{,}000/440 = 100$ A

$\therefore$ Current in first section $\quad I = 100 + 50 = 150$ A

Let $\quad V =$ voltage drop across first section ; R = resistance of the first section

$\quad A =$ cross-sectional area of the first section

Then $\quad R = V/I = V/150\ \Omega$

Now, $\quad A = \dfrac{\rho l}{R} = \rho l \dfrac{I}{V} = 80{,}000 \times 2 \times 10^{-6} \times 150/\text{V} = 24/\text{V}\text{cm}^2$

Now, $\quad$ maximum allowable drop $= 5\%$ of $440 = 22$ V

$\therefore$ voltage drop along other sections $= (22 - V)$ volt

Hence, cross-sectional area of 350-m section is

$$A_1 = 35{,}000 \times 2 \times 10^{-6} \times 50/(22 - V) = 3.5/(22 - V) \text{ cm}^2$$

Also, cross-sectional area of 1500-m section is

$$A_2 = 150{,}000 \times 2 \times 10^{-6} \times 100/(22 - V) = 30/(22 - V) \text{ cm}^2$$

Now, total weight of copper required is proportional to the total volume.

$$\therefore \quad W = K[(800 \times 24/V) + 350 \times 3.5/(22 - V) + 1500 \times 30/(22 - V)]$$
$$= K[1.92/V + 4.62/(22 - V)] \times 10^4$$

Weight of copper required would be minimum when $dW/dV = 0$

$$\therefore \qquad \frac{dW}{dV} = K\left[\frac{-1.92}{V^2} + \frac{4.62}{(22-V)^2}\right]\times 10^4 = 0$$

$$\text{or} \qquad \frac{1.92}{V^2} = \frac{4.62}{(22-V)^2} \qquad \text{or} \qquad (22-V)^2 = 2.4\,V^2$$

$$\text{or} \qquad V = 22/2.55 = \mathbf{8.63\ volt} \qquad \therefore \qquad A = 24/8.63 = \mathbf{2.781\ cm^2}$$

$$A_1 = 3.5/(22-8.63) = \mathbf{0.2618\ cm^2} \qquad A_2 = 30/(22-8.63) = \mathbf{2.246\ cm^2}$$

Example 40.15. *A d.c. two-wire distributor AB is 450 m long and is fed at both ends at 250 volts. It is loaded as follows : 20A at 60 m from A, 40A at 100m from A and a uniform loading of 1.5 A/m from 200 to 450 m from A. The resistance of each conductor is 0.05 Ω/km. Find the point of minimum potential and its potential.* **(Electrical Power-II, Bangalore Univ. 1993)**

Solution. In Fig. 40.27, let *D* be the point of minimum potential and let *i* be the current in distributor section *CD*. Then, current supplied to load *D* from end *B* is (40 – *i*). If *r* is the resistance of the distributor/metre (both go and return), then

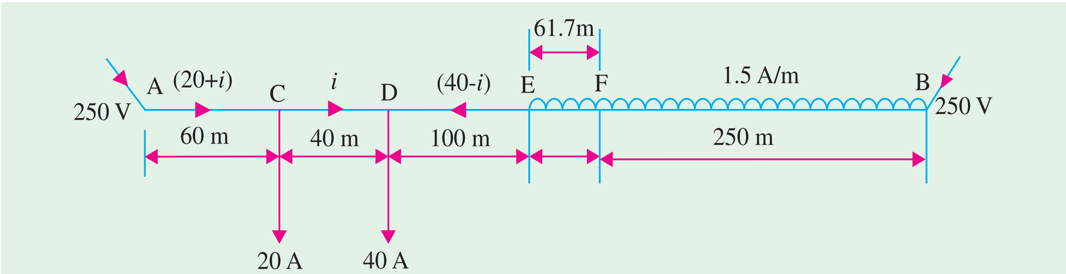


Fig. 40.27

$$\text{Drop over } AD = (20+i)\times 60r + i\times 40r$$

$$\text{Drop over } BD = \text{drop due to concentrated load + drop due to distributed load}$$

$$= (40-i)\times 350r + 1.5\times r\times 250^2/2 \qquad \text{—Art. 40.6}$$

Since the two feeding points *A* and *B* are at the same potential of 250 V, the two drops must be equal.

$$\therefore \qquad (20+i)\times 60r + i\times 40r = (40-i)\times 350r + 1.5\times r\times 250^2/2 \qquad \therefore \qquad i = 132.6\ \text{Å}$$

Since (40 – *i*) comes out to be negative, it means that *D* is not the point of minimum potential. The required point is somewhat nearer the other end *B*. Let it be *F*. Obviously, current in section *DF* = 132.6 – 40 = 92.6 A. Hence, distance of minimum potential point *F* from end *A* is

$$= 60 + 40 + 100 + 92.6/1.5 = \mathbf{261.7\ m}$$

Total voltage drop over section *AF* is

$$= 2\times 0.05\times 10^{-3}\,(152.6\times 60 + 132.6\times 40 + 92.6\times 100 + 92.6 + 61.7/4) = 2.65\ \text{V}$$

$$\therefore \quad \text{potential of point } F = 250 - 2.65 = \mathbf{247.35\ V.}$$

Example 40.16. *A two-wire d.c. distributor AB, 1000 metres long, is supplied from both ends, 240 V at A and 242 V at B. There is a concentrated load of 200 A at a distance of 400 metre from A and a uniformly distributed load of 1.0 A/m between the mid-point and end B. Determine* **(i)** *the currents fed at A and B* **(ii)** *the point of minimum potential and* **(iii)** *voltage at this point. Take cable resistance as 0.005 Ω per 100 metre each core.*

Solution. The resistance per 100 metres of both cores = 0.005 × 2 = 0.01 Ω.

Let us take 100 m as the unit of length. Let current fed at end *B* be I_B as shown in Fig. 40.28.

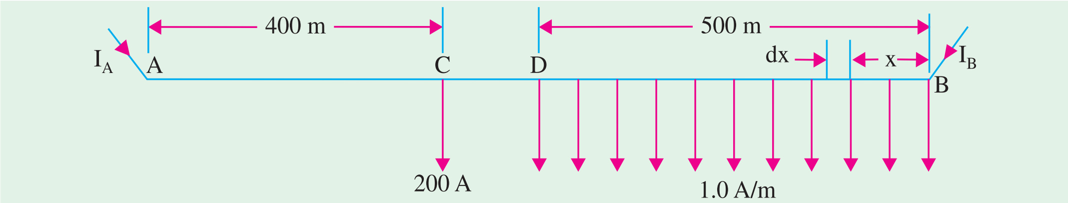


Fig. 40.28

Consider any element dx anywhere within distance BD *i.e.* distance over which uniformly distributed load is applied. Suppose it is at a distance of x units (of 100 metre each). The current through dx is $I = (I_B - 100 \times 1.0 \times x) = (I_B - 100x)$

$\therefore$ voltage drop over $dx = (I_B - 100x) \times 0.01 \times dx$

Voltage drop over distance BD

$$= \int_0^5 (I_B - 100x) \times 0.01 \times dx = 0.01 \int_0^5 (I_B - 100x)\, dx$$

$$= 0.01 \left| (I_B x - 500x^2) \right|_0^5 = (0.05\, IB - 12.5) \text{ V}$$

Voltage drop over $DC = (I_B - 500) \times 0.01$; Voltage drop over $CA = (I_B - 700) \times 0.01 \times 4$

Since total drop over $AB = 242 - 240 = 2$ volt

$\therefore$ $0.05\, I_B - 12.5 + 0.01\, I_B - 5 + 0.04\, I_B - 28 = 2$ $\qquad \therefore$ I_B = **455 A**

Now, total current is = 500 + 200 = 700 A $\qquad \therefore$ I_A = 700 − 455 = **245 A**

It is obvious that (245 − 200) = 45 A is fed into the distributed load at D. Hence, point of minimum potential M will be 45/1.0 = 45 metres from D. Its distance from B = 500 − 45 = 455 metres or 4.55 units of length.

Voltage drop upto this point from end $B = \int_0^{4.55} (455 - 100x) \times 0.01 \times dx = 10.35$ V

$\therefore$ potential of M = 242 − 10.35 = **231.65 V**

Tutorial Problem No. 40.1

1. A 2-wire direct-current distributor PQ is 500 metres long. It is supplied by three feeders entering at P, Q and R where R is midway between P and Q. The resistance of the distributor, go and return, is 0.05 Ω per 100 metres. The distributor is loaded as follows :

Point	*A*	*B*
Distance from P (metres)	*100*	*350*
Load (ampere)	*30*	*40*

In addition, a distributed load of 0.5 A/metre exists from P to Q. Calculate (*a*) the current supplied by each feeder if P and R are maintained at 220 V while Q is at 215 V (*b*) the voltage at a point between P and Q and 50 metres from P.

[(*a*) current at P = 18 A ; at Q = 28.5 A ; at R = 98.5 A (*b*) voltage at stated point = 214.8 V]

2. A section of a 2-wire distributor network is 1,200 metres long and carries a uniformly distributed load of 0.5 A/ metre. The section is supplied at each end by a feeder from a distribution centre at which the voltage is maintained constant. One feeder is 900 and the other 600 metres long and each has a cross-sectional area 50% greater than that of the distributor. Find the current in each feeder cable and the distance from one end of the distributor at which p.d. is a minimum. **(*London Univ.*)**

[Current in 900 metres: 272.7 A; current in 600 metres: 327.3 A; 545.5 metres from the 900 metre feeder point]

3. A pair of distributing mains of uniform cross-section 1,000 metres in length having a resistance of 0.15 Ω each, are loaded with currents of 50, 100, 57.5, 10 and 75 A at distances measured from one end where the voltage between mains is 211.6, of 100, 300, 540, 740 and 850 metres respectively. If the voltage of the other end is maintained at 210, calculate the total current entering the system at each end of the mains and the position of the point of minimum potential. ***(I.E.E. London)***

[160.63 A and 131.87 A ; 540 metre load]

4. A 2-core distribution cable *AB*, 400 metre long, supplies a uniformly distributed lighting load of 1 A/m. There are concentrated loads of 120, 72, 48 and 120 A at 40, 120, 200 and 320 metres respectively from end *A*. This cable has a resistance of 0.15 Ω per 1,000 m run. Calculate the voltage and the position of the lowest run lamp when the cable is fed at 250 Volts (*a*) from both ends *A* and *B* (*b*) from end *A* only. **[(*a*) 239.1 volt at 200 m (*b*) 207.6 volt at B]**

5. A 2-wire d.c. distributor *AB*, 300 m long, is fed at both ends *A* and *B*. The distributor supplies a uniformly distributed load of 0.25 A per m together with concentrated loads of 40 A at *C* and 60 A at *D*, *AC* and *BD* being 120 m each. *A* and *B* are maintained at 300 V, the loop resistance of the distributor is 0.1 ohm/100 m. Determine the current fed at *A* and *B* and also the potential of points *C* and *D*. **[85.5 A, 89.5 A, 291.54 V, 291.06 V]**

40.11. Ring Distributor

A ring distributor is a distributor which is arranged to form a closed circuit and which is fed at one or more than one points. For the purpose of calculating voltage distribution, it can be looked upon as consisting of a series of open distributors fed at both ends. By using a ring distributor fed properly, great economy in copper can be affected.

If the ring distributor is fed at one point then, for the purposes of calculation, it is equivalent to a straight distributor fed at both ends with equal voltages (Ex. 40.17).

Example 40.17. *A 400-metre ring distributor has loads as shown in Fig. 40.29 (a) where distances are in metres. The resistance of each conductor is 0.2 Ω per 1,000 metres and the loads tapped off at points B, C and D are as shown. If the distributor is fed at A, find voltages at B,C and D.*

Solution. Let us assume a current of I in section *AD* [Fig. 40.29 (*a*)] and then find the total drop which should be equated to zero.

$\therefore \quad 70\, I + 90\,(I - 50) + 80\,(I - 120) + 60\,(I - 220) = 0 \qquad \therefore \quad 300\, I = 27{,}300$ or $I = 91$ A

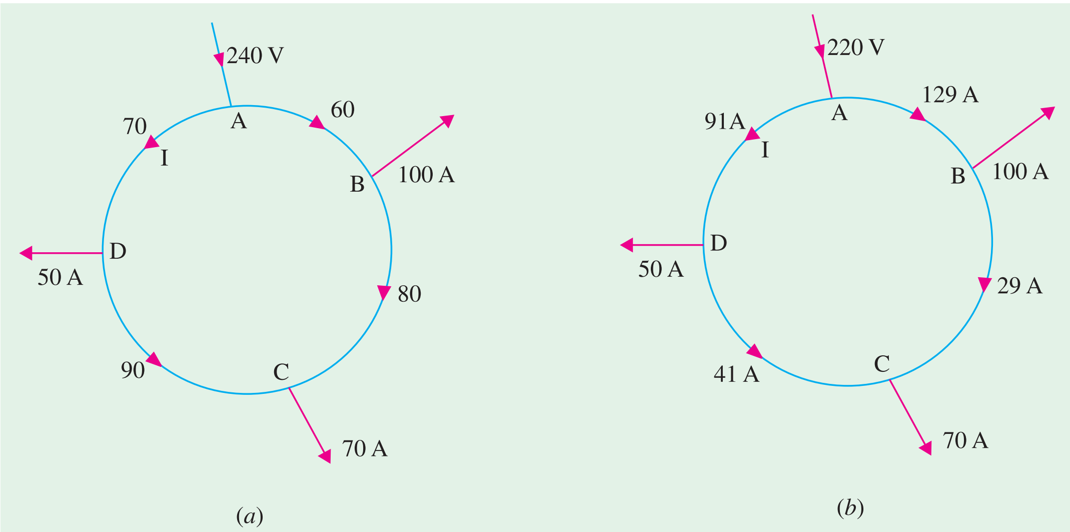


Fig. 40.29

The current distribution becomes as shown in Fig. 40.29 (*b*) from where it is seen that *C* is the point of minimum potential.

Drop in AD = $2(91 \times 70 \times 0.2/1{,}000) = 2.55$ V ; Drop in $DC = 2(41 \times 90 \times 0.2/1{,}000) = 1.48$ V

Drop in CB = $2(29 \times 80 \times 0.2/1{,}000) = 0.93$ V ; Drop in $BA = 2(129 \times 60 \times 0.2/1{,}000) = 3.1$ V

Voltage at D = $240 - 2.55$ = **237.45 V** ; Voltage at $C = 237.45 - 1.48$ = **235.97 V**

Voltage at $B = 240 - 3.1$ = **236.9 V**

Example 40.18. *In a direct current ring main, a voltage of 400 V is maintained at A. At B, 500 metres away from A, a load of 150 A is taken and at C, 300 metres from B, a load of 200 A is taken. The distance between A and C is 700 metres. The resistance of each conductor of the mains is 0.03 Ω per 1,000 metres. Find the voltage at B and C and also find the current in the section BC.*

(Elect. Technology, Kerala Univ.)

Solution. Let us assume a current of I in section AB, then find the total drop round the ring main and equate it to zero. As seen from Fig. 40.30 (*a*).

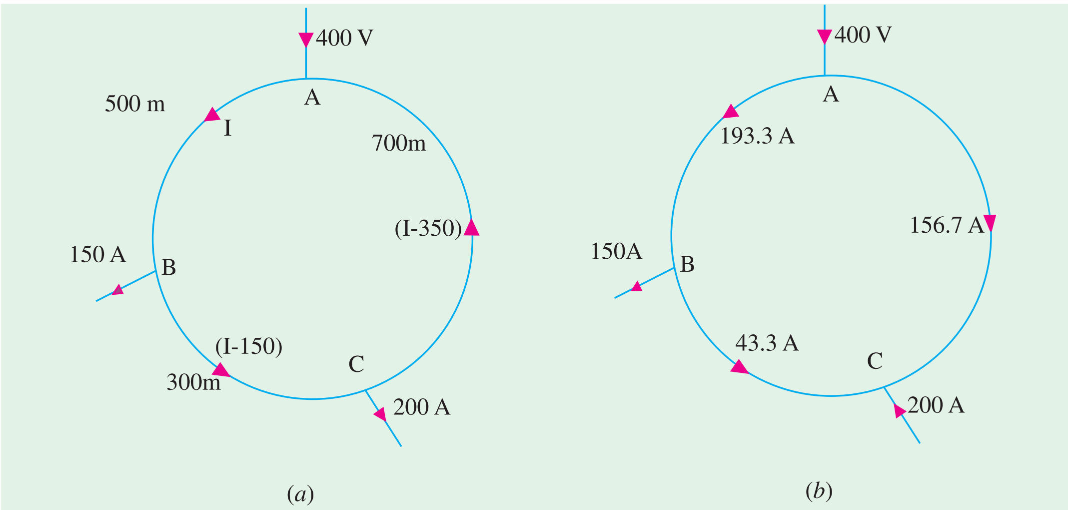


Fig. 40.30

$$500\, I + 300\,(I - 150) + 700\,(I - 350) = 0 \qquad \therefore \quad I = 193.3 \text{ A}$$

The current distribution becomes as shown in Fig. 40.30 (*b*) from where it is seen that *C* is the point of minimum potential.

Drop over AB = $2(193.3 \times 50 \times 0.03/1{,}000) = 5.8$ V

Drop over BC = $2(43.3 \times 300 \times 0.03/1{,}000) = 0.78$ V

Voltage at B = $400 - 5.8 = 394.2$ V ; Voltage at $C = 394.2 - 0.78$ = **393.42 V**

The current in section BC is as shown in Fig. 40.30 (*b*).

Example 40.19. *A d.c. ring main ABCDE is fed at point A from a 220-V supply and the resistances (including both lead and return) of the various sections are as follows (in ohms) : AB = 0.1 ; BC = 0.05 ; CD = 0.01 ; DE = 0.025 and EA = 0.075. The main supplies loads of 10 A at B ; 20 A at C ; 30 A at D and 10 A at E. Find the magnitude and direction of the current flowing in each section and the voltage at each load point.*

If the points C and E are further linked together by a conductor of 0.05 Ω resistance and the output currents from the mains remain unchanged, find the new distribution of the current and voltage in the network. **(London Univ.)**

Solution. The ring main is shown in Fig. 40.31.

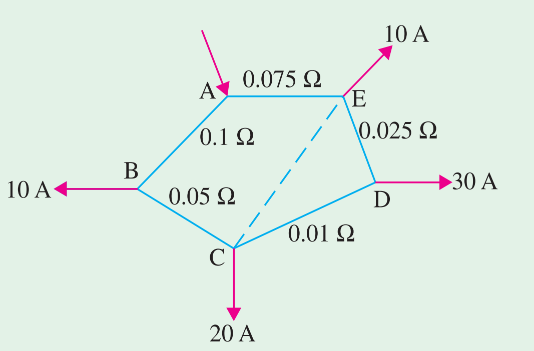


Fig. 40.31

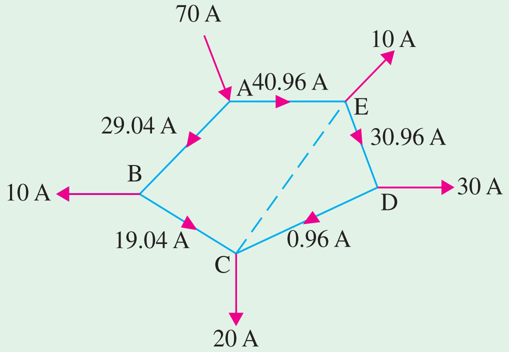


Fig. 40.32

Let us assume a current of I amperes in section AB and put the total drop round the ring equal to zero.

$\therefore \quad 0.1\,I + 0.05(I - 10) + 0.01(I - 30) + 0.025(I - 60) + 0.075(I - 70) = 2$ or $I = 29.04$ A

Current distribution now becomes as shown in Fig. 40.32.

Drop in AB = $29.04 \times 0.1 = 2.9$ V; Drop in BC = $19.04 \times 0.05 = 0.95$ V

Drop in ED = $30.96 \times 0.025 = 3.77$V; Drop in AE = $40.96 \times 0.075 = 3.07$ V

$\therefore$ Potential of B = 217.1 V, Potential of C = **216.15 V**

Potential of E = 216.93 V, Potential of D = **216.16 V**

The interconnector between points C and E is shown in Fig. 40.31. It may be noted here that the function of the interconnector is to reduce the drop of voltage in various sections. For finding current in the interconnector, first p.d. across its ends is calculated. Then we calculate the resistance, viewed from points E and C of the network composed resistances of the distribution lines only, ignoring the load (Art, 2-22). Then current through the interconnector is given by

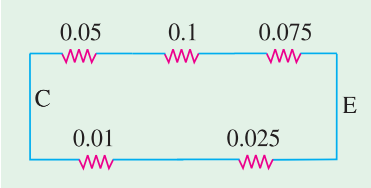


Fig. 40.33

$$I = \frac{\text{p.d. between points } E \text{ and } C}{\text{resistance of distribution network + interconnector}}$$

P.D. between points E and C = 216.93 – 216.15 = 0.78 V

To determine the resistance viewed from C and E, the network is drawn as shown in Fig. 40.33. Since the two branches are in parallel, the combined resistance is

$$= 0.225 \times 0.035/(0.225 + 0.035) = 0.03\ \Omega$$

$\therefore$ current in interconnector $= 0.78/(0.03 + 0.05) = 9.75$ A from E to C.

The currents in the other sections must be calculated again.

Let us assume a current I_1 and ED, then since the voltage round the closed mesh EDC is zero, hence

$-0.025\,I_1 - 0.01\,(I_1 - 30) + 0.05 \times 9.75 = 0$ or $0.035\,I_1 = 0.7875$ $\therefore$ $I_1 =$ **22.5 A**

Current in AE = 10 + 22.5 + 9.75 = **42.25 A;** Current in AB = 70 – 42.25 = **27.75 A**

Drop in AB = $27.75 \times 0.1 =$ **2.775 V ;** Drop in BC = $17.75 \times 0.05 =$ **0.888 V**

Drop in ED = $32.25 \times 0.025 =$ **0.806 V ;** Drop in AE = $42.25 \times 0.075 =$ **3.169 V**

Potential of B = 220 – 2.775=**217.225 V;** Potential of C = 217.225 – 0.888 = **216.337 V**

Potential of E = 220 – 3.169=**216.83 V ;** Potential of D = 216.83 – 0.806 = **216.024 V**

Tutorial Problem No. 40.2

1. Four power loads *B*, *C*, *D* and *E* are connected in this order to a 2-core distributor cable, arranged as a ring main, and take currents of 20, 30, 25 and 30 A respectively. The ring is supplied from a substation at the point *A* between *B* and *E*. An interconnector cable joins the points *C* and *E* and from a point *F* on this inter-connector cable a current of 20 A is taken.
The total resistance of the cable between the load points is : AB = 0.04 Ω ; BC = 0.03 Ω ; CD = 0.02 Ω ; DE = 0.03 Ω ; EA = 0.04 Ω ; CF = 0.02 Ω and EF = 0.01 Ω. Calculate the current in each section of the ring and the interconnector. ***(London Univ.)***
[AB = 53.94 A ; BC = 33.94 A ; CD = 8.35 A ; ED = 16.65 A ; AE = 71.06 A ; FC = 4.42 A ; EF = 24.42 A]

2. A 2-core ring feeder cable *ABCDEA* is connected to a sub-station at *A* and supplies feeding points to a distribution network at *B*, *C*, *D* and *E*. The points *C* and *E* are connected by an inter-connecter *CFE* and a load is taken at *F*. The total resistance in ohms of both conductors in the several sections is AB = 0.05 ; BC = 0.4 ; CD = 0.03 ; DE = 0.04 ; EA = 0.05 ; CF = 0.02 ; FE = 0.1. The currents taken at the load points are B = 12 A ; C = 15 A ; D = 12 A; E = 15 A and F = 10 A. Calculate the current in each section of the cable and the p.d. at each load point, if the p.d. at *A* is maintained constant at 250 V. ***(City & Guilds, London)***
[Currents : AB = 27.7 A ; FC = 3.3 A ; P.D. s at B = 248.6 V ; C = 248 V ; D = 247.87 V ; E = 248.18 V ; F = 248 V]

3. A distributor cable in the form of a ring main *ABCDEA*, supplies loads of 20, 60, 30, and 40 A taken at the points *B*, *C*, *D* and *E* respectively, the feeding point being at *A*. The resistances of the sections are AB = 0.1 Ω, BC = 0.15 Ω, CD = 0.1 Ω, DE = 0.05 Ω and EA = 0.1 Ω. The points *E* and *C* are connected by a cable having a resistance of 0.3 Ω. Calculate the current in each section of the network. ***(City & Guilds, London)***
[A to B : 60 A ; B to C : 40 A ; E to C : 10 A ; D to C : 10 A ; E to D : 40 A ; A to E : 90 A]

40.12. Current Loading and Load-point Voltages in a 3-wire System

Consider a 3-wire, 500/250-V distributor shown in Fig. 40.34. The motor requiring 500 V is connected across the outers whereas other loads requiring lower voltage of 250 V are connected on both sides of the neutral.

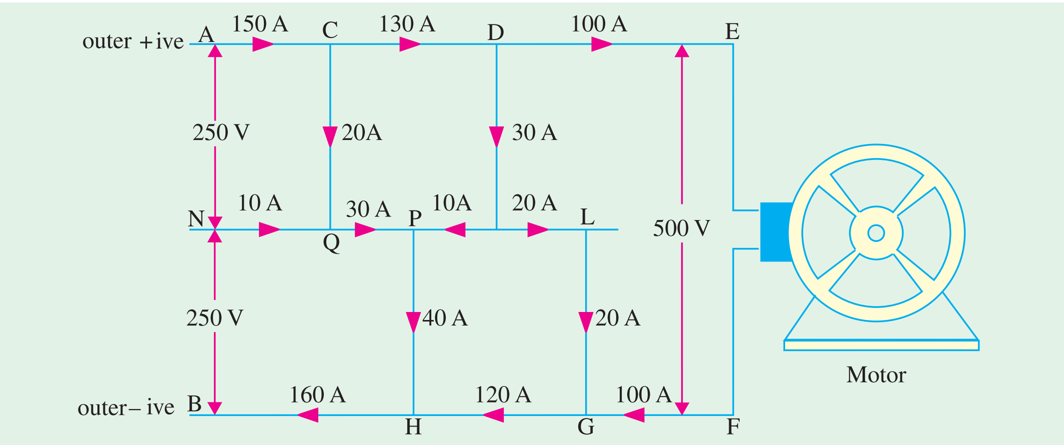


Fig. 40.34

The current in positive outer *AE* flows from the left to right and in negative outer *FB* from right to left. The current through the various sections of the neutral wire *NL* may flow in either direction depending on the load currents on the positive side and negative side but is independent of the loads connected between outers.

Since 150 A enters the +ve outer but 160 A comes out of the network, it means that a current of 10 A must flow into the neutral at point *N*. Once the direction and magnitude of current in *NQ* is known, the directions and magnitudes of currents in other sections of the neutral can be found very easily. Since *PH* takes 40 A, currents meeting at *P* should add up to 40 A. As seen, 20 A of *CQ* and 10A of *NQ* flow towards *P*, the balance of 10 A flows from point *M*. Then, 30 A current of *DM* is divided into two parts, 10 A flowing along *MP* and the other 20 A flowing along *ML* to feed the load *LG*.

Knowing the values of currents in the various conductors, voltage drops can be calculated provided resistances are known. After that, voltages at different load points can be calculated as illustrated in Ex. 40.20.

Example 40.20. *In a 3-wire distribution system, the supply voltage is 250 V on each side. The load on one side is a 3 ohm resistance and on the other, a 4 ohm resistance. The resistance of each of the 3 conductors is 0.05 Ω. Find the load voltages.*

(Elements of Elect. Engg-I, Bangalore Univ.)

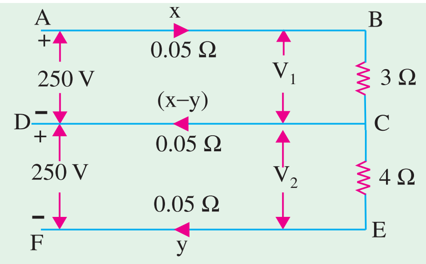


Fig. 40.35

Solution. Let the assumed directions of unknown currents in the three conductors be as shown in Fig. 40.35. Applying KVL (Art. 2.2) to closed circuit *ABCDA*, we have

$$-3.05x - 0.05(x - y) + 250 = 0$$

$$\text{or} \quad 310x - 5y = -25{,}000 \quad \ldots(i)$$

Similarly, circuit *DCEFD* yields

$$0.05\,(x - y) - 4.05y + 250 = 0$$

$$\text{or} \quad 5x - 410y = -25{,}000 \quad \ldots(ii)$$

From (*i*) and (*ii*), we get $x = 81.64$ A, $y = 61.97$ A. Since both currents come out to be positive, it means that their assumed directions of flow are correct.

$$\therefore \quad V_1 = 250 - 0.05 \times 81.64 - 0.05\,(81.64 - 61.97) = \mathbf{244.9\ V}$$

$$V_2 = 250 + 0.05(81.64 - 61.97) - 0.05 \times 61.97 = \mathbf{247.9\ V}$$

Example 40.21. *A 3-wire d.c. distributor PQ, 250 metres long, is supplied at end P at 500/250 V and is loaded as under :*

Positive side: 20 A 150 metres from P ; 30 A 250 metres from P.

Negative side: 24 A 100 metres from P ; 36 A 220 metres from P.

The resistance of each outer wire is 0.02 Ω per 100 metres and the cross-section of the middle wire is one-half that of the outer. Find the voltage across each load point.

Solution. The current loading is shown in Fig. 40.36.

The current flowing into the positive side is 30 + 20 = 50 A. Since current flowing out of the negative side is 36 + 24 = 60 A, it means that 10 A must be flowing in the section *NC* of the neutral. To make up 24 A in load *CD*. 10 A are contributed by NC and 14 A by *BC*. The balance of 6 A flows through *BF* and adds up with 30 A of *KF* to make up 36A through load *FE*.

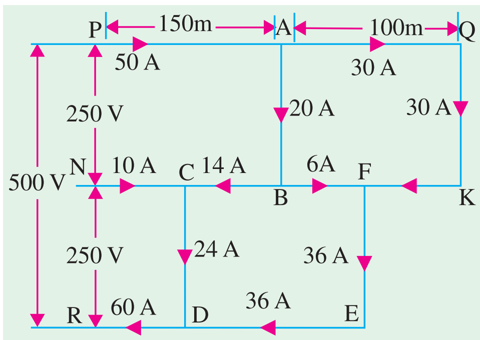


Fig. 40.36

It is given that the resistance of outers is 0.02 Ω per 100 m. Since neutral is of half the cross-section, its resistance is 0.04 Ω per 100 m. Knowing them, voltage at load points can be determined as under. Let us see how we will calculate voltage across load *A B*.

Voltage at $AB = 250$ – drop in PA – drop in BC + drop in CN.

It should be particularly noted that drop in CN has been added instead of being ***subtracted.*** The reason is this : as we start from P and go round $PABCN$, we go along the current over section PA *i.e.* we go 'downstream' hence drop is taken negative, but along CN we go 'upstream', hence the drop as taken as positive*. Proceeding in this way, we tabulate the currents, resistances and voltage drops in various sections as given below :

Section	*Resistance (W)*	*Current (A)*	*Drop (V)*
PA	0.03	50	1.5
AQ	0.02	30	0.6
KF	0.012	30	0.36
BC	0.02	14	0.28
BF	0.028	6	0.168
NC	0.04	10	0.4
ED	0.024	36	0.864
DR	0.02	60	1.2

P.D. across AB = 250 – 1.5 – 0.28 + 0.4 = **248.62 V**

P.D. across QK = 248.62 – 0.6 – 0.36 + 0.168 = **247.83 V**

P.D. across CD = 250 – 0.4 – 1.2 = **248.4 V**

P.D. across FE = 248.4 + 0.28 – 0.168 – 0.864 = **247.65 V**

Tutorial Problem No. 40.3

1. A 3-wire system supplies three loads (*a*) 10 (*b*) 20 and (*c*) 30 amperes situated at distances of 100, 150 and 300 metres respectively from the supply point on the positive side of the neutral wire. Connected between negative and neutral wires are two loads (*d*) 30 A and (*e*) 20A, situated 120 and 200 metres respectively from the supply point. Give a diagram showing the values and directions of the currents in various parts of the neutral wire.

If the resistance of the outers is 0.05 Ω per 1000 metres and that of the neutral 0.1 Ω per 1000 metres, calculate the potential difference at the load points (*b*) and (*e*); the pressure at the supply point being 100 V between outers and neutral. **[10A and 5A; Volts at (*b*) = 99.385 V ; at (*e*) = 99.86 V]**

2. A 3-wire d.c. distributor 400 metres long is fed at both ends at 235 volts between each outer and neutral. Two loads P and Q are connected between the positive and neutral and two loads R and S are connected between the negative outer and the neutral. The loads and their distances from one end (X) of the distributor are as follows : Load P, 50 A, 100 metres from X ; Load Q, 70 A, 300 metres from X; Load R, 60 A, 150 metres from X; Load S, 60 A,350 metres from X. Determine the p.d. at each load point and the current at each feeding point. The resistance of each outer is 0.25 Ω per 1000 metres and that of the neutral is 0.5 W per 1000 metres.

[Current into the +ve outer at X = 55 A ; other end Y = 65 A; Out from –ve outer at X = 45 A; other end Y = 75 A ; P.D. s at P = 233.125 V; Q = 232.375 V; R = 232.812 V; S = 233.812 V]

40.13. Three-wire System

As already mentioned briefly in Art. 40.2, it consists of the 'outer' conductors (between which the voltage is twice the normal value for lighting) and the third wire which is called the middle or neutral wire. It is of half the cross-section as compared to any one of the two outers and is earthed at the generator end. The voltage of the neutral is thus approximately half way between that of the outers.

* Hence, sign convention in this : While going 'upstream' take the drop as positive and while going 'downstream' take the drop as negative. (Art. 2-3).

If the total voltage between outers is 460 V, then the positive outer is 230 V ***above*** the neutral and the negative outer 230 V ***below*** the neutral. Motor loads requiring higher voltage are connected across the outers directly whereas lighting and heating loads requiring lesser voltage of 230 V are connected between any one of the outers and the neutral. If the loads on both sides of the neutral are equal *i.e.* balanced as shown in Fig. 40.37, then there is no current in the middle wire and the effect is as if the different loads were connected in series across the outers. However, in practice, although effort is made to distribute the various loads equally on the two sides of the neutral, yet it is difficult to achieve exact balance, with the result that some out-of-balance current does flow in the neutral as shown in Fig. 40.38. In the present case as viewed from generator end, the load is equivalent to a resistance of 25/3 ohms in series with a resistance of 25/2 ohms (Fig. 40.39). Obviously, the voltages across the two will becomes unequal. The voltage across the positive side will fall to $\dfrac{500 \times 25/2}{(25/2 + 25/3)}$ = 200 V and on the negative side, it will rise to $\dfrac{500 \times 25/3}{(25/2 + 25/3)}$ = 300 V. This difficulty of unequal voltages with unbalanced loads is removed by using balancers as discussed below in Art. 40.14.

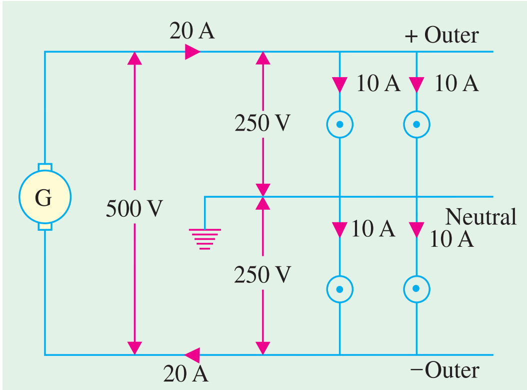


Fig. 40.37

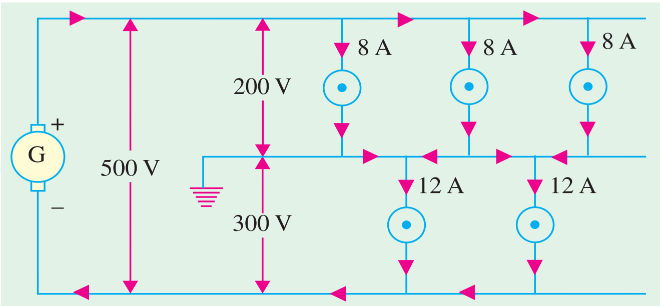


Fig. 40.38

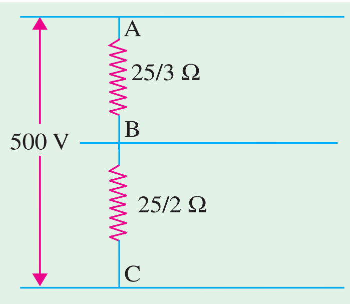


Fig. 40.39

40.14. Balancers

In order to maintain p.d.s on the two sides of neutral equal to each other, a balancer set is used. The commonest form of balancer consists of two identical shunt-wound machines which are not only coupled mechanically but have their armatures and field circuits joined in series across the outers. The neutral is connected to the junction of the armatures as shown. When the system is unloaded or when the loads on the two sides are balanced, then

1. both machines run as unloaded motors and
2. since their speeds and field currents are equal, their back e.m.fs. are the same.

When the two sides are unbalanced *i.e.* when the load supplied by + ve outer is different from that

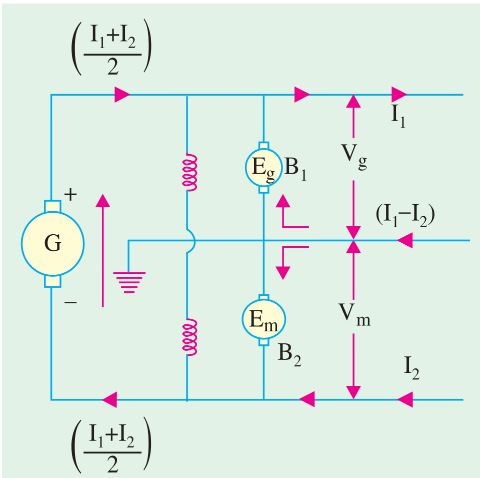


Fig. 40.40

supplied by negative outer, then out-of-balance current $(I_1 - I_2)$ flows through the mid-wire to the balancers where it divides into two halves each equal to $(I_1 - I_2)/2$ as shown in Fig. 40.40.

If I_1 is greater than I_2, then +ve side is more heavily loaded (than –ve side) hence the p.d. on this side tends to fall below the e.m.f. of the balancer on this side so that B_1 runs as a generator. However, the p.d. on the lightly-loaded side rises above the e.m.f. of the balancers on that side, hence B_2 runs as a motor. In this way, energy is transferred through balancers from the lightly-loaded side to the heavily-loaded side. In Fig. 40.40, machine B_2 is running as a motor and driving machine B_1 as a generator.

Let R_a = armature resistance of each machine

V_g = terminal p.d. of machine running as a generator *i.e.* B_1

E_g = induced e.m.f. of B_1

V_m = terminal p.d. of motoring machine *i.e.* B_2

E_m = induced e.m.f. of B_2

then $$V_g = E_g - \frac{(I_1 - I_2)}{2} R_a \quad \text{and} \quad V_m = E_m + \frac{(I_1 - I_2)}{2} R_a$$

$$\therefore \quad V_m - V_g = (E_m - E_g) + (I_1 - I_2) R_a \qquad \text{...(i)}$$

Since the speed and excitation of the two machines are equal,

$$\therefore \quad E_g = E_m \qquad \therefore \quad V_m - V_g = (I_1 - I_2) R_2 \qquad \text{...(ii)}$$

Hence, we find that the difference of voltages between the two sides of the system is proportional to —

(*i*) out of balance current $(I_1 - I_2)$ and (*ii*) armature resistance of the balancer.

For this reason, R_a is kept very small and effort is made to arrange the loads on the two sides such that out-of-balance current is as small as possible.

The value $(V_m - V_g)$ can be still further reduced *i.e.* the voltages on the two sides can be more closely balanced by cross-connecting the balancer fields as shown in Fig. 40.41. In this way, generator draws its excitation from the lightly-loaded side which is at a higher voltage, hence E_g is increased. The motoring machine draws its excitation from the heavily loaded side which is at a little lower voltage, hence E_m is decreased. In this way, the difference $(E_m - E_g)$ is decreased and so is $(V_m - V_g)$. Further, regulation of the voltage can be accomplished by connecting an adjustable regulator in series with the two balancer fields as shown in Fig. 40.42.

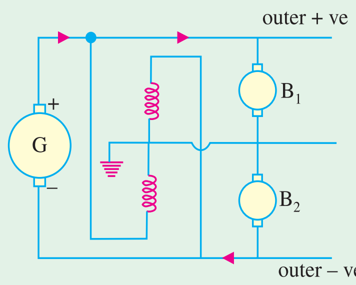


Fig. 40.41

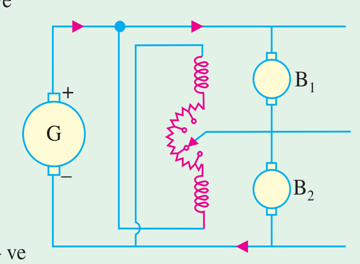


Fig. 40.42

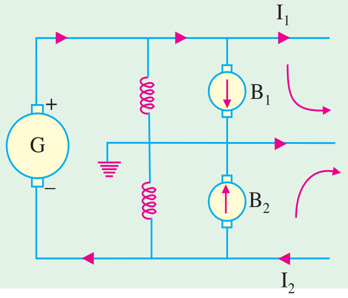


Fig. 40.43

It should be noted that since machine B_1 is running as a generator and B_2 as a motor, the directions of currents in B_1, B_2 and the neutral are as shown in Fig. 40.40. If, however, B_2 runs as a generator and B_1 as a motor *i.e.* if negative side is more heavily loaded than the +ve side, then directions of currents through B_1, B_2 and the neutral are as shown in Fig. 40.43. In particular, the change in the direction of the current through midwire should be noted.

Example 40.22. *A d.c. 3-wire system with 500-V between outers has lighting load of 100 kW on the positive and 50 kW on the negative side. If, at this loading, the balancer machines have each a loss of 2.5 kW, calculate the kW loading of each balancer machine and the total load on the system.*

Solution. The connections are shown in Fig. 40.44.

Total load on main generator $= 100 + 50 + (2 \times 2.5)$
$=$ **155 kW**

Output current of main generator $= 155 \times 1000/500$
$= 310$ A

Load current on +ve side, $I_1 = 100 \times 1000/250$
$= 400$ A

Load current on – ve side, $I_2 = 50 \times 1000/250$
$= 200$A

Out-of-balance current $= 400 - 200 = 200$ A

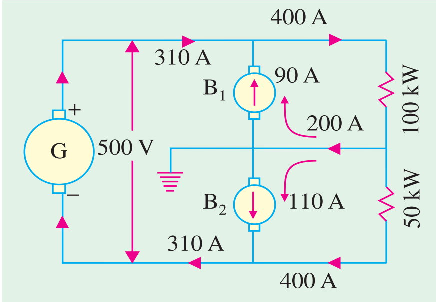


Fig. 40.44

Since +ve side is more heavily loaded , B_1 is working as a generator and B_2 as motor.

$\therefore$ current of $B_1 = 400 - 310 = 90$ A
current of $B_2 = 200 - 90 = 110$ A
Loading of $B_1 = 250 \times 90/1000 =$ **22.5 kW**
Loading of $B_2 = 250 \times 110/1000 =$ **27.5 kW**

Example 40.23. *In a 500/250-V d.c. 3-wire system, there is a current of 2000 A on the +ve side, 1600 A on the negative side and a load of 300 kW across the outers. The loss in each balancer set is 8 kW. Calculate the current in each armature of the balancer set and total load on the main generator.*

Solution. Connections are shown in Fig. 40.45.

It should be noted that loading across 'outers' directly in no way determines the current in the neutral.

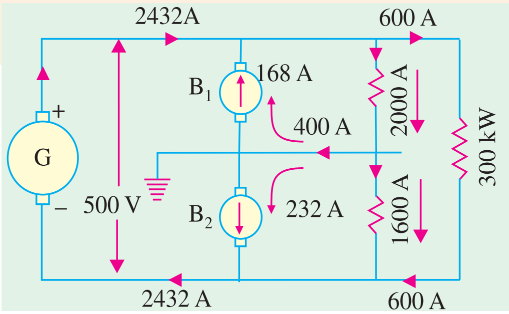


Fig. 40.45

+ve loading $= 2000 \times 250/1000$
$= 500$ kW
– ve loading $= 1600 \times 250/1000$
$= 400$ kW

Total loading on main generator is
$= 500 + 400 + 300 + (2 \times 8) =$ **1216 kW**

$\therefore$ Current of main generator $= 1216 \times 1000/500 = 2{,}432$ A

Out-of-balance current $= 2000 - 1600 = 400$ A

Current through $B_1 = 2{,}600 - 2{,}432 =$ **168 A**

Current through $B_2 = 400 - 168 =$ **232 A**

Example 40.24. *On a 3-wire d.c. distribution system with 500 V between outers, there is a load of 1500 kW on the positive side and 2,000 kW on the negative side. Calculate the current in the neutral and in each of the balancer armatures and the total current supplied by the generator. Neglect losses.*
(Electrical Engineering ; Madras Univ.)

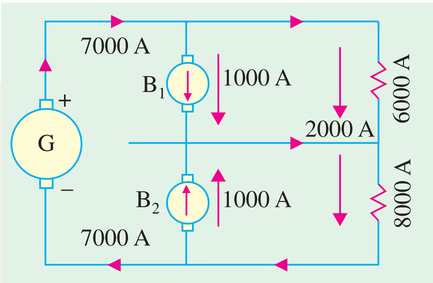


Fig. 40.46

Solution. Since negative side is more heavily loaded than the positive side, machine B_2 runs as a generator and B_1 as a motor. The directions of current through B_1 and B_2 are as shown in Fig. 40.46. Total loading on the main generator
$= 2{,}000 + 1{,}500 = 3{,}500$ kW

Current supplied by main generator $= 3{,}500 \times 1000/500 =$ **7,000 A**

Current on +ve side $= 1500 \times 1000/250 =$ **6,000 A**

Current on –ve side $= 2000 \times 1000/250 =$ **8,000 A**

Out-of-balance current $= 8{,}000 - 6000 =$ **2,000 A**

Current through the armature of each machine $=$ **1000 A**

Example 40.25. *A 125/250 V, 3-wire distributor has an out-of-balance current of 50 A and larger load of 500 A. The balancer set has a loss of 375 W in each machine. Calculate the current in each of the balancer machines and output of main generator.*

(Electrical Technology-II, Gwalior Univ.)

Solution. As shown in Fig. 40.47, let larger load current be I_1 = 500 A. Since $(I_1 - I_2) = 50$

$\therefore \quad I_2 = 450$ A

Larger load $= 500 \times 125/1000 = 62.5$ kW

Smaller load $= 450 \times 125/1000 = 56.25$ kW

Balancer loss $= 2 \times 375 = 0.75$ kW

Output of main generator

$= 62.5 + 56.25 + 0.75 =$ **119.5 kW**

Current of main generator

$= 119.5 \times 1000/250 = 478$ A

As seen from Fig. 40.47, current of

$B_1 = (500 - 478) =$ **22 A**

and current of $B_2 = (50 - 22) =$ **28 A**

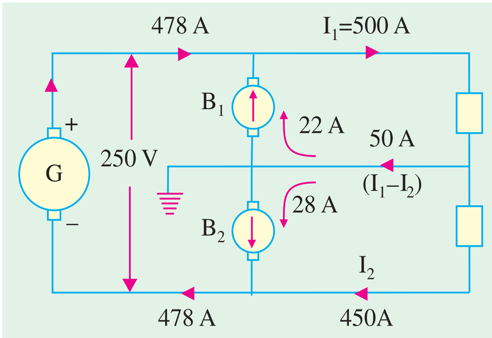


Fig. 40.47

Example 40.26. *The load on a d.c. 3-wire system with 500 V between outers consists of lighting current of 1500 A on the positive side and 1300 A on the negative side while motors connected across the outers absorb 500 kW. Assuming that at this loading, the balancer machines have each a loss of 5 kW, calculate the load on the main generator and on each of the balancer machines.*

(Electrical Engineering ; Madras Univ.)

Solution. Connections are shown in Fig. 40.48.

Positive loading $= 1500 \times 250/1000 = 375$ kW

Negative loading $= 1300 \times 250/1000 = 325$ kW

Total load on the main generator is

$= 375 + 325 + 500 + (2 \times 5) =$ **1210 kW**

Current supplied by the main generator is

$= 1210 \times 1000/500 = 2{,}420$ A

Out-of-balance current $= 1500 - 1300 = 200$ A

Current through $B_1 = 2500 - 2420 = 80$ A

Current through $B_2 = 200 - 80 = 120$ A

Loading of $B_1 = 80 \times 250/1000 =$ **20 kW**

Loading of $B_2 = 120 \times 250/1000 =$ **30 kW**

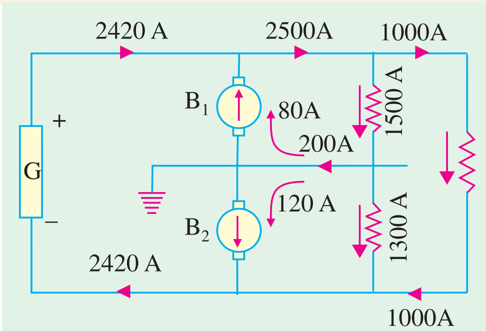


Fig. 40.48

Example 40.27. *A d.c. 3-wire system with 480 V across outers supplies 1200 A on the positive and 1000 A on the negative side. The balancer machines have each an armature resistance of 0.1W and take 10 A on no-load. Find*

(a) *the voltage across each balancer and*

(b) *the total load on the main generator and the current loading of each balancer machine.*

The balancer field windings are in series across the outers.

Solution. As shown in Fig. 40.49, B_1 is generating and B_2 is motoring.

The out-of-balance current is (1200 – 1000) = 200 A. Let current through the motoring machine be I_m, then that through the generating machine is $(200 - I_m)$. Let V_g and V_m be p.ds. of the two machines.

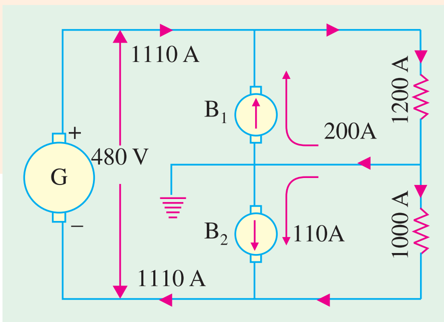


Fig. 40.49

Since B_2 is driving B_1, output of the motor supplies the losses in the set plus the output of the generator. Total losses in the set

$$= \text{no-load losses} + \text{Cu losses in two machines}$$
$$= 480 \times 10 + 0.1\, I_m^{\,2} + (200 - I_m)^2 \times 0.1$$
$$V_m I_m = V_g (200 - I_m) + \text{total losses}$$
$$= V_g (200 - I_m) + 4800 + 0.1\, I_m^2 + (200 - I_m)^2 \times 0.1$$

Now $\qquad V_m = E_b + I_m R_a \quad$ and $\quad V_g = E_b - I_g R_a$

Now, back e.m.f. $\quad E_b = (240 - 0.1 \times 10) = 239$ V

$\therefore \qquad V_m = (239 + 0.1\, I_m)$ and $V_g = 239 - (200 - I_m) \times 0.1$

$\therefore \quad (239 + 0.1\, I_m)\, I_m = [239 - (200 - I_m) \times 0.1]\,(200 - I_m)^2 + 4800 + 0.1\, I_m^{\,2} + (200 - I_m)^2 \times 0.1$

$\therefore \qquad I_m = 110$ A $\quad$ and $\quad I_g = 200 - 110 = 90$ A

(a) $\therefore \quad V_m = 239 + 0.1 \times 110 =$ **250 V** $\quad$ and $\quad V_g = 239 - (0.1 \times 90) =$ **230 V**

(b) Load on main generator $= 1200 - 90 =$ **1110 A**

Example 40.28. *A d.c. 3-wire system with 460 V between outers supplies 250 kW on the positive and 400 kW on the negative side, the voltages being balanced. Calculate the voltage on the positive and negative sides respectively, if the neutral wire becomes disconnected from the balancer set.*

(Electrical Power-III, Bangalore Univ.)

Solution. Before the disconnection of the mid-wire, voltages on both sides of the system are equal *i.e.* 230 V. The loads are 250 kW on the +ve side and 400 kW on the –ve side. If R_1 and R_2 are the resistances of the two loads, then

$$230^2/R_1 = 250{,}000\,; \quad R_1 = 0.2116\ \Omega$$

Similarly, $\qquad R_2 = 2302/400{,}000 = 0.1322\ \Omega$

When the mid-wire is disconnected from the balancer set *i.e.* from the generator side, then the two resistances R_1 and R_2 are put in series across 460 V as shown in Fig. 40.50 (*b*).

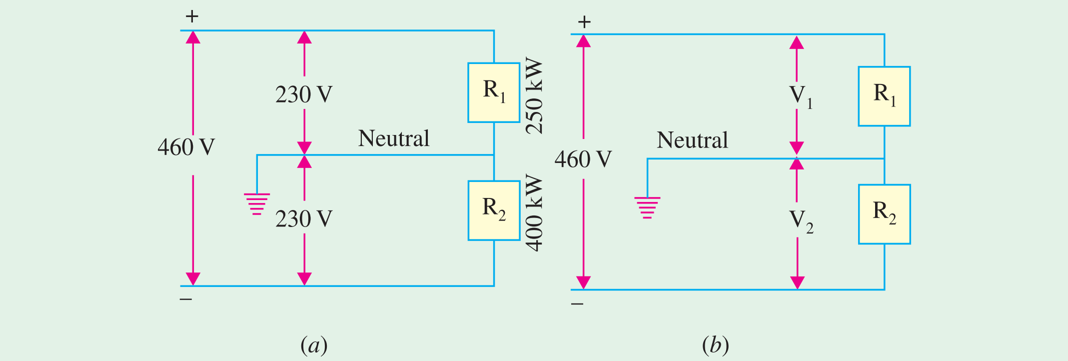


Fig. 40.50

$$\therefore \qquad V_1 = \frac{R_1}{R_1 + R_2} \times 460 = \frac{0.02116 \times 460}{(0.2116 + 0.1322)} = \mathbf{283\ V}$$

$$V_2 = \frac{R_1}{R_1 + R_2} \times 460 = \frac{0.1322 \times 460}{0.3438} = \mathbf{177\ V}\ (\text{or } V_2 = 460 - V_1)$$

Tutorial Problem No. 40.4

1. In a 500/250-V d.c. 3-wire system there is an out-of-balance load of 125 kW on the positive side. The loss in each balancer machine is 7.5 kW and the current in the negative main is 1500 A. Calculate the total load on the generator and the current in each armature of the balancer set.

[890 kW ; 220 A ; 280 A]

2. A 460-V d.c. 2-wire supply is converted into 3-wire supply with the help of rotary balancer set, each machine having a no-load loss of 2.3 kW. If the load on the positive side of 69 kW and on the negative side 57.5 kW, calculate the currents flowing in each of the balancer machines. **[15 A ; 35 A]**
3. In a 500/250 volt 3-wire d.c. system there is an out-of-balance load of 200 kW on the positive side. The loss in each balancer is 10 kW and the current in the negative main is 2800 A. Calculate the current in each armature of the balancer set and the total load on the generators. ***(I.E.E. London)***

[Motoring machine = 440 A; Generating machine = 360 A ; Total load = 1620 kW]

40.15. Boosters

A booster is a generator whose function is to add to or inject into a circuit, a certain voltage that is sufficient to compensate for the I_R drop in the feeders etc.

In a d.c. system, it may sometimes happen that a certain feeder is much longer as compared to others and the power supplied by it is also larger. In that case, the voltage drop in this particular feeder will exceed the allowable drop of 6% from the declared voltage. This can be remedied in two ways ***(i)*** by increasing the cross-section of the feeder, so that its resistance and hence I_R drop is decreased ***(ii)*** or by increasing the voltage of the station bus-bars.

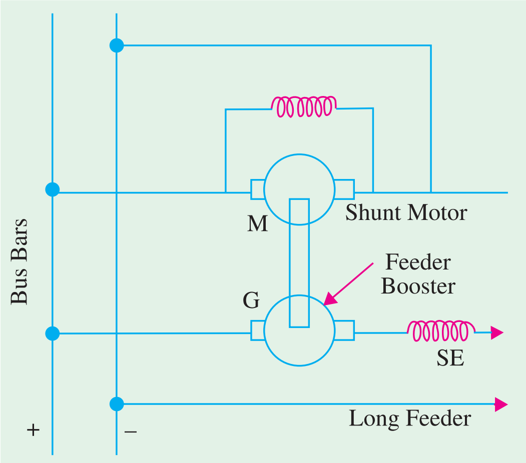


Fig. 40.51

The second method is not practicable because it will disturb the voltage of other feeders, whereas the first method will involve a large initial investment towards the cost of increased conductor material.

To avoid all these difficulties, the usual practice is to install a booster in series with this longer feeder as shown in Fig. 40.51. Since it is used for compensating drop in a feeder, it is known as feeder booster. It is a (series) generator connected in series with the feeder and driven at a constant speed by a shunt-motor working from the bus-bars. The drop in a feeder is proportional to the load current, hence the voltage injected into the feeder by the booster must also be proportional to the load current, if exact compensation is required. In other words, the booster must work on the straight or linear portion of its voltage characteristic.

Example 40.29. *A 2-wire system has the voltage at the supply end maintained at 500. The line is 3 km long. If the full-load current is 120 A, what must be the booster voltage and output in order that the far end voltage may also be 500 V.*

Take the resistance of the cable at the working temperature as 0.5 ohm/kilometre.

(Elect. Machinery-I, Calcutta Univ.)

Solution. Total resistance of the line is = $0.5 \times 3 = 1.5\ \Omega$

Full-load drop in the line is = $1.5 \times 120 = 180$ V

Hence, the terminal potential difference of the booster is **180 V** (*i.e.* 180/120 = 1.5 volt per ampere of line current).

$$\text{Booster-output} = 120 \times 180/1000 = \mathbf{21.6\ kW}$$

40.16. Comparison of 2-wire and 3-wire Distribution Systems

We will now compare the 2-wire and 3-wire systems from the point of view of economy of conductor material. For this comparison, it will be assumed that—

1. the amount of power transmitted is the same in both cases.
2. the distance of transmission is the same.

3. the efficiency of transmission (and hence losses) is the same.
4. voltage at consumer's terminals is the same.
5. the 3-wire system is balanced and
6. in the 3-wire system, the mid-wire is of half the cross-section of each outer.

Let W be the transmitted power in watts and V the voltage at the consumer's terminals. Also, let

R_2 = resistance in ohms of each wire of 2-wire system.
R_3 = resistance in ohms of each outer in 3-wire system.

The current in 2-wire system is W/V and the losses are $2\,(W/V)^2\,R_2$.

In the case of 3-wire system, voltage between outers is $2\,V$, so that current through outers is $(W/2V)$, because there is no current in the neutral according to our assumption (5) above. Total losses in the two outers are $2(W/2V)^2\,R_3$.

Since efficiencies are the same, it means the losses are also the same.

$$\therefore \qquad 2(W/V)^2 R_2 = 2(W/2V)^2 \times R_3 \quad \text{or} \quad \frac{R_3}{R_2} = \frac{4}{1}$$

Since the cross-section and hence the volume of a conductor of given length, is inversely proportional to its resistance,

$$\therefore \qquad \frac{\text{volume of each 3-wire conductor}}{\text{volume of each 2-wire conductor}} = \frac{1}{4}$$

Let us represent the volume of copper in the 2-wire system by 100 so that volume of each conductor is 50.

Then, volume of each outer in 3-wire system = 50/4 = 12.5

volume of neutral wire ,, ,, = 12.5/2 = 6.25

∴ total volume of copper in 3-wire system = 12.5 + 6.25 + 12.5 = 31.25

$$\therefore \qquad \frac{\text{total coper vol. in 3-wire feeder}}{\text{total copper vol. in 2-wire feeder}} = \frac{31.25}{100} = \frac{5}{6}$$

Hence, a 3-wire system requires only 5/16th (or 31.25%) as much copper as a 2-wire system.

OBJECTIVE TESTS – 40

1. If in a d.c. 2-wire feeder, drop per feeder conductor is 2%, transmission efficiency of the feeder is .... percent.
   (*a*) 98 (*b*) 94
   (*c*) 96 (*d*) 99
2. Transmitted power remaining the same, if supply voltage of a dc 2-wire feeder is increased by 100 percent, saving in copper is .... percent.
   (*a*) 50 (*b*) 25
   (*c*) 100 (*d*) 75
3. A uniformly-loaded d.c. distributor is fed at both ends with equal voltages. As compared to a similar distributor fed at one end only, the drop at middle point is
   (*a*) one-half (*b*) one-fourth
   (*c*) one-third (*d*) twice.
4. In a d.c. 3-wire distributor using balancers and having unequal loads on the two sides
   (*a*) both balancers run as motors
   (*b*) both balancers run as generators
   (*c*) balancer connected to heavily-loaded side runs as a motor
   (*d*) balancer connected to lightly-loaded side runs as a motor.
5. As compared to a dc 2-wire distributor, a 3-wire distributor with same maximum voltage to earth uses only...................... percent of copper.
   (*a*) 66.7 (*b*) 33.3
   (*c*) 31.25 (*d*) 150
6. In a d.c. 3-wire distribution system, balancer fields are cross-connected in order to
   (*a*) make both machines run as unloaded motors.
   (*b*) equalize voltages on the positive and negative outers
   (*c*) boost the generated voltage
   (*d*) balance loads on both sides of the neutral.

ANSWERS

1. (*c*) **2.** (*a*) **3.** (*b*) **4.** (*d*) **5.** (*c*) **6.** (*a*)

CHAPTER 41

Learning Objectives

- General Layout of the System
- Power System and System Networks
- Systems of A.C. Distribution
- Effect of Voltage on Transmission Efficiency
- Comparison of Conductor Materials Required for Various Overhead Systems
- Reactance of an Isolated Single-Phase Transmission Line
- A.C. Distribution Calculations
- Load Division Between Parallel Lines
- Suspension Insulators
- Calculation of Voltage Distribution along Different Units
- Interconnectors
- Voltage Drop Over the Interconnectors
- Sag and Tension with Support at Unequal Levels
- Effect of Wind and Ice

A.C. TRANSMISSION AND DISTRIBUTION

The above figure shows a mini AC power plant. AC has distinct advantages over DC in generation as well as transmission.

41.1. General Layout of the System

The conductor system by means of which electric power is conveyed from a generating station to the consumer's premises may, in general, be divided into two distinct parts *i.e.* transmission system and distribution system. Each part can again be sub-divided into two—primary transmission and secondary transmission and similarly, primary distribution and secondary distribution and then finally the system of supply to individual consumers. A typical layout of a generating, transmission and distribution network of a large system would be made up of elements as shown by a single-line diagram of Fig. 41.1 although it has to be realized that one or more of these elements may be missing in any particular system. For example, in a certain system, there may be no secondary transmission and in another case, when the generating station is nearby, there may be no transmission and the distribution system proper may begin at the generator bus-bars.

Now-a-days, generation and transmission is almost exclusively three-phase. The secondary transmission is also 3-phase whereas the distribution to the ultimate customer may be 3-phase or single-phase depending upon the requirements of the customers.

In Fig. 41.1, *C.S.* represents the central station where power is generated by 3-phase alternators at 6.6 or 11 or 13.2 or even 32 kV. The voltage is then stepped up by suitable 3-phase transformers for transmission purposes. Taking the generated voltage as 11 kV, the 3-phase transformers step it up to 132 kV as shown. Primary or high-voltage transmission is carried out at 132 kV*. The transmission voltage is, to a very large extent, determined by economic considerations. High voltage transmission requires conductors of smaller cross-section which results in economy of copper or aluminium. But at the same time cost of insulating the line and other expenses are increased. Hence, the economical voltage of transmission is that for which the saving in copper or aluminium is not offset ***(i)*** by the increased cost of insulating the line ***(ii)*** by the increased size of transmission-line structures and ***(iii)*** by the increased size of generating stations and sub-stations. A rough basis of determining the most economical transmission voltage is to use 650 volt per km of transmission line. For example, if transmission line is 200 km, then the most economical transmission voltage will be $200 \times 650 \cong 132{,}000$ V or 132 kV.

The 3-phase, 3-wire overhead high-voltage transmission line next terminates in step-down transformers in a sub-station known as Receiving Station (*R.S.*) which usually lies at the outskirts of a city because it is not safe to bring high-voltage overhead transmission lines into thickly-populated areas. Here, the voltage is stepped down to 33 kV. It may be noted here that for ensuring continuity of service ***transmission is always by duplicate lines***.

From the Receiving Station, power is next transmitted at 33 kV by underground cables (and occasionally by overhead lines) to various sub-stations (*SS*) located at various strategic points in the city. This is known as secondary or low-voltage transmission. From now onwards starts the primary and secondary distribution.

At the sub-station (*SS*) voltage is reduced from 33kV to 3.3kV 3-wire for primary distribution. Consumers whose demands exceeds 50 kVA are usually supplied from *SS* by special 3.3 kV feeders.

The secondary distribution is done at 400/230 V for which purpose voltage is reduced from 3.3 kV to 400 V at the distribution sub-stations. Feeders radiating from distribution sub-station supply power to distribution networks in their respective areas. If the distribution network happens to be at a great distance from sub-station, then they are supplied from the secondaries of distribution transformers which are either pole-mounted or else housed in kiosks at suitable points of the distribution networks. The most common system for secondary distribution is 400/230-V, 3-phase 4-wire system. The single-phase residential lighting load is connected between any one line and the neutral

* High voltages like 750 kV are in use in USSR (Konakovo-Moscow line) and 735 kV in Canada (Montreal-Manicoagan Scheme).

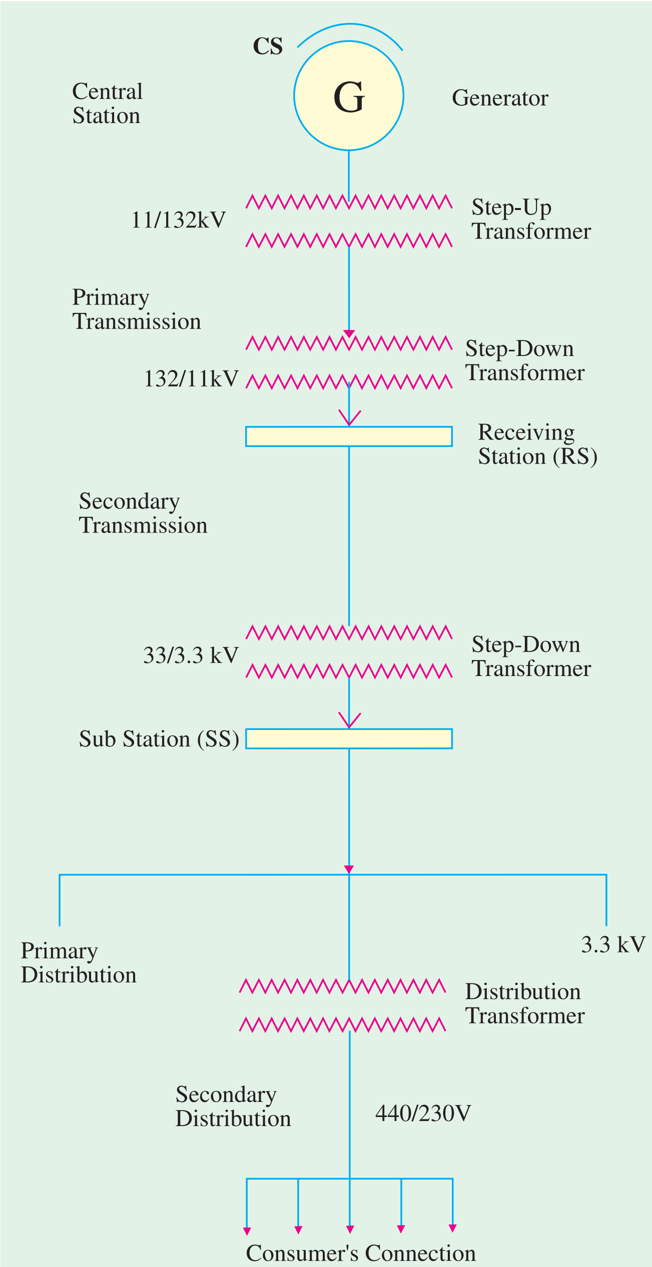


Fig. 41.1

whereas 3-phase, 400-V motor load is connected across 3-phase lines directly.

It should be noted that low-voltage distribution system is sub-divided into feeders, distributors and service mains. No consumer is given direct connection from the feeders, instead consumers are connected to distribution network through their service mains. The a.c. distributors are, in many ways, similar to the d.c. distributors as regards their constructional details and restrictions on drops in voltage.

Summarizing the above, we have

1. Generating voltage :
   6.6, 11, 13.2 or 33 kV.
2. High voltage transmission :
   220 kV, 132 kV, 66kV.
3. High voltage or primary
   distribution : 3.3, 6.6 kV.
4. Low-voltage distribution :
   A.C. 400/230, 3-phase, 4-wire
   D.C. 400/230 ; 3-wire system

The standard frequency for a.c. working is 50 Hz or 60 Hz (as in U.S.A.). For single-phase traction work, frequencies as low as 25 or 16 2/3Hz are also used.

41.2. Power Systems and System Networks

It is a common practice now-a-days to interconnect many types of generating stations (thermal and hydroelectric etc.) by means of a common electrical network and operate them all in parallel.

This combination of generating stations forms what is known as ***power system***. The various elements of such a system like generating stations, transmission lines, the substations, feeders and distributors etc. become tied into a whole by the integrated process of continuous generation and consumption of electric energy.

***A system network* (*or grid*)** is the name given to that part of power system which consists of the sub-stations and transmission lines of various voltage rating.

Fig. 41.2 shows single-line diagram representing the main connections of a power system consisting of a heating and power central station (*HCS*), a large-capacity hydro-electric station (*HS*) and

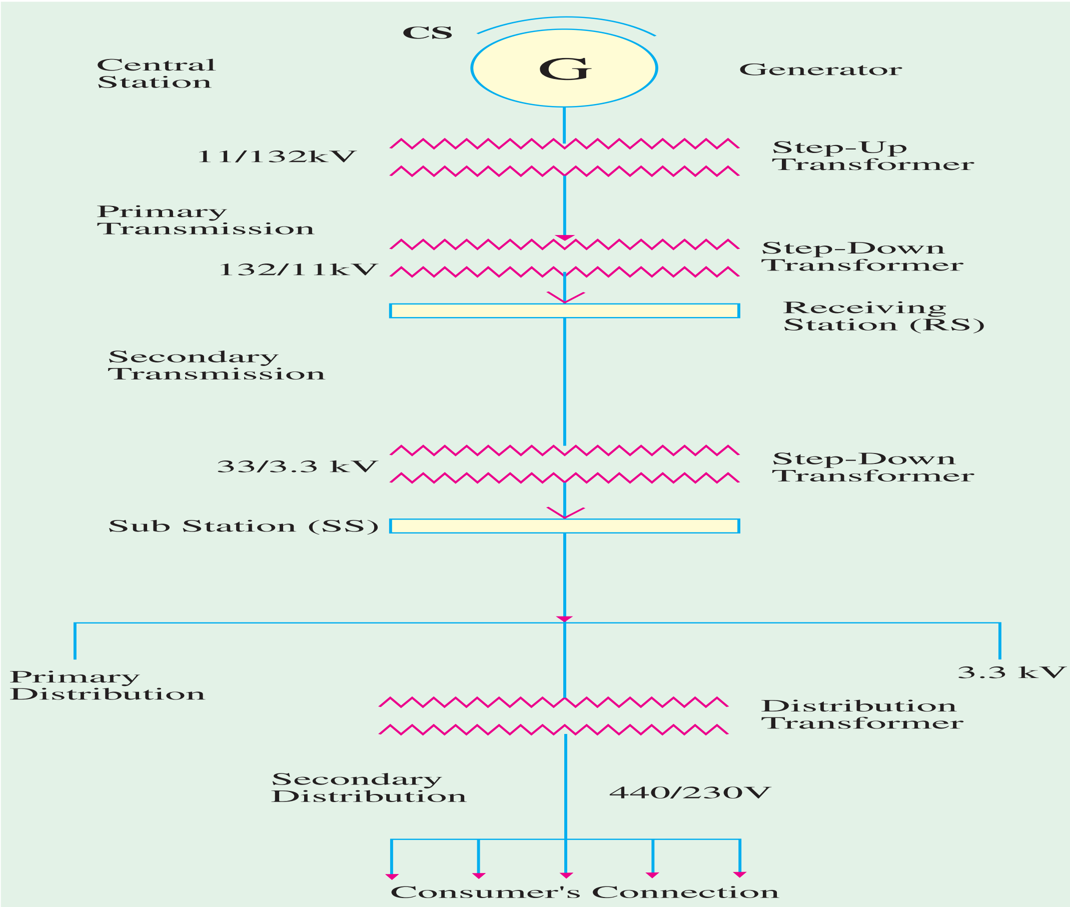


Fig. 41.2

two regional thermal power stations (*RTS*-1 and *RTS*-2). The stations *HS*, *RTS*-1 and *RTS*-2 are situated at large distances from the consumers, hence voltage of the electric power generated by them has to be stepped up by suitable transformers before it is fed into the system transmission network.

As shown in Fig. 41.2, *HS* is connected with the main 110-kV network of the system with the help of **(*i*)** two-220 kV transmission lines *L*-1 and **(*ii*)** main (regional) sub-station *A* which houses two 220/110 kV, 2-winding transformers for interconnecting the two transmission lines.

Transmission lines *L*-2, *L*-3 and *L*-4 constitute a high-voltage loop or ring mains. As seen, disconnnection of any one of the lines will not interrupt the connections between the elements of the system. Station *RTS*-1 feeds directly into the 110-kV line loop whereas *RTS*-2 is connected to the main network of the system by lines *L*-5 and *L*-6 through the buses of substations *B* and *C*. *HCS* is interconnected with 110-kV system through the buses of substation A by means of 10/110-kV transformers and line *L*-7.

It may be pointed out here that the main substations of the system are *A* and *B*. Substation *B* houses 3-winding transformers. The 35-kV supply is provided over quite large areas whereas 6-10 kV supply is supplied to consumers situated with a limited radius from the substations. Substations *C* and *D* are equipped with 2-winding transformers which operate at voltages indicated in the diagram. Substation C is known as a ***through substation*** whereas *D* is known as a *spur* or ***terminal*** substation.

Obviously, Fig. 41.2 shows only part of 220-kV and 110-kV lines and leaves out the 35, 10 and 6-kV circuits originating from the buses of the substations. Also, left out are low-voltage circuits for transmission and distribution (see Fig. 41.9).

41.3. Systems of A.C. Distribution

As mentioned earlier, a.c. power transmission is always at high voltage and mostly by 3-phase system. The use of single-phase system is limited to single-phase electric railways. Single-phase power transmission is used only for short distances and for relatively low voltages. As will be shown later, 3-phase power transmission requires less copper than either single-phase or 2-phase power transmission.

The distribution system begins either at the sub-station where power is delivered by overhead transmission lines and stepped down by transformers or in some cases at the generating station itself. Where a large area is involved, primary and secondary distributions may be used.

With respect to phases, the following systems are available for the distribution of a.c. power.

1. Single-phase, 2-wire system.
2. Single-phase, 3-wire system.
3. Two-phase, 3-wire system.
4. Two-phase, 4-wire system.
5. Three-phase, 3-wire system.
6. Three-phase, 4-wire system.

The above figure shows an electrical substation. A substation is a high-voltage electric system facility. It is used to switch generators, equipment, and circuits or lines in and out of a system. It is also used to change AC voltages from one level to another, and/or change alternating current to direct current or direct current to alternating current. Some substations are small with little more than a transformer and associated switches.

41.4. Single-phase, 2-wire System

It is shown in Fig. 41.3 (*a*) and (*b*). In Fig. 41.3 (*a*), one of the two wires is earthed whereas in Fig. 41.3 (*b*) mid-point of the phase winding is earthed.

41.5. Single-phase, 3-wire System

The 1-phase, 3-wire system is identical in principle with the 3-wire d.c. system. As shown in Fig. 41.4, the third wire or neutral is connected to the centre of the transformer secondary and earthed for protecting personnel from electric shock should the transformer insulation break down or the secondary main contact high voltage wire.

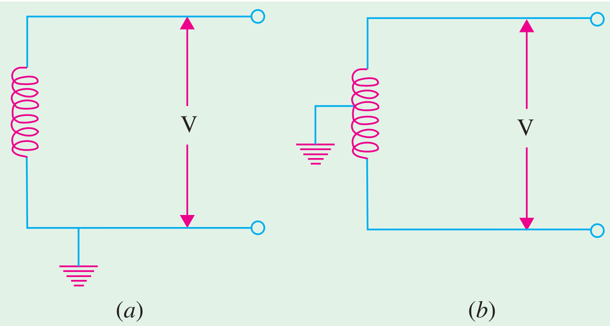


Fig. 41.3

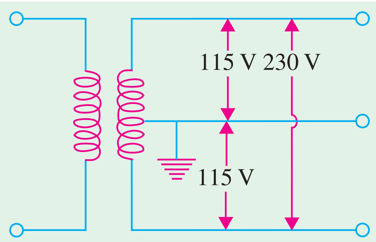


Fig. 41.4

41.6. Two-phase, 3-wire System

This system is still used at some places. The third wire is taken from the junction of the two-phase windings I and II, whose voltages are in quadrature with each other as shown in Fig. 41.5. If the voltage between the third or neutral wire and either of the two wires is V, then the voltage between the outer wires is V as shown. As compared to 2-phase, 4-wire system, the 3-wire system suffers from the defect that it produces voltage unbalance because of the unsymmetrical voltage drop in the neutral.

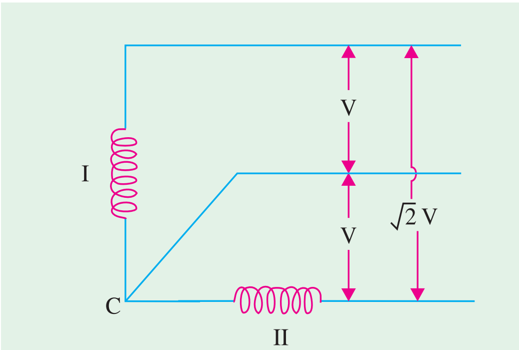


Fig. 41.5

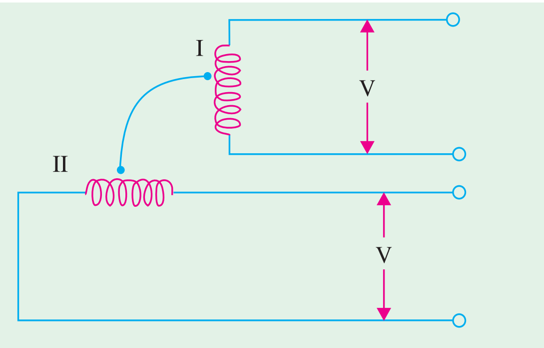


Fig. 41.6

41.7. Two-phase, 4-wire System

As shown in Fig. 41.6, the four wires are taken from the ends of the two-phase windings and the mid-points of the windings are connected together. As before, the voltage of the two windings are in quadrature with each other and the junction point may or may not be earthed. If voltage between the two wires of a phase winding be V, then the voltage between one wire of phase I and one wire of phase II is 0.707 V.

41.8. Three-phase, 3-wire System

Three-phase systems are used extensively. The 3-wire system may be delta-connected or star-connected whose star point is usually earthed. The voltage between lines is V in delta-connection and $\sqrt{3}\ V$ in case of star connection where V is the voltage of each phase as shown in Fig. 41.7 (*a*) and (*b*) respectively.

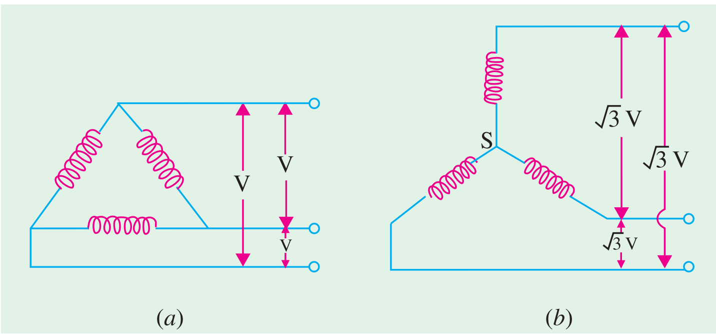


Fig. 41.7

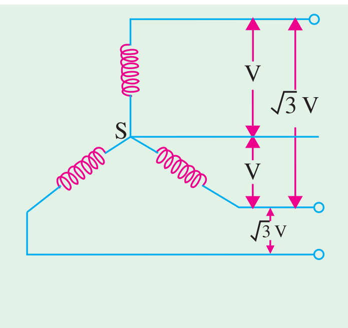


Fig. 41.8

41.9. Three-phase, 4-wire System

The 4th or neutral wire is taken from the star point of the star-connection as shown in Fig. 41.8 and is of half the cross-section of the outers or line conductors. If V is the voltage of each winding, then line voltage is $\sqrt{3}\ V$. Usually, phase voltage *i.e.* voltage between any outer and the neutral for a symmetrical system is 230 V so that the voltage between any two lines or outers is $\sqrt{3} \times 230 = 400$ V.

Single-phase residential lighting loads or single-phase motors which run on 230 V are connected between the neutral and any one of the line wires. These loads are connected symmetrically so that line wires are loaded equally. Hence, the resultant current in the neutral wire is zero or at least minimum. The three phase induction motors requiring higher voltages of 400 V or so are put across the lines directly.

41.10. Distribution

The distribution system may be divided into feeders, distributors, sub-distributors and service mains. As already explained in Art. 41.1, feeders are the conductors which connect the sub-station (in some cases the generating station) to the distributors serving a certain allotted area. From distributors various tappings are taken. The connecting link between the distributors and the consumers' terminals are the service mains. The essential difference between a feeder and a distributor is that whereas the current loading of a feeder is the same throughout its length, the distributor has a distributed loading which results in variations of current along its entire length. In other words, no direct tappings are taken from a feeder to a consumer’s premises.

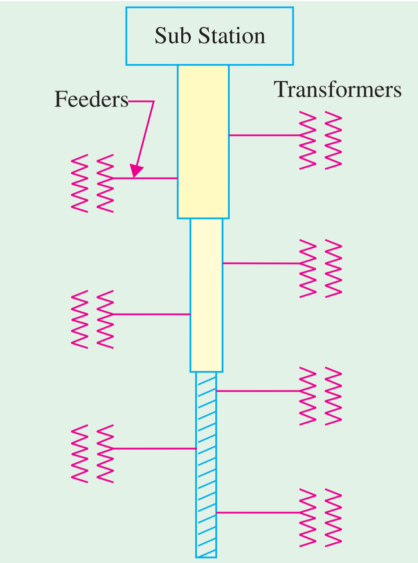


Fig. 41.9

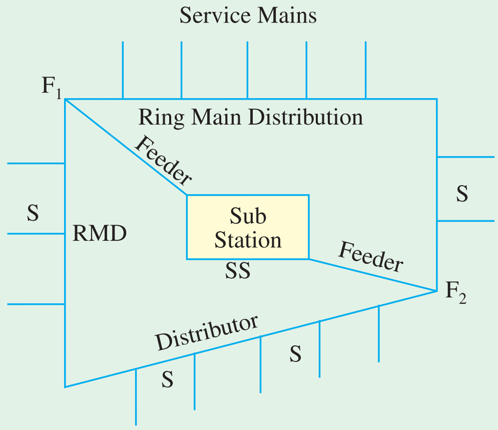


Fig. 41.10

In early days, ***radial*** distribution of tree-system type, as shown in Fig. 41.9, was used. In this system, a number of independent feeders branch out radially from a common source of supply *i.e.* a sub-station or generating station. The distribution transformers were connected to the taps along the length of the feeders. One of the main disadvantages of this system was that the consumer had to depend on one feeder only so that if a fault or breakdown occurred in his feeder, his supply of power was completely cut off till the fault was repaired. Hence, there was no absolute guarantee of continuous power supply.

For maintaining continuity of service, ring-main distributor (*R.M.D.*) system as shown in Fig. 41.10, is employed almost universally. *SS* represents the sub-station from which two feeders supply power to the ring-main distributor at feeding points F_1 and F_2. The ring-main forms a complete loop and has ***isolating*** switches provided at the poles at strategic points for isolating a particular section in case of fault. In this way, continuity of service can be maintained to other consumers on healthy sections of the ring-main. The number of feeders of the ring-main depends on **(*i*)** the nature of loading—heavy or light **(*ii*)** the total length of the *R.M.D.* and **(*iii*)** on the permissible/allowable drop of voltage. Service mains (*S*) are taken off at various points of the *R.M.D.* Sometimes sub-distributors are also used. Since a loop or closed ring-main can be assumed to be equivalent to a number of straight distributors fed at both ends, the voltage drop is small which results in economy of conductor material. The service mains are the connecting link between the consumer's terminals and the *R.M.D.* or sub-distributor.

41.11. Effect of Voltage on Transmission Efficiency

Let us suppose that a power of W watt is to be delivered by a 3-phase transmission line at a line voltage of *E* and power factor cos ϕ.

The line current $$I = \frac{W}{\sqrt{3}\,E\cos\phi}$$

Let l = length of the line conductor ; σ = current density

ρ = specific resistance of conductor material, A = cross-section of conductor

then $$A = \frac{I}{\sigma} = \frac{W}{\sqrt{3}\,E.\sigma.\cos\phi}$$

Now $$R = \frac{\rho l}{A} = \frac{\sqrt{3}\sigma\,\rho l\,E\cos\phi}{W}$$

Line loss $= 3 \times$ loss per conductor $= 3\,I^2R$

$$= 3\,\frac{W^2}{3E^2\cos^2\phi} \times \frac{\sqrt{3}\sigma\,\rho l\cos\phi}{W} = \frac{\sqrt{3}\sigma\rho l W}{E\cos\phi} \qquad \text{...(1)}$$

Line intake or input $$= \text{output} + \text{losses} = W + \frac{\sqrt{3}\rho\sigma l W}{E\cos\phi} = W\left(1 + \frac{\sqrt{3}\sigma\rho l}{E\cos\phi}\right)$$

∴ efficiency of transmission

$$= \frac{\text{output}}{\text{input}} = \frac{W}{W\left(1 + \dfrac{\sqrt{3}\sigma\rho l}{E\cos\phi}\right)} = \left(1 - \frac{\sqrt{3}\sigma\rho l}{E\cos\phi}\right)\text{approx} \qquad \text{...(2)}$$

Voltage drop per line $$= IR = \frac{\sqrt{3}\,\sigma\rho l\,E\cos\phi}{W} \times \frac{W}{\sqrt{3}\,E\cos\phi} = \sigma\rho l \qquad \text{...(3)}$$

Volume of copper $$= 3\,lA = \frac{3\,Wl}{\sqrt{3}\,\sigma E\cos\phi} = \frac{\sqrt{3}\,Wl}{\sigma E\cos\phi} \qquad \text{...(4)}$$

For a given value of transmitted power W, line length *l*, current density σ and specific resistance ρ of the given conductor material, the effect of supply voltage on transmission can be seen as follows :

1. From equation (1), line loss is inversely proportional to *E*. It is also inversely proportional to power factor, cos ϕ.
2. Transmission efficiency increases with voltage of transmission and power factor as seen from equation (2).
3. As seen from equation (3) for a given current density, the resistance drop per line is constant (since ρ and *l* have been assumed fixed in the present case). Hence, percentage drop is decreased as *E* is increased.
4. The volume of copper required for a transmission line is inversely proportional to the voltage and the power factor as seen from equation (4).

It is clear from the above that for long distance transmission of a.c. power, high voltage and high power factors are essential. But as pointed out earlier in Art. 41.1, economical upper limit of voltage is reached when the saving in cost of copper or aluminium is offset by the increased cost of insulation and increased cost of transformers and high-voltage switches etc. Usually, 650 volt per route km is taken as a rough guide.

41.12. Comparison of Conductor Materials Required for Various Overhead Systems

We will now calculate the amounts of conductor material required for various systems of a.c. power transmission. To do it without prejudice to any system, we will make the following assumptions :

1. Amount of power transmitted by each system is the same.
2. Distance of transmission is the same in each case.
3. Transmission efficiency is the same *i.e.* the losses are the same in each case.
4. Loads are balanced in the case of 3-wire systems.
5. Cross-section of the neutral wire is half that of any outer.
6. Maximum voltage to earth is the same in all cases.

By way of illustration a few cases will be compared, although the reader is advised to attempt others as well.

(*i*) Two-wire d.c. System and Three-phase, 3-wire System

Let in both cases, E be the maximum voltage between conductors and W the power transmitted.

In d.c. system, let R_1 be the resistance of each conductor and I_1 the current, then $I_1 = (W/E)$, hence losses in the two conductors are $= 2I_1^2 R_1 = 2\,(W/E)^2 R_1$ ...(*i*)

For a.c. system, let resistance per conductor be R_2 and power factor cos ϕ, then since E is the maximum voltage, the $R.M.S.$ voltage is $E/\sqrt{2}$.

Current in each line, $$I_2 = \frac{W}{\sqrt{3}\,(E/\sqrt{2})\cos\phi}$$

Total losses in three conductors are,

$$3I_2^2 R_2 = \left[\frac{W}{\sqrt{3}\,(E/\sqrt{2})\cos\phi}\right]^2 R_2 = \frac{2W^2}{E^2}\times\frac{1}{\cos^2\phi}\,.\,R_2 \qquad ...(ii)$$

Since transmission efficiency and hence losses are the same, therefore equating (*i*) and (*ii*), we get

$$\frac{2W^2R_1}{E^2} = \frac{2W^2}{E^2}\cdot\frac{1}{\cos^2\phi}R_2 \quad \text{or} \quad \frac{R_1}{R_2} = \frac{1}{\cos^2\phi}$$

Since area of cross-section is inversely proportional to the resistance.

$$\therefore \qquad \frac{\text{area of one a.c. conductor}}{\text{area of one d.c. conductor}} = \frac{1}{\cos^2\phi}$$

Now, for a given length, volumes are directly proportional to the areas of cross-section.

$$\therefore \qquad \frac{\text{volume of one a.c. conductor}}{\text{volume of one d.c. conductor}} = \frac{1}{\cos^2\phi}$$

Keeping in mind the fact, that there are two conductors in d.c. system and three in a.c. system under consideration, we have

$$\frac{\text{total volume in 3-wire a.c. system}}{\text{total volume in 2-wire d.c. system}} = \frac{1}{\cos^2\phi}\cdot\frac{3}{2} = \frac{1.5}{\cos^2\phi}$$

(*ii*) Three-phase, 4-wire and 3-wire d.c. Systems

The neutral conductor of each system is earthed. Let E be the maximum voltage to earth and W the power to be transmitted in both cases.

For d.c. 3-wire system, voltage between outers is $2E$. If I_1 and R_1 are the current in and resistance of each conductor, then $I_1 = W/2E$. Assuming a balanced load (in which case there would be no current flowing in the neutral conductor), the value of loss in two conductors is

$$2I_1^2R_1 = 2(W/2E)^2R_1 = W^2R_1/2E^2$$

In the case of a.c. system, if E is the maximum voltage between any wire and neutral, then its R.M.S. value is $E/\sqrt{2}$. Hence, the line voltage = $\sqrt{3}\,.\,E/\sqrt{2}$. The line current is

$$I_2 = \frac{W}{\sqrt{3}\times(\sqrt{3}\,E/\sqrt{2})\cos\phi}$$

where $\cos\phi$ is the power factor. If R_2 is the resistance of each line, then total loss in 3 lines is

$$= 3I_2^2R_2 = 3\times\left[\frac{W}{\sqrt{3}\,(\sqrt{3}E/\sqrt{2})\cos\phi}\right]^2\times R_2 = \frac{2W^2R}{3E^2\cos^2\phi}$$

For equal transmission efficiencies, the two losses should be the same.

$$\therefore \qquad \frac{W^2R_1}{2E^2} = \frac{2W^2R_1}{3E^2\cos^2\phi} \quad \text{or} \quad \frac{R_1}{R_2} = \frac{4}{3\cos^2\phi}$$

Since volume is directly proportional to the area of cross-section which is itself inversely proportional to the resistance, hence

$$\frac{\text{volume of one a.c. conductor}}{\text{volume of one d.c. conductor}} = \frac{4}{3\cos^2\phi}$$

Assuming the neutral wires of each system to have half the cross-section of the outers, there is 3.5 times the quantity of one conductor in a.c. and 2.5 times in the d.c. system. Hence

$$\frac{\text{total volume in the a.c. system}}{\text{total volume in the d.c. system}} = \frac{3.5}{2.5}\times\frac{4}{3\cos^2\phi} = \frac{1.867}{\cos^2\phi}$$

In Table 41.1 are given the ratios of copper in any system as compared with that in the corresponding d.c. 2-wire system which has been alloted a basic number of 100.

Table No. 41.1

System	*With same maximum voltage to earth*	*With same maximum voltage between conductors*
D.C. 2-wire	100	100
D.C. 2-wire Mid-point earthed	25	100
D.C. 3-wire Neutral = ½ outer	31.25	125
D.C. 3-wire Neutral = ½ outer	37.5	150
Single-phase, 2-wire	$200/\cos^2\phi$	$200/\cos^2\phi$
Single-phase, 2-wire Mid-point earthed	$50/\cos^2\phi$	$200/\cos^2\phi$
Single-phase, 3-wire Neutral = ½ outer	$62.5/\cos^2\phi$	$250/\cos^2\phi$
Two-phase, 4-wire	$50/\cos^2\phi$	$200/\cos^2\phi$

Two-phase, 3-wire	$146/\cos^2\phi$	$291/\cos^2\phi$
Three-phase, 3-wire	$50/\cos^2\phi$	$150/\cos^2\phi$
Three-phase, 4-wire Neutral = outer	$67/\cos^2\phi$	$200/\cos^2\phi$

A comparison of costs for underground systems employing multicore cables can also be worked out on similar lines. In these case, however, maximum voltage between the conductors is taken as the criterion.

It would be clear from the above discussion, that for a.c. systems, the expression for volume contains $\cos^2\phi$ in denominator. Obviously, reasonable economy in copper can be achieved only when $\cos\phi$ approaches unity as closely as possible. For equal line loss, the volume of copper required for $\cos\phi = 0.8$ will be $1/0.8^2 = 1.6$ times that required for $\cos\phi = 1$.

Since the cost of distribution system represents a large percentage of the total capital investment of power supply companies, it is quite reasonable that a consumer should be penalized for low power factor of his load. This point is fully dealt with in Chapt. 47.

Example 41.1. *A 3-phase, 4-wire system is used for lighting. Compare the amount of copper required with that needed for a 2-wire D.C. system with same line voltage. Assume the same losses and balanced load. The neutral is one half the cross-section of one of the respective outers.*

Solution. (*a*) Two-wire *DC* Let V = voltage between conductors

P = power delivered, R_1 = resistance/conductor

Current $I_1 = P/V$

$$\text{power loss} = 2I_1^2R_1 = 2P^2R_1/V^2$$

(*b*) Three-phase, 4-wire

Let V be the line-to-neutral voltage and R_2 resistance of each phase wire.

$$P = 3VI_2\cos\phi = 3VI_2 \qquad \text{—if } \cos\phi = 1$$

$$\text{Power loss} = 3I_2^2R_2 = 3(P/3V)^2R_2 = P^2R_2/3V^2$$

Since power losses in both systems are equal

$$\therefore \qquad 2P^2R_1/V^2 = P^2R_2/3V^2 \quad \text{or} \quad R_1/R_2 = 1/6$$

If A_1 and A_2 are the cross-sectional areas of conductors in the two systems, then $A_1/A_2 = 6$. Because $R \propto l/A$

$$\therefore \qquad \text{Cu reqd. for 2-wire system} = 2A_1l$$

$$\text{Cu reqd. for 3-}\phi\text{, 4-wire system} = (3A_2l + A_2l/2)$$

$$\therefore \qquad \frac{\text{Cu for } 3-\phi \text{ system}}{\text{Cu for d.c. system}} = \frac{3.5\,A_2l}{2\,A_1l} = \frac{3.5}{2}\times\frac{1}{6} = 0.292$$

Cu for 3-ϕ system = **0.292 × Cu for d.c. system.**

Example 41.2. *Estimate the weight of copper required to supply a load of 100 MW at upf by a 3-phase, 380-kV system over a distance of 100 km. The neutral point is earthed. The resistance of the conductor is 0.045 ohm/cm²/km. The weight of copper is 0.01 kg/cm³. The efficiency of transmission can be assumed to be 90 percent.* **(Power Systems, AMIE, Sec. B, 1994)**

Solution. Power loss in the line = (1 – 0.9) × 100 = 10 MW

$$\text{Line current} = 100\times10^6/\sqrt{3}\times380\times10^3\times1 = 152\text{ A}$$

Since I^2R loss in 3-conductors is 10×106 W, loss per conductor is $= 10\times10^6/3 = 3.333\times10^6$ W

$$\text{Resistance per conductor} = 10 \times 10^6/3 \times 152^2 = 144.3\ \Omega$$

Resistance per conductor per km = 144.3/100 = 1.443 Ω

∴ Conductor cross-section = 0.045/1.443 = 0.03 m^3

Volume of copper per meter run = 0.03 × 100 = 3 cm^3

Weight of copper for 3-conductor for 100 km length = 3 × (3 × 0.01) × 100 × 1000 = **9000 kg**

Example 41.3. *A d.c. 2-wire distribution system is converted into a.c. 3-phase, 3-wire system by adding a third conductor of the same size as the two existing conductors. If voltage between conductors and percentage power loss remain the same, calculate the percentage additional balanced load which can now be carried by the conductors at 0.95 p.f.*

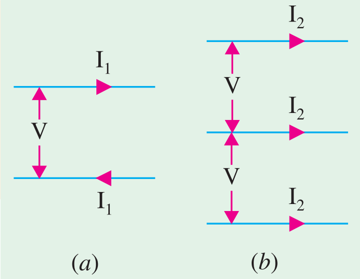


Fig. 41.11

Solution. (*a*) *DC* 2-wire system [Fig. 40.11 (*a*)]

If R is the resistance per conductor, then power transmitted is $P = VI_1$ and power loss = $2I_1 2R$. Percentage power loss

$$= 2I_1^2 R \times 100/VI_1$$
$$= 2I_1 R \times 100/V$$

(*b*) as 3-phase, 3-wire system [Fig. 40.11 (*b*)]

$$P_2 = \sqrt{3}\, VI_2 \cos\phi, \text{ power loss} = 3I_2^2 R$$
$$\%\text{ power loss} = 3I_2^2 R/P_2 = \sqrt{3} I_2 R \times 100\ V \cos\phi$$

Since losses in the two cases are the same

$$\therefore \quad 2I_1 R \times 100/V = \sqrt{3} I_2 R \times 100/V \cos\phi \text{ or } \quad I_2 = 2\cos\phi \times I_1/\sqrt{3}$$
$$\therefore \quad P_2 = \sqrt{3}.V.2\cos^2\phi \times I_1/\sqrt{3} = 2VI_1 \cos^2\phi = 2VI_1 (0.95)^2 = 1.8\ VI_1$$

Percentage additional power transmitted in a 3-phase, 3-wire system

$$= \frac{P_2 - P_1}{P_1} \times 100 = \frac{1.8\,VI_1 - VI_1}{VI_1} \times 100 = \mathbf{80\%}$$

Example 41.4. *A 2-phase, 3-wire a.c. system has a middle conductor of same cross-sectional area as the outer and supplies a load of 20 MW. The system is converted into 3-phase, 4-wire system by running a neutral wire. Calculate the new power which can be supplied if voltage across consumer terminal and percentage line losses remain the same. Assume balanced load.*

Solution. The two systems are shown in Fig. 41.12. Let R be the resistance per conductor.

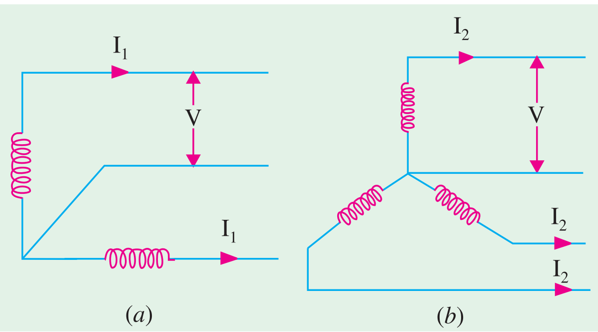


Fig. 41.12

$P_1 = 2VI_1 \cos\phi$; Cu loss, $W_1 = 2I_1^2 R$

Percentage Cu loss

$$= (W_1/P_1) \times 100$$
$$= (2I^2 R/2VI_1 \cos\phi) \times 100$$
$$= I_1 R \times 100/V \cos\phi$$
$$P_2 = 3VI_2 \cos\phi$$
$$W_2 = 3I_2^2 R$$
$$\%\text{ line loss} = (W_2/P_2) \times 100$$
$$= (3I_2^2 R/3VI_2 \cos\phi) \times 100$$
$$= I_2 R \times 100/V \cos\phi$$

Since percentage line losses are the same in both cases

$$\therefore \quad I_1 R \times 100/V \cos\phi = I_2 R \times 100/V \cos\phi \qquad \therefore \quad I_1 = I_2$$

Now, $$P_1 = 2VI_1 \cos\phi = \mathbf{20\ MW}\ ; \qquad \therefore \quad VI_1 \cos\phi = 10\text{ MW}$$

$$P_2 = 3VI_2 \cos\phi = 3VI_1 \cos\phi = 3 \times 10 = \mathbf{30\ MW}$$

41.13. Constants of a Transmission Line

A transmission line not only has an ohmic resistance but also inductance and capacitance between its conductors. These are known as the constants of a transmission line. While calculating the drop in a.c. transmission and distribution circuits, we will have to consider ***(i)*** resistive or ohmic drop—in phase with the current ***(ii)*** inductive drop—leading the current by 90° and ***(iii)*** the capacitive drop and charging current taken by the capacitance of the line. The capacitance and hence the charging current is usually negligible for short transmission lines.

41.14. Reactance of an Isolated Single-Phase Transmission Line

In Fig. 41.13 (*a*) are shown the cross-sections of two conductors of a single-phase line spaced *D* from centre to centre.

Since currents through the two conductors will, at any time, always be flowing in opposite directions, the fields surrounding them always reinforce each other in the space between them as shown in Fig. 41.13.

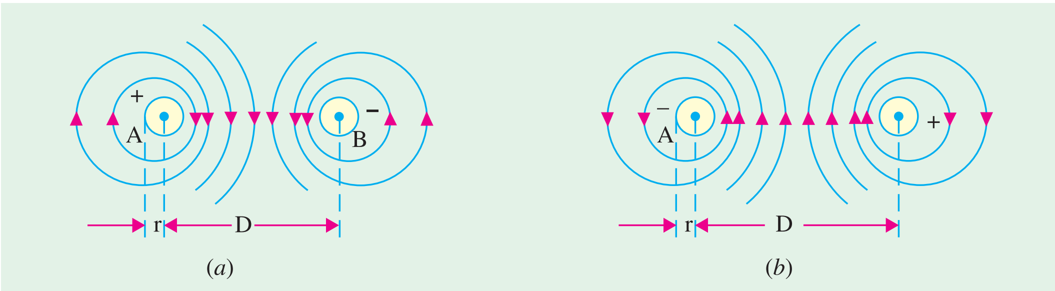


Fig. 41.13

The two parallel conductors form a rectangular loop of one turn through which flux is produced by currents in the two conductors. Since this flux links the loop, the loop possesses inductance. It might be thought that this inductance is negligible because the loop has only one turn and the entire flux-path lies through air of high reluctance. But as the cross-sectional area of the loop is large, from 1 to 10 metre wide and several km long, even for a small flux density, the total flux linking the loop is large and so inductance is appreciable.

It can be proved that inductance per loop metre (when $r \le D$) is ...***(i)***

$$L = \frac{\mu}{\pi} \log_e D/r + \frac{\mu_i}{4\pi} \text{ henry/metre}$$

where μ = absolute permeability of the surrounding medium

μ_i = absolute permeability of the conductor material

Now, $\mu = \mu_0\,\mu_r$ and $\mu_i = \mu_0\,\mu_r$ where μ_r and μ_r' are the relative permeabilities of the surrounding medium and the conductor material. If surrounding medium is air, then $\mu_r = 1$. Also, if conductor is made of copper, then $\mu_r' = 1$. Hence, the above expression becomes.

$$L = \left(\frac{\mu_0\,\mu_r}{\pi} \log h\, D/r + \frac{\mu_0\,\mu_r'}{4\pi}\right) \text{ H/m}$$

$$= \frac{\mu_0}{4\pi}\,(\log h\,D/r + 1)\ H/m = 10^{-7}\,(1 + \log h\,Dr)\ \text{H/m}$$

Loop reactance $X = 2\pi f L$ ohm/metre

Obviously, the inductance of each single conductor is half the above value*.

$$\text{inductance/conductor} = \frac{1}{2}\,(1 + \log h\,D/r) \times 10^{-7}\ \text{H/m} \quad ...(ii)$$

$$\text{reactance/conductor} = 2\,\pi\,f \times \frac{1}{2}\,(1 + \log h\,D/r) \times 10^{-7}\ \Omega/\text{m}$$

41.15. Reactance of 3-phase Transmission Line

In 3-phase transmission, it is more convenient to consider the reactance of each conductor instead of the looped line or of the entire circuit. Two cases will be considered for 3-phase lines.

(*i*) Symmetrical Spacing

In Fig. 41.14 (*a*) are shown the three conductors spaced symmetrically *i.e.* each conductor is at the apex of the same equilateral triangle *ABC* of side *D*. The inductance per conductor per metre is found by using equation **(*ii*)** in Art 41.14 because it can be shown that inductance per km of one conductor of 3-phase circuit is the same as the inductance per conductor per km of single-phase circuit with equivalent spacing.

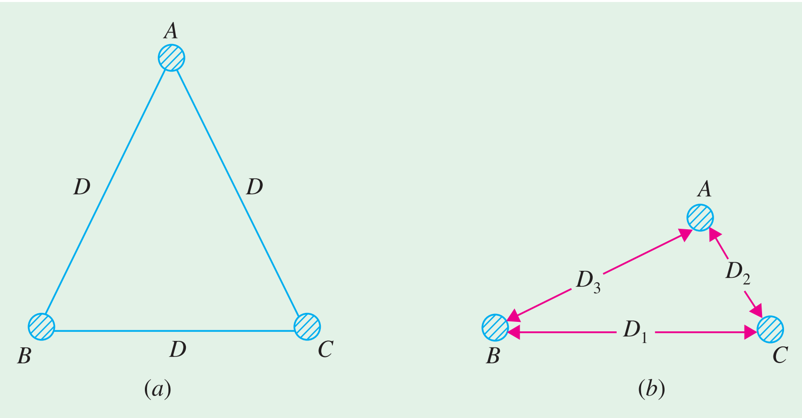


Fig. 41.14

(*ii*) Unsymmetrical Spacing

In Fig. 41.14 (*b*) is shown a 3-phase circuit with conductors placed unsymmetrically. In this case also, the inductance is given by equation **(*ii*)** of Art. 41.14 with the only modification that *D* is put equal to $\sqrt[3]{(D_1\,D_2\,D_3)}$.

41.16. Capacitance of a Single-phase Transmission Line

We know that any two conductors which are separated by an insulating medium constitute a capacitor. When a potential difference is established across two such conductors, the current flows in at one conductor and out at the other so long as that p.d. is maintained. The conductors of an overhead transmission line fulfil these conditions, hence when an alternating potential difference is applied across a transmission line, it draws a leading current even when it is unloaded. This leading current is in quadrature with the applied voltage and is known as the ***charging current***. Its value depends upon voltage, the capacitance of the line and the frequency of alternating current.

As shown in Art 5.10, the capacitance between conductors of a single-phase line is approximately given by

* It may be noted that standard conductors have a slightly higher inductance.

$$C = \frac{\pi \in}{\log h\, D/r} = \frac{\pi \in_0 \in_r}{2.3 \log_{10} D/r} \quad F/m = \frac{0.0121 \in_r}{\log_{10} D/r}\ \mu F/\text{km}$$

Here, D is the distance between conductor centres and r the radius of each conductor, both expressed in the same units (Fig. 41.15).

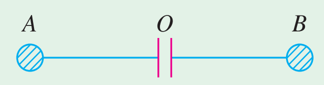


Fig. 41.15

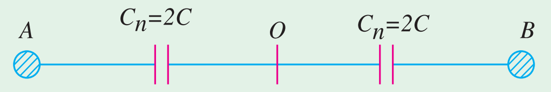


Fig. 41.16

As shown in Fig. 41.16, the capacitance to neutral $C_n = 2C$ where point O is the neutral of the system. Obviously, the total capacitance between conductors A and B is given by finding the resultant of two capacitances each of value C_n joined in series, the resultant, obviously, being equal to C.

It is important to remember that if capacitance to neutral is used for calculating the charging current, then voltage to neutral must also be used.

$$\therefore \quad C_n = 2C = \frac{2\,\pi \in}{\log_e D/r}\ \text{F/m} \qquad \ldots(ii)$$

$$\therefore \quad \text{line charging current} = \frac{V}{X_C} = \frac{V}{1/(2\pi f C_n)} = 2\pi\ f C_n V \ \text{A/m}$$

where V is the ***voltage to neutral***.

However, it may be noted that ground effect has been neglected while deriving the above expression. This amounts to the tacit assumption that height h of the conductors is very large as compared to their spacing 'd'. In case ground effect is to be taken into account, the expression for capacitance becomes.

$$C_n = \frac{2\,\pi \in}{\log h \dfrac{d}{r\sqrt{1 + d^2/4h^2}}}\ \text{F/m} \qquad \text{(Ex. 41.7)}$$

Example 41.5. *What is the inductance per loop metre of two parallel conductors of a single phase system if each has a diameter of 1 cm and their axes are 5 cm apart when conductors have a relative permeability of* ***(a)*** *unity and* ***(b)*** *100. The relative permeability of the surrounding medium is unity in both cases. End effects may be neglected and the current may be assumed uniformly distributed over cross-section of the wires.*

Solution. (*a*) Here, $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m ; $\mu_i = 1$

$$\therefore \quad L = \frac{4\pi \times 10^{-7}}{\pi}\left(\log h^5/0.5 + \frac{1}{4}\right) = \mathbf{1.02\ \mu H/m}$$

(*b*) Here, $\mu = \mu_0 = 4\pi \times 10^{-7}$; $\mu_i = 100\ \mu_0$

$$L = \frac{4\pi \times 10^{-7}}{\pi}\left(\log h^5/0.5 + \frac{100}{4}\right) = \mathbf{10.9\ \mu H/m}$$

Example 41.6. *A 20-km single-phase transmission line having 0.823 cm diameter has two line conductors separated by 1.5 metre. The conductor has a resistance of 0.311 ohm per kilometre. Find the loop impedance of this line at 50 Hz.* **(Gen. Trans. & Dist. Bombay Univ. 1992)**

Solution. Loop length $= 20 \text{ km} = 2 \times 10^4$ m

Total loop inductance is $L = 2 \times 10^4 \left(\frac{\mu}{\pi} \log_e D/r + \frac{\mu_i}{4\pi}\right)$ henry

Here, $D = 1.5$ m ; $r = 0.823/2 = 0.412$ cm $= 4.12 \times 10^{-3}$ m

Assuming $\mu_r = 1$ for copper conductors since they are non-magnetic and also taking $\mu_r = 1$ for air, we have

$$L = 2\times10^4\left(\frac{\mu}{\pi}\log_e D/r + \frac{\mu_i}{4\pi}\right) = \frac{2\times10^4\times4\pi\times10^{-7}}{\pi}\left(\log_e^{1.5/4.12\times10^{-3}} + \frac{1}{4}\right)\text{ H}$$

$$= 8\times10^{-3}\,(5.89 + 0.25) = 49.12\times10^{-3}\text{ H}$$

Reactance $X = 2\pi\times50\times49.12\times10^{-3} = 15.44\ \Omega$; Loop resistance $= 2\times20\times0.311 = 12.44\ \Omega$

$$\text{Loop impedance} = \sqrt{12.44^2 + 15.44^2} = \mathbf{19.86\ \Omega}$$

Example 41.7. *The conductors in a single-phase transmission line are 6 m above ground. Each conductor has a diameter of 1.5 cm and the two conductors are spaced 3 m apart. Calculate the capacitance per km of the line **(i)** excluding ground effect and **(ii)** including the ground effect.*

Solution. The line conductors are shown in Fig. 41.17.

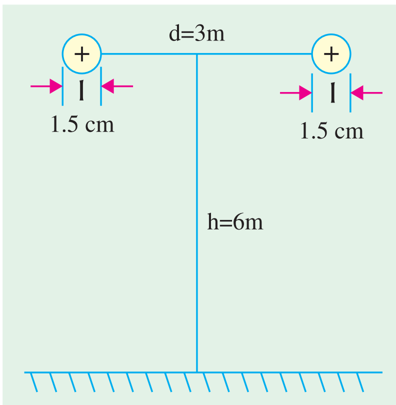


Fig. 41.17

(i)
$$C_n = \frac{2\,\pi\epsilon_0}{\log h\,d/r} = \frac{2\pi\times8.854\times10^{-12}}{\log h\,3/0.75\times10^{-2}}\text{ F/m}$$

$$= 9.27\times10^{-12}\text{ F/m} = \mathbf{9.27\times10^{-3}\ \mu F/km}$$

(ii) In this case,

$$C_n = \frac{2\,\pi\epsilon_0}{\log h\dfrac{d}{r\sqrt{1+d^2/4h^2}}}\text{ F/m}$$

$$= \frac{2\pi\times8.854\times10^{-12}}{\log h\dfrac{3}{0.75\times10^{-12}\sqrt{1+3^2/4\times6^2}}}$$

$$= 9.33\times10^{-12}\text{ F/m} = \mathbf{9.33\times10^{-3}\ mF/km}$$

It is seen that line capacitance when considering ground effect is more (by about 0.64% in the present case).

41.17. Capacitance of a Three-phase Transmission Line

In Fig. 41.18 (*a*) are shown three conductors spaced symmetrically and having a capacitance of C between each pair. The value of ***C i.e. line-to-line*** capacitance is the same as given by equation ***(i)*** in Art. 41.16.

Considering the two conductors A and B, their line capacitance C can be regarded as consisting of two line-to-neutral capacitances connected in series, so that for the line-to-neutral capacitance of A with respect to neutral plane 1, as in Art. 41.16, we have

$$C_{n1} = \frac{2\pi\epsilon}{\log_e D/r}\text{ F/m}$$

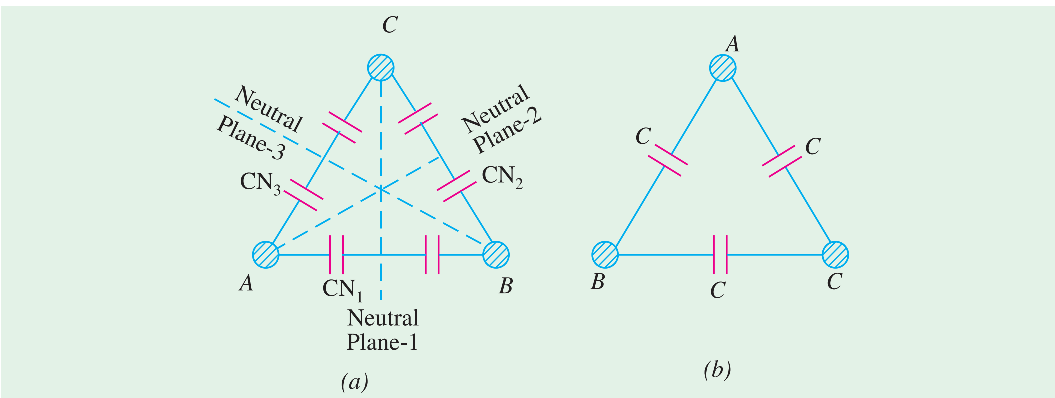


Fig. 41.18

Similarly, line-to-neutral capacitance of conductor A with respect to neutral plane 3 is

$$C_{n3} = \frac{2\pi \in}{\log_e D/r} \text{ F/m}$$

The vector sum of these capacitances is equal to either because their phase angle is 120º *i.e.* the phase angle of the voltages which cut across them.

Hence, the capacitance to neutral per metre of one conductor of a 3-phase transmission line is

$$C_n = \frac{2\pi \in}{\log_e D/r} \text{ F/m}$$

In case the conductors are placed asymmetrically, the distance used is $D = \sqrt[3]{(D_1 D_2 D_3)}$.

41.18. Short Single-phase Line Calculations

Single-phase circuits are always short and work at relatively low voltages. In such cases, the line constants can be regarded as 'lumped' instead of being distributed and capacitance can be neglected.

Let E_S = voltage at sending end ; E_R = voltage at receiving end
I = line current $\cos\phi_R$ = power factor at receiving end
R = resistance of both conductors; X = reactance of both conductors = ωL

Then, the equivalent circuit of a short single-phase line is as shown in Fig. 41.19.

Resistive drop = IR —in phase with I
Reactive drop = IX —in quadrature with I

The vector diagram is shown in Fig. 41.20. From the right-angled triangle OMA, we get

$$OM^2 = OA^2 + AM^2 = (OK + KA)^2 + (AC + CM)^2$$

$$E_S = \sqrt{[(E_R \cos\phi_R + IR)^2 + (E_R \sin\phi_R + IX)^2]}$$

An approximate expression for the voltage drop is as follows :

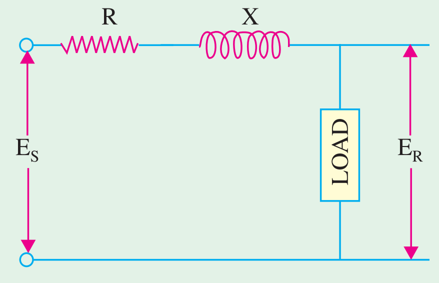


Fig. 41.19

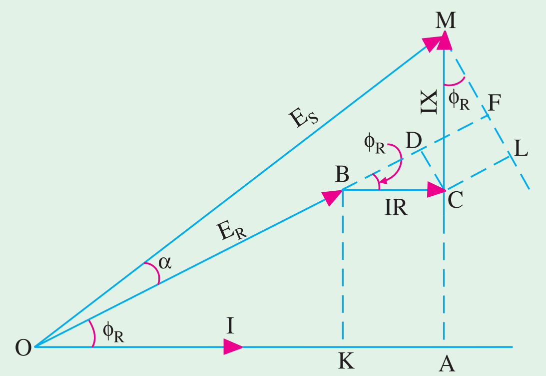


Fig. 41.20

Draw the various dotted lines as in Fig. 41.20. Here, ML is $\perp$ OF, CL is $\|$ OF and CD is $\perp$ OF. As seen

$$OM = OF \text{ (approximately).}$$
$$= OD + DF = OB + BD + DF$$

or $$E_S = E_R + BD + DF = E_R + BD + CL = E_R + IR\cos\phi_R + IX\sin\phi_R$$

or $$E_S - E_R = IR\cos\phi_R + IX\sin\phi_R \quad \therefore \text{ drop} = I(R\cos\phi_R + X\sin\phi_R) \text{ (approx.)}$$

$$\text{Voltage regn. of line} = [(E_S - E_R)/E_R] \times 100$$

Solution in Complex Notation

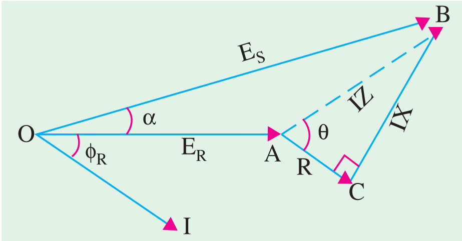


Fig. 41.21

Let us take E_R as reference vector as shown in Fig. 41.21 so that $E_R = (E_R + j_0)$

As seen from Δ *OAB*, E_S is equal to the vector sum of E_R and *IZ* or $\mathbf{E_S = E_R + IZ}$

Now, $\quad \mathbf{I} = I \angle -\phi_R = I(\cos\phi_R - j\sin\phi_R)$

Similarly, $\quad \mathbf{Z} = Z\angle\theta = (R + jX)$

$$\therefore \quad \mathbf{E_S} = \mathbf{E_R} + \mathbf{I_Z}\angle\theta - \phi_R$$

$$\text{or} \quad \mathbf{E_S} = \mathbf{E_R} + \mathbf{I}(\cos\phi_R - j\sin\phi_R)(R + jX)$$

$$= \mathbf{E_R} + (IR\cos\phi_R + IX\sin\phi_R) + j(IX\cos\phi_R - I_R\sin\phi_R)^2$$

$$\therefore \quad E_S = \sqrt{(E_R + IR\cos\phi_R + IX\sin\phi_R)^2 + (IX\sin\phi_R - IR\sin\phi_R)^2}$$

If the p.f. is leading, then

$$\mathbf{I} = I\angle\phi_R = I(\cos\phi_R + j\sin\phi_R)$$

$$\therefore \quad \mathbf{E_S} = \mathbf{E_R} + \mathbf{IZ}\angle\theta + \phi_R = \mathbf{E_R} + I(\cos\phi_R + j\sin\phi_R)(R + jX)$$

It may be noted that the sending end power factor angle is $\phi_s = (\phi_R + \alpha)$.

Example 41.8. *A single-phase line has an impedance of $5\angle 60°$ and supplies a load of 120 A, 3,300 V at 0.8 p.f. lagging. Calculate the sending-end voltage and draw a vector diagram.*

(City & Guides, London)

Solution. The vector diagram is similar to that shown in Fig. 41.21. Here

$$\mathbf{E_R} = 3{,}300\angle 0°, \quad \mathbf{Z} = 5\angle 60°$$

Since $\quad \phi_R = \cos^{-1}(0.8) = 36°52', \quad \therefore \quad \mathbf{I} = 120\angle -36°52'$

Voltage drop $\quad = \mathbf{IZ} = 120 \times 5\angle 60° - 36°52'$

$$= 600\angle 23°8' = 600(0.9196 + j0.3928) = 551.8 + j\,235.7$$

$$\therefore \quad \mathbf{E_S} = (3{,}300 + j_0) + (551.8 + j235.7) = 3{,}851.8 + j235.7$$

$$\mathbf{E_S} = \sqrt{3851.8^2 + 235.7^2} = \mathbf{3{,}860\ V}$$

Example 40.9. *An overhead, single-phase transmission line delivers 1100 kW at 33 kV at 0.8 p.f. lagging. The total resistance of the line is 10 Ω and total inductive reactance is 15 Ω. Determine **(i)** sending-end voltage **(ii)** sending-end p.f. and **(iii)** transmission efficiency.*

(Electrical Technology-I, Bombay Univ.)

Solution. Full-load line current is $I = 1100/33 \times 0.8 = 41.7$ A

$$\text{Line loss} = I^2R = 41.7^2 \times 10 = 17{,}390\ \text{W} = 17.39\ \text{kW}$$

(iii) Transmission efficiency $= \dfrac{\text{output}}{\text{output + losses}} = \dfrac{1100 \times 100}{1100 + 17.39} = \mathbf{98.5\%}$

(i) Line voltage drop $= \mathbf{IZ} = 41.7(0.8 - j\,0.6)(10 + j\,15) = 709 + j\,250$

Sending-end voltage is

$$\mathbf{E_S} = \mathbf{E_R} + \mathbf{I_Z} = (33{,}000 + j\,0) + (709 + j\,250)$$

$$= 33{,}709 + j\,250 = 33{,}710\angle 25°$$

Hence, sending-end voltage is 33.71 kV

(ii) As seen from Fig. 41.22, α= 0°25′

Sending-end p.f. angle is

$$\theta + \alpha = 36°52' + 0°25' = 37°17'$$

$$\therefore \quad \text{p.f.} = \cos 37°17' = \mathbf{0.795\ (lag).}$$

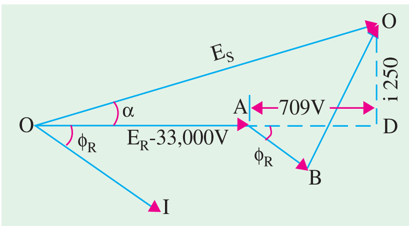


Fig. 41.22

Note. As seen from Fig. 41.22, approximate line drop is

$$= I(R\cos\phi_R + X\sin\phi_R) = 41.7\,(10\times 0.8 + 15\times 0.6) = 709 \text{ V}$$

$$\therefore \quad E_S = 33{,}000 + 709 = 33{,}709 \text{ V} \quad \text{—as above}$$

Example 41.10. *What is the maximum length in km for a 1-phase transmission line having copper conductors of 0.775 cm² cross-section over which 200 kW at unity power factor and at 3300 V can be delivered ? The efficiency of transmission is 90 per cent. Take specific resistance as $(1.725 \times 10^{-8})\ \Omega$-m.* **(Electrical Technology, Bombay Univ.)**

Solution. Since transmission efficiency is 90 per cent, sending-end power is 200/0.9 kW.

Line loss $= (200/0.9 - 200) = 22.22$ kW $= 22{,}220$ W

Line current $= 200{,}000/3300 \times 1 = 60.6$ A

If R is the resistance of one conductor, then

$$2I^2R = \text{line loss} \quad \text{or} \quad 2\times 60.6^2 \times R = 22{,}220 \quad \text{or} \quad R = 3.03 \text{ W}$$

Now, $R = \rho\dfrac{1}{A}$ $\quad\therefore\quad 3.03 = 1.725\times 10^{-8} \times l/0.775\times 10^{-4}$ $\quad\therefore\ l = 13{,}600,\ m =$ **13.6 km**

Example 41.11. *An industrial load consisting of a group of induction motors which aggregate 500 kW at 0.6 power factor lagging is supplied by a distribution feeder having an equivalent impedance of (0.15 + j0.6) ohm. The voltage at the load end of the feeder is 2300 volts.*

(a) *Determine the load current.*

(b) *Find the power, reactive power and voltampere supplied to the sending end of the feeder.*

(c) *Find the voltage at the sending end of the feeder.*

(Electrical Technology, Vikram Univ., Ujjain)

Solution. (a) $I = 500 \times 103/\sqrt{3} = 209$ A

(c) Let $V_R = (2300 + j\,0)$; $\quad I = 209\,(0.6 - j\,0.8)$

Voltage drop $=$ **IZ** $= 209\,(0.6 - j\,0.8)\,(0.15 + j\,0.6) = 119 + j50$

$$\mathbf{V}_S = \mathbf{V}_R + \mathbf{IZ} = 2300 + 119 + j\,50 = 2419 + j\,50 = 2420 \angle 1.2'$$

(b) Sending power $= \sqrt{3}\times 2420\times 209\times 0.5835 =$ **511.17 kW**

Sending-end reactive power $= \sqrt{3}\times 2420\times 209\times 0.8121 =$ **711.42 kVAR**

Sending-end volt ampere kVA $= \sqrt{3}\times 2420\times 209 =$ **876 kVA**

or $\quad$ kVA $= \sqrt{\text{kW}^2 + \text{kVAR}^2} = \sqrt{511.17^2 + 711.42^2} =$ **876**

41.19. Short Three-phase Transmission Line Constants

Three-phase calculations are made in the same way as single-phase calculations because a 3-phase unit can be regarded as consisting of 3 single-phase units each transmitting one-third of the total power. In Fig. 41.23 is shown a 3-phase system in which each conductor has a resistance of $R\Omega$ and inductance of $X\Omega$. In the case of a 3-phase system, it is advantageous to work in phase instead of in line-to-line values. In Fig. 41.24, one phase is shown separately. Since in a balanced load, there is no current in the neutral wire hence no current flows through the ground.

The resistive and reactive drops are calculated as before. By adding these drops ***vectorially*** to the receiving-end voltage, the sending-end voltage can be calculated. The line voltage at the sending-end can be found by multiplying this value by $\sqrt{3}$.

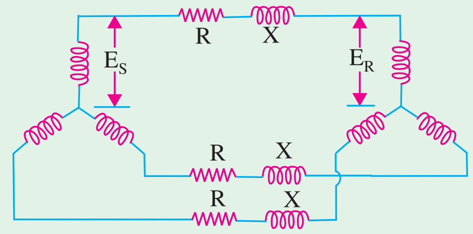


Fig. 41.23

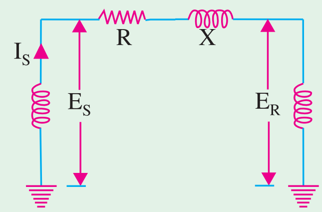


Fig. 41.24

In the figure, a star-connected load is assumed. But if it is a delta-connected load then it can be replaced by an equivalent star-connected load.

Example 41.12. *A 33-kV, 3-phase generating station is to supply 10 MW load at 31 kV and 0.9 power factor lagging over a 3-phase transmission line 3 km long. For the efficiency of the line to be 96% , what must be the resistance and reactance of the line?*

(Electrical Power-III, Bangalore Univ.)

Solution. Power output = 10 MW ; η = 0.96 ; Power input = 10/0.96 = 10.417 MW

Total loss = 0.417 MW

Now, I = 10 × 106/√3 × 31 × 103 × 0.9 = 207 A

If R is the resistance per phase, then 3 × 2072 × R = 0.417 × 106 ∴ R = **3.24 Ω**

Now, V_S per phase = 33/√3 = 19.052 kV and V_R per phase = 31/√3 = 17.898 kV

Using the approximate relation of Art. 36-18, we get

$$V_S = V_R + I(R\cos\phi_R + X\sin\phi_R)$$

$$19{,}052 = 17{,}898 + 207(3.24 \times 0.9 + X \times 0.4368) \quad \therefore X = \mathbf{6.1\ \Omega/phase.}$$

Example 41.13. *A balanced Y-connected load of (300 + j100) Ω is supplied by a 3-phase line 40 km long with an impedance of (0.6 + j0.7) Ω per km (line-to-neutral). Find the voltage at the receiving end when the voltage at the sending end is 66 kV. What is the phase angle between these voltages? Also, find the transmission efficiency of the line.*

(Elect. Power Systems, Gujarat Univ.)

Solution. The circuit connections are shown in Fig. 41.25.

Resistance for 40 km conductor length = 40 × 0.6 = 24 Ω

Reactance for 40 km conductor length = 40 × 0.7 = 28 Ω

Total resistance/phase = 300 + 24 = 324 Ω

Total reactance/phase = 100 + 28 = 128 Ω

Total impedance/phase $= \sqrt{324^2 + 128^2} = 348\ \Omega$

Line current $= \dfrac{66{,}000/\sqrt{3}}{348} = \dfrac{38{,}100}{348} = 110\ \text{A}$

Now, $\tan\phi_R$ = 00/300 ; ϕ_R = 18.4′ ; $\cos\phi_R$ = 0.949, $\sin\phi_R$ = 0.316

Voltage drop in conductor resistance = 110 × 24 = 2640 V

Voltage drop in conductor reactance = 110 × 28 = 3080 V

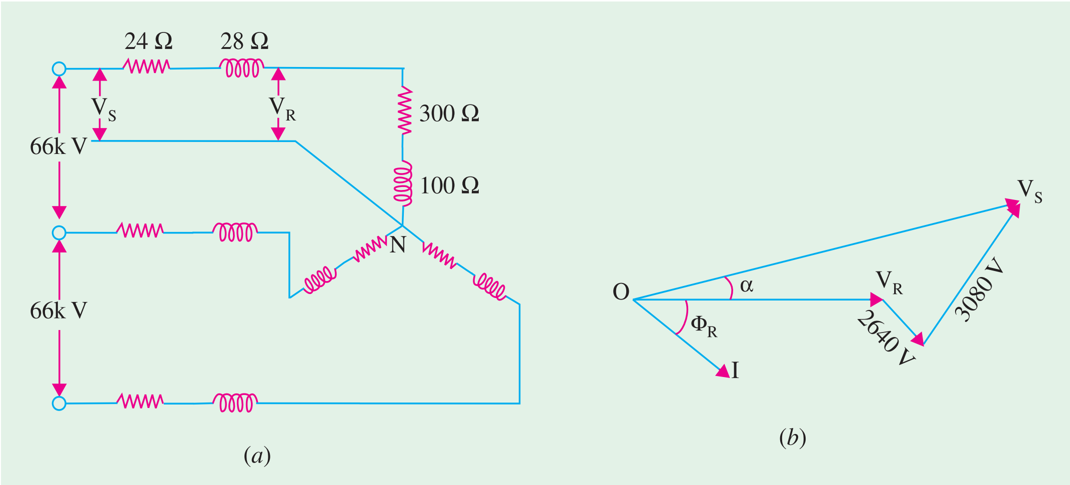


Fig. 41.25

It is seen from Art. 41.18 that

$$\mathbf{V_S} = (V_R + IR\cos\phi_R + IX\sin\phi_R) + j\,(IX\cos\phi_R - IR\sin\phi_R)$$
$$= (V_R + 2640\times 0.949 + 3080\times 0.316) + j\,(3080\times 0.949 - 2640\times 0.316)$$
$$= (V_R + 3475) + j\,2075$$

Now, $\mathbf{V_S} = 66000/\sqrt{3} = 38{,}100$ V

$\therefore$ $38{,}100^2 = (V_R + 3475)^2 + 2075^2$, $V_R = 34{,}585$ V

Line voltage across load $= 34{,}585 \times \sqrt{3}$ = **59.88 kV**

$\therefore$ $\mathbf{V_S} = (34{,}585 + 3475) + j\,2075 = 38{,}060 + j\,2075 = 38{,}100\angle 3.1°$

Obviously, V_S leads V_R by 3.1° as shown in Fig. 41.25 (*b*).

Power delivered to the load/phase $= 110^2 \times 300$ W

Power transmitted/phase $= 110^2 \times 324$ W

$\therefore$ transmission efficiency $\eta = (300/324)\times 100$ = **92.6%**

Example 41.14. *Define 'regulation' and 'efficiency' of a short transmission line.*

A 3-phase, 50-Hz, transmission line having resistance of 5Ω per phase and inductance of 30 mH per phase supplies a load of 1000 kW at 0.8 lagging and 11 kV at the receiving end. Find.

(*a*) *sending end voltage and power factor* **(*b*)** *transmission efficiency* **(*c*)** *regulation.*

(Electrical Engineering-III. Poona Univ. 1990)

Solution. VR per phase $= 11{,}000/\sqrt{3} = 6{,}350$ V

Line current, $= 1000\times 10^3/\sqrt{3}\times 11{,}000\times 0.8 = 65.6$ A

$X_L = 2\pi\times 50\times 30\times 10^{-3} = 9.4\ \Omega$, $R = 5\Omega$; $Z_{ph} = (5 + j\,9.4)\ \Omega$

$\therefore$ drop per conductor $= 65.6\,(0.8 - j\,0.6)\,(5 + j\,9.4) = 632 + j\,296$

(*a*) $V_S = 6350 + (632 + j\,296) = 6{,}982 + j\,296 = 6{,}988\angle 2.4°$

Sending-end line voltage $= 6{,}988\times\sqrt{3} = 12{,}100$ V = **12.1 kV**

As seen from Fig. 41.25 (*b*), $\alpha = 2.4°$, Now, $\cos\phi_R = 0.8$, $\phi_R = 36.9°$

$\phi_S = 36.9° + 2.4° = 39.3°$, $\cos\phi_S$ = **0.774 (lag).**

(*b*) Total power loss $= 3\,I^2R = 3\times 65.62\times 5 = 64{,}550$ W $= 64.55$ kW

Input power $= 1000 + 64.55 = 1064.55$ kW

$\therefore$ power transmission η = 1000/1064.55 = 0.9394 or **93.94%**

(*c*) Now, line regulation is defined as the rise in voltage when full-load is thrown off the line divided by voltage at the load end.

$$\therefore \qquad \% \text{ regn.} = \frac{(12.1 - 11)}{11} \times 100 = \mathbf{10\%}$$

Example 41.15. *A short 3-φ line with an impedance of (6 + j8) Ω per line has sending and receiving end line voltages of 120 and 110 kV respectively for some receiving-end load at a p.f. of 0.9. Find the active power and the reactive power at the receiving end.*

(Transmission and Distribution, Madras Univ.)

Solution. As seen from Art. 41.18, considering phase values, we have

$$V_S = V_R + I\,(R \cos \phi R + X \sin \phi R)$$

Now, V_S per phase = $120/\sqrt{3}$ = 69,280 V

Similarly, V_R per phase = $110/\sqrt{3}$ = 63,507 V

$\therefore$ 69,280 = 63,507 + I (6 × 0.9 + 8 × 0.435)

$\therefore$ Line current I = 5,773/8.88 = 650 A

Active power at receiving end = $\sqrt{3} V_L I_L \cos \phi = \sqrt{3} \times 110 \times 650 \times 0.9$ = **111,400 kW**

Reactive power at receiving end = $\sqrt{3}\, V_L\, I_L \sin \phi = \sqrt{3} \times 110 \times 650 \times 0.435$ = **53,870 kVAR**

Example 41.16. *A 3-phase, 20 km line delivers a load of 10 MW at 11 kV having a lagging p.f. of 0.707 at the receiving end. The line has a resistance of 0.02 Ω/km phase and an inductive reactance of 0.07 Ω/km/phase. Calculate the regulation and efficiency of the line. If, now, the receiving-end p.f. is raised to 0.9 by using static capacitors, calculate the new value of regulation and efficiency.* **(Electrical Engg. ; Bombay Univ.)**

Solution. (*i*) When p.f. = 0.707 (lag)

Line current = $10 \times 10^6/\sqrt{3} \times 11{,}000 \times 0.707$ = 743 A

V_R per phase = $11{,}000/\sqrt{3}$ = 6,352 V

Total resistance/phase for 20 km = 20 × 0.02 = 0.4 W

Total reactance/phase for 20 km = 20 × 0.07 = 1.4 W

$\therefore$ Total impedance/phase = $(0.4 + j\,1.4)\ \Omega$

If V_R is taken as the reference vector, then drop per phase

$$= 743\,(0.707 - j\,0.707)\,(0.4 + j\,1.4) = (945 + j\,525)$$

$$\therefore \quad V_S = 6{,}352 + 945 + j\,525 = 7{,}297 + j\,525$$

$$\text{or} \quad V_S = \sqrt{7297^2 + 525^2} = 7{,}315 \text{ V}$$

$$\therefore \quad \% \text{ regulation} = \frac{7{,}315 - 6{,}352}{6{,}352} \times 100 = \mathbf{15.1\ \%}$$

Total line loss = $3I^2R$ = 3 × 7432 × 0.4 = 662 kW

Total output = 10 + 0.662 = 10.662 MW $\therefore$ η = 10 × 100/10.662 = **94%**

(*ii*) When p.f. = 0.9 (lag)

Line current = $10^7/\sqrt{3} \times 11{,}000 \times 0.9$ = 583 A

Drop/phase = 583 (0.9 − j 0.435) (0.4 + j 1.4) = 565 + j 633

$$V_S = 6{,}352 + (565 + j\,633) = 6{,}917 + j\,633$$

$$\therefore \quad V_S = \sqrt{6{,}917^2 + 633^2} = 6{,}947 \text{ V}$$

$\therefore$ % regulation $= \dfrac{6,947 - 6,352}{6,352} \times 100$ = **9.37%**

Total line loss $= 3 \times 5832 \times 0.4 = 408$ kW ; Total output = 10.408 MW

$\therefore \quad \eta = 10 \times 100/10.408 =$ **96.1%**

Example 41.17. *A load of 1,000 kW at 0.8 p.f. lagging is received at the end of a 3-phase line 10 km long. The resistance and inductance of each conductor per km are 0.531 W and 1.76 mH respectively. The voltage at the receiving end is 11 kV at 50 Hz. Find the sending-end voltage and the power loss in the line. What would be the reduction in the line loss if the p.f. of the load were improved to unity?* **(Elect. Power Systems, Gujarat Univ.)**

Solution. Line current $= 1,000 \times 1,000/\sqrt{3} \times 11 \times 1,000 \times 0.8 = 65.6$ A

Voltage/phase $= 11,000/\sqrt{3} = 6,352$ V

$\mathbf{X} = 2\pi \times 50 \times 176 \times 10\sqrt{3} \times 10 = 5.53\ \Omega$; $\mathbf{R} = 0.531 \times 10 = 5.31\ \Omega$ $\quad\therefore\quad$ $\mathbf{Z} = (5.31 + j\,5.53)$

Voltage drop/phase $= 65.6\ (0.8 - j\,0.6)\ (5.31 + j\,5.53) = 496.4 + j\,81.2$

$\therefore$ $\mathbf{V}_S = 6,352 + 496.4 + j\,81.2 = 6,848 + j\,81 = 6,848\ V$ numerically (approx.)

$\therefore$ line-to-line sending-end voltage $= 6,848 \times \sqrt{3} = 11,860\ V =$ **11.86 kV**

Total loss $= 3 \times 5.31 \times 65.62 = 68,550$ W = **68.55 kW**

Line current for unity p.f. $= 1,000/11 \times \sqrt{3} = 52.49$ A

$\therefore$ New losses $= 3 \times 5.31 \times 52.492 = 43.89$ kW

$\therefore$ reduction $= 68.55 - 43.89 =$ **24.66 kW**

Example 41.18. *Estimate the distance over which a load of 15,000 kW at 0.85 p.f. can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance 0.905 W per kilometre. The voltage at the receiving end is to be 132 kV and the loss in transmission is to be 7.5% of the load.* **(Transmission and Distribution, Madras Univ.)**

Solution. Line current $= 15,000/132 \times \sqrt{3} \times 0.85 = 77.2$ A

Total loss = 7.5% of 15,000 = 1,125 kW

If R is the resistance of one conductor, then

$$3\,I^2R = 1,125,000 \quad \text{or} \quad 3 \times (77.2)^2 \times R = 1,125,000 \ ; \ R = 62.94\ \Omega$$

Length of the line $= 62.94/0.905 =$ **69.55 km.**

Example 41.19. *A 3-ϕ line has a resistance of 5.31Ω and inductance of 0.0176 H. Power is transmitted at 33 kV, 50-Hz from one end and the load at the receiving end is 3,600 kW at 0.8 p.f. lagging. Find the line current, receiving-end voltage, sending-end p.f. and efficiency of transmisson.* **(Transmission and Distribution-I, Madras Univ.)**

Solution. $V_S = V_R + I_R \cos\phi_R + IX \sin\phi_R$ approximately

Now, power delivered/phase $= V_R\, I \cos\phi_R$

$\therefore$ $1,200 \times 1,000 = VR\,.\,I. \times 0.8 \quad \therefore \quad I = 15 \times 105/V_R$

Also, V_S per phase $= 33,000/\sqrt{3} = 19,050$ V

$R = 5.31\ \Omega$; $X = 0.0176 \times 314 = 5.54\ \Omega$

$$\therefore \quad 19,050 = V_R + \frac{15 \times 10^5}{V_R} \times 5.31 \times 0.8 + \frac{15 \times 10^5}{V_R} \times 5.54 \times 0.6$$

$$\therefore \quad V_R^{\,2} - 19,050\,V_R + 11,358,000 = 0 \text{ or } V_R = \frac{19.050 \pm 17,810}{2} = \frac{36.860}{2} = 18,430 \text{ V}$$

Line voltage at the receiving end $= 18,430 \times \sqrt{3} =$ **32 kV**

$I = 15 \times 105/18,430 =$ **81.5 A**

Now, $V_S = V_R + I(\cos\phi_R - j\sin\phi_R)(R + jX)$

$= 18{,}430 + 81.5\,(0.8 - j\,0.6)\,(5.31 + j\,5.54)$

$= 18{,}430 + 615 + j\,100 = 19{,}050 \angle 0.3°$

$\phi_R = \cos^{-1}(0.8) = 36°52'$;

$\phi_S = 36°52' + 18' = 37°10'$

$\therefore$ sending-end p.f. $= \cos\phi_S = \cos 37°10' =$ **0.797 A**

Power lost in line $= 3\,I^2R = 3 \times 81.5^2 \times 5.31 = 106$ kW

Power at sending end $= 3{,}600 + 106 = 3{,}706$ kW

transmission $\eta = 3{,}600 \times 100/3{,}706 =$ **97.2%**

Example 41.20. *A 3-phase short transmission line has resistance and reactance per phase of 15Ω and 20 Ω respectively. If the sending-end voltage is 33 kV and the regulation of the line is not to exceed 10%, find the maximum power in kW which can be transmitted over the line. Find also the k VAR supplied by the line when delivering the maximum power.*

Solution. As seen from Art. 41.18

$$V_S^2 = (V_R + I_R\cos\phi_R + IX\sin\phi_R)^2 + (IX\cos\phi_R - I_R\,\phi_R)^2$$

$$= V_R^{\,2} + 2\,IV_R\,(R\cos\phi_R + X\sin\phi_R) + I^2(R^2 + X^2)$$

real power/phase, $P = V_R I\cos\phi_R$; reactive power/ phase, $Q = V_R I\sin\phi_R$

$\therefore$ $P^2 + Q^2 = V_R^{\,2} I^2$ or $I^2 = (P^2 + Q^2)/V_R^{\,2}$

Substituting this value above, we get

$$V_S^2 = V_R^{\,2} + 2PR + 2QX + \frac{P^2 + Q^2}{V_R^2}(R^2 + X^2)$$

or
$$V_S^2 = V_R^{\,2} - 2PR - 2QX - \frac{P^2 + Q^2}{V_R^2}(R^2 + X^2) = 0 \qquad \ldots(i)$$

To find the maximum power transmitted by the line, differentiate the above equation *w.r.t.* Q and put $dP/dQ = 0$ (treating V_S and V_R as constants).

$$\therefore \quad -2R\frac{dP}{dQ} - 2X - 2P\,.\frac{(R^2 + X^2)}{V_R}\,.\frac{dP}{dP} - 2Q\frac{(R^2 + X^2)}{V_R^2} = 0$$

Since $dP/dQ = 0 \;\therefore\; 2X + 2Q\dfrac{(R^2 + X^2)}{V_R^2} = 0$ or $Q = \dfrac{-V_R^2 X}{R^2 + X^2} = \dfrac{-V_R^2 X}{Z^2}$

Putting this value of Q in Eq. *(i)* above we have

$$\frac{P^2Z^2}{V_R^2} + 2\,PR - V_S^2 + V_R^2 - \frac{V_R^2\,X^2}{Z^2} = 0$$

or
$$\frac{P^2Z^2}{V_R^2} + 2\,PR - V_S^2 + V_R^2\left(1 - \frac{X^2}{Z^2}\right) = 0$$

or
$$\frac{P^2Z^2}{V_R^2} + 2\,PR + \frac{V_S^2 + V_R^2}{Z^2} - V_S = 0$$

Solving for P, we get $P_{max} = \dfrac{V_R^2}{Z^2}\left(Z\,.\dfrac{V_S}{V_R} - R\right)$ watts/phase

In the present case, $V_S = 33{,}000/\sqrt{3} = 19{,}050$ V ; Since regulation is limited to 10%,

$\therefore$ $V_R + 10\%\,V_R = V_S$ or $1.1\,V_R = 19{,}050$; $V_R = 17{,}320$ V ; $Z = \sqrt{15^2 + 20^2} = 25\ \Omega$

$$\therefore \qquad P_{max} \text{ per phase} = \left(\frac{17.320}{25}\right)^2 \left(25 \times \frac{19,050}{17,320} - 15\right) = 6 \times 10^6 \text{ W} = 6 \text{ MW}$$

Total maximum power $= 3 \times 6 =$ **18 MW**

$$\text{kVAR supplied/phase } Q = \frac{V_R^2\, X}{Z^2} = -\frac{17,320^2 \times 20}{25^2} \times 10^{-3} = 9,598$$

Total kVAR supplie $= 3 \times 9,598 =$ **28,794.**

Example 41.21. *A 3- ϕ, 50-Hz generating station supplies a load of 9,900 kW at 0.866 p.f. (lag) through a short overhead transmission line. Determine the sending-end voltage if the receiving-end voltage is 66 kV and also the efficiency of transmission. The resistance per km is 4Ω and inductance 40 mH. What is the maximum power in kVA that can be transmitted through the line if both the sending and receiving-end voltages are kept at 66 kV and resistance of the line is negligible.*

Solution. $R = 4\,\Omega$; $X = 40 \times 10^{-3} \times 314 = 12.56\,\Omega$

Line current $I = 9,900/\sqrt{3} \times 66 \times 0.866 = 100$ A

$V_R = 66,000/\sqrt{3} = 38,100$ V ; $\cos\phi_R = 0.866$; $\sin\phi_R = 0.5$

$$V_S = V_R + I(R\cos\phi_R + X\sin\phi_R) = 38,100 + 100(4 \times 0.866 + 12.56 \times 0.5) = 39,075 \text{ V}$$

Line value of sending-end voltage $= 39,075 \times \sqrt{3} =$ **67.5 kV**

Total line loss $= 3I^2R = 3 \times 100^2 \times 4 = 120$ kW

$\therefore \qquad \eta = 9,900/(9,900 + 120) = 0.988$ or 98.8%

Max, value of Q for 3-phases $= \dfrac{3 . V_R^2}{Z^2} . X$

Now, $V_S = V_R = 38,100$ V and resistance is negligible.

$\therefore$ Max. value of $Q = \dfrac{3 . V_R^2}{Z^2} . X \;\; 10^{-3} = 3 \times 3,810 \times 10^{-3}/12.56 =$ **3,48,000 kVA**

Example 41.22. *A 3-phase load of 2,000 kVA, 0.8 p.f. is supplied at 6.6 kV, 50-Hz by means of a 33 kV transmission line 20 km long and a 5 : 1 transformer. The resistance per km of each conductor is 0.4 Ω and reactance 0.5Ω. The resistance and reactance of the transformer primary are 7.5 Ω and 13.2 Ω, whilst the resistanceof the secondary is 0.35 Ω and reactance 0.65 Ω. Find the voltage necessary at the sending end of transformission line when 6.6 kV is maintained at the load-end and find the sending-end power factor. Determine also the efficiency of transmission.*

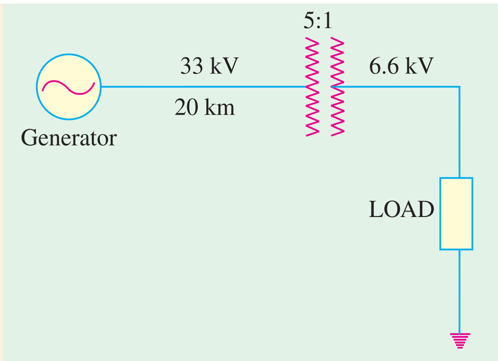


Fig. 41.26

Solution. One phase of the system is shown in Fig. 41.26. Impedance per phase of high voltage line = (8 + j 10)

Impedance of the transformer primary

$= (7.5 + j\,13.2)$ ohm

Total impedance on the high-tension side

$= (8 + j\,10) + (7.5 + j\,13.2) = 15.5 + j\,23.2$

This impedance can be transferred to secondary side by using the relation given in Art. 30.12.

Hence, impedance as referred to secondary side is

$= (15.5 + j\,23.2)/52 = 0.62 + j\,0.928$

Adding to them the impedance of the transformer secondary, we get the total impedance as referred to low-voltage side

$$= (0.62 + j\,0.928) + (0.35 + j\,0.65) = 0.97 + j\,1.578$$

Now, kVA load per phase $= 2{,}000/3.0 = 667$

Receiving-end voltage/phase $= 6.6/\sqrt{3} = 3.81$ kV

$\therefore$ current in the line $= 667/3.81 = 175$ A

Drop per conductor $= I(R\cos\phi + X\sin\phi) = 175(0.97\times 0.8 + 1.578\times 0.6) = 302$ V

Now, $E_S = E_R + I(R\cos\phi + X\sin\phi)$

Hence, sending-end voltage (phase to neutral as referred to the lower voltage side) is 3,810 + 302 = 4,112 V. As referred to high-voltage side, its value = 4,112 × 5 = 20,560 V

Line voltage $= 20{,}560\times\sqrt{3}/1000 = 35.6$ kV

If ϕ_S is the power factor angle at the sending-end, then

$$\tan\phi_S = \frac{\sin\phi + (IX/E_B)}{\cos\phi + (IR/E_R)} = \frac{0.6 + (175\times 1.578/3810)}{0.8 + (175\times 0.97/3.10)} = 0.796$$

$\therefore\ \phi_S = \tan^{-1}(0.796) = 38°31'$ $\quad\therefore\ \cos\phi_S = \cos 38°31' =$ **0.782**

Power loss/phase $= 1752\times 0.97/1000 = 29.7$ kW

Power at the receiving end/phase $= 2000\times 0.8/3 = 533.3$ kW

$\therefore$ transmission efficiency $= \dfrac{533.3\times 100}{533.3 + 29.7} =$ **94.7 %**

Tutorial Problem No. 41.1

1. 500 kW at 11kV are received from 3-phase transmission line each wire of which has a resistance of 1.2 Ω and a reactance of 1 Ω. Calculate the supply pressure when the power factor of the load is *(i)* unity and *(ii)* 0.5 leading. **[(*i*) 11,055 V (*ii*) 10,988 V]**

2. What load can be delivered by a 3-phase overhead line 5 km long with a pressure drop of 10%. Given that the station voltage is 11 kV, resistance per km of each line 0.09 Ω, reactance per km 0.08 Ω and the power factor of the load 0.8 lagging. **[14,520 kW]**

3. Estimate the distance over which a load of 15,000 kW at 0.85 power factor can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance 0.56 Ω per km. The p.d. at the receiving end is to be 132 kV and the loss in transmission is not to exceed 7.5% **[121.9 km] (*I.E.E. London*)**

4. A d.c. 2-wire system is to be converted into 3-phase, 3-wire a.c. system by adding a third conductor of the same size as the two existing conductors. Calculate the percentage additional balanced load that can now be carried by the conductors at 0.96 p.f. lagging. Assume the same voltage between the conductors and the same percentage power loss. **[84%]**

5. A 3-phase short transmission line of resistance 8 Ω and reactance 11 Ω per phase is supplied with a voltage of 11 kV. At the end of the line is a balanced load of P kW per phase at a p.f. of 0.8 leading. For what value of P is the voltage regulation of the line zero? **[210 kW] (*Electrical Technology, M.S. Univ. Baroda*)**

6. A 3-ph, 50-Hz transmission line 10 km long delivers 2,500 kVA at 10 kV. The p.f. of the load is 0.8 (lag). The resistance of each conductor is 0.3 W/km and the inductance 1.82 mH/km. Find *(a)* the voltage and p.f. at the sending end *(b)* the efficiency of transmission and (*c*) the percentage regulation of the line. **[(*a*) 11.48 kV ; 0.763 (*b*) 91.4% (*c*) 14.8%]**

7. The conductors in a single-phase transmission line are 6 m above the ground. Each conductor has 1.5 cm diameter and the conductors are spaced 3 m apart. Starting from the fundamentals, determine the capacitance per kilometre of the line first including and then excluding the effect of ground. What do you conclude ? **[4.60 × 10^{-3} μF/km ; 4.63 × 10^{-3} μF/km] (*Ranchi Univ.*)**

41.20. Effect of Capacitance

So far we have neglected the effect of capacitance on the line regulation because the capacitances of short lines transmitting at relatively low voltages (up to 20 kV) are negligible. But as the voltage and length of the transmission line increase, the capacitance gradually becomes of greater importance. Similarly, the leakage across insulators also assumes greater importance. Hence, exact calculations of regulation for long lines take into consideration the capacitance and leakage reactance of the lines and are quite elaborate, the amount of elaboration depending on the transmitting voltage.

(i) In the case of short lines, ordinarily, the capacitance is negligible. But if in a problem, the line capacitance is given and if the line is less than 80 km, then the line capacitance can be lumped at the receiving or load end as shown in Fig. 41.27 (*a*) although this method of localizing the line capacitance at the load end over-estimates the effect of capacitance. In that case, the line current $\mathbf{I}_S$ is the vector sum of the load current $\mathbf{I}_R$ and the charging current $\mathbf{I}_C$ of the capacitance. Hence, $\mathbf{I}_S = \mathbf{I}_R + \mathbf{I}_C$.

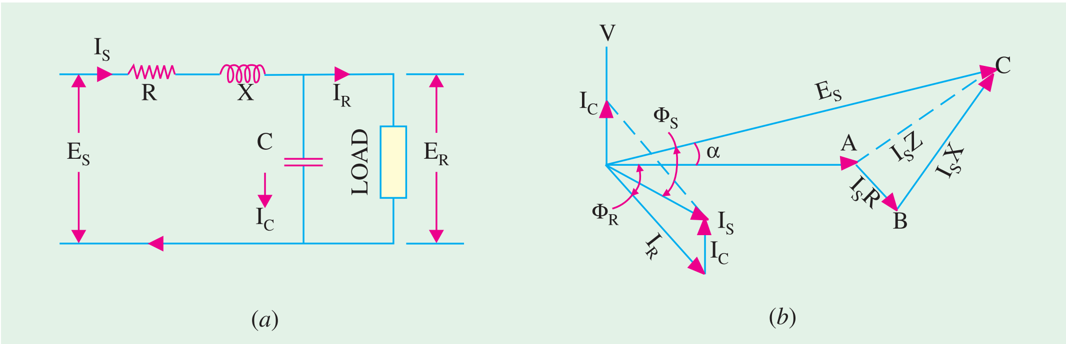


Fig. 41.27

Now, charging current $\quad \mathbf{I}_C = j\omega CE_R \quad \therefore \quad \mathbf{I}_R = I_R(\cos\phi_R - j\sin\phi_R)$

$\therefore \quad \mathbf{I}_S = I_R\cos\phi_R - jI_R\sin\phi_R + j \quad CE_R = I_R\cos\phi R + j(-I_R\sin\phi_R + \omega CE_R)$

$$\text{Line drop} = I_S(R + jX) \quad \therefore \quad \mathbf{E}_S = \mathbf{E}_R + \mathbf{I}_S(\boldsymbol{R} + \boldsymbol{J}_X)$$

The vector diagram is shown in Fig. 41.27 (*b*).

(ii) In the case of lines with voltages up to 100 kV and 150 km in length,satisfactory solutions can be obtained by the so-called *T*-method and π-or pi method as described below.

Example 41.23. *A (medium) single-phase transmission line 50 km long has the following constants :*

resistance/km $= 0.5\ \Omega$; reactance/km $= 1.6\ \Omega$

susceptance/km $= 28 \times 10^{-6}$ S; receiving-end line voltage = 66,000 V

Assuming that total capacitance of the line is located at receiving end alone, determine the sending-end voltage, the sending-end current and regulation. The line is delivering 15,000 kW at 0.8 p.f. lagging. Draw a vector diagram to illustrate your answer.

Solution. Let E_S and E_R be sending-end and receiving-end voltages respectively as shown in Fig. 41.28.

Load current at the receiving-end is $I_R = 15 \times 10^4/66 \times 10^3 \times 0.8 = 284$ A

Total resistance $= 0.5 \times 50 = 25\ \Omega$; Total resistance $= 1.6 \times 50 = 80\ \Omega$

Susceptance $B = 28 \times 10^{-6} \times 50 = 14 \times 10^{-4}$ Siemens

Capacitive admittance $\quad Y = B = 14 \times 10^{-4}$ Siemens

As seen from vector diagram of Fig. 41.29, sending-end current I_S is the vector sum of load current I_R and capacitive current I_C.

Now, $\quad I_C = E_R\ Y = 66{,}000 \times 14 \times 10^{-4} = 92$ A

Let $\quad \mathbf{E}_R = (66{,}000 + j\,0)$

$\mathbf{I}_R = 284\,(0.8 - j\,0.6) = 227 - j190;\ \mathbf{I}_C = j\,92$

$\mathbf{I}_S = \mathbf{I}_R + \mathbf{I}_C = (227 - j\,190 + j\,92) = 240 \angle -18°57' \,;\ Z = 25 + j\,80 = 84 \angle 72°36'$

Line drop $= \mathbf{I}_S\mathbf{Z} = 240 \angle{-18°57'} \times 84 \angle 72°36' = 20{,}160 \angle 53°39' = 11{,}950 + j\,16{,}240$

$\mathbf{E}_S = \mathbf{E}_R + \mathbf{I}_S\mathbf{Z} = 66{,}000 + (11{,}950 + j\,16{,}240) = 79{,}500 \angle 11°44'$

$$\text{Regulation} = \frac{E_S - E_R}{E_S} \times 100 = \frac{79{,}500 - 66{,}000}{66{,}000} \times 100 = \mathbf{20.5\%}$$

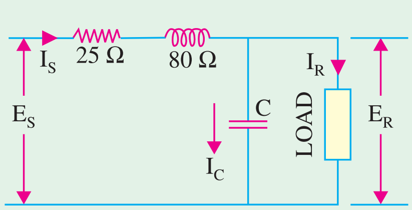


Fig. 41.28

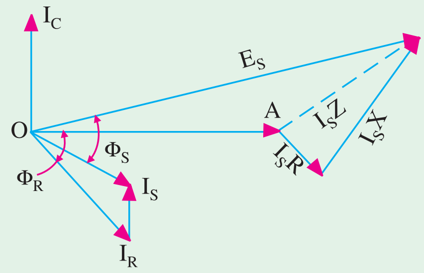


Fig. 41.29

41.21. "Nominal" T-method

In this *T*-method, also known as mid-capacitor method, the whole of the line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. 41.30.

It is seen that $\quad \mathbf{E}_1 = \mathbf{E}_R + \mathbf{I}_R\mathbf{Z}_{BC}$. $\quad$ Knowing E_1, we can find $\mathbf{I}_C$ as under.

$\mathbf{I}_C = j\omega C_{E1} \quad \therefore \quad \mathbf{I}_S = \mathbf{I}_C + \mathbf{I}_R$

Obviously, current through portion *A B* is I_S ; hence voltage drop $= I_S\,Z_{AB}$

$$\therefore \quad \boxed{\mathbf{E}_S = \mathbf{E}_1 + \mathbf{I}_S\,\mathbf{Z}_{AB}}$$

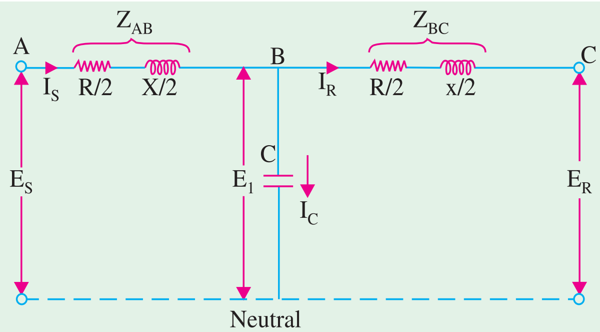


Fig. 41.30

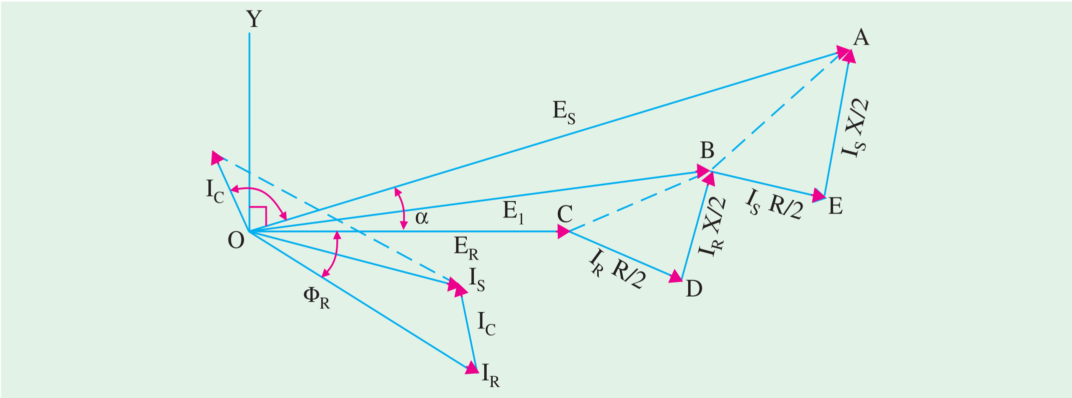


Fig. 41.31

The vector diagram is shown in Fig. 41.31. Receiving-end voltage E_R is taken as the reference vector. It may be pointed out here that all values are phase values *i.e.* line to neutral values. I_R is the load current lagging E_R by ϕ_R, $CD = I_R.R/2$ and parallel to I_R. $BD = I_R\ X/2$ and perpendicular to I_R. OB represents E_1 *i.e.* the voltage across the middle capacitor. BE represents $I_S.R/2$ and is parallel to I_S. Similarly, $EA = I_S.X/2$ and perpendicular to I_S. OA represents the voltage at the sending end.

It may be noted that if leakage is appreciable, then leakage conductance G can be assumed to be concentrated at the middle point of the line and can be represented by non-inductive conductance G shunting the middle capacitor as shown in Fig. 41.32.

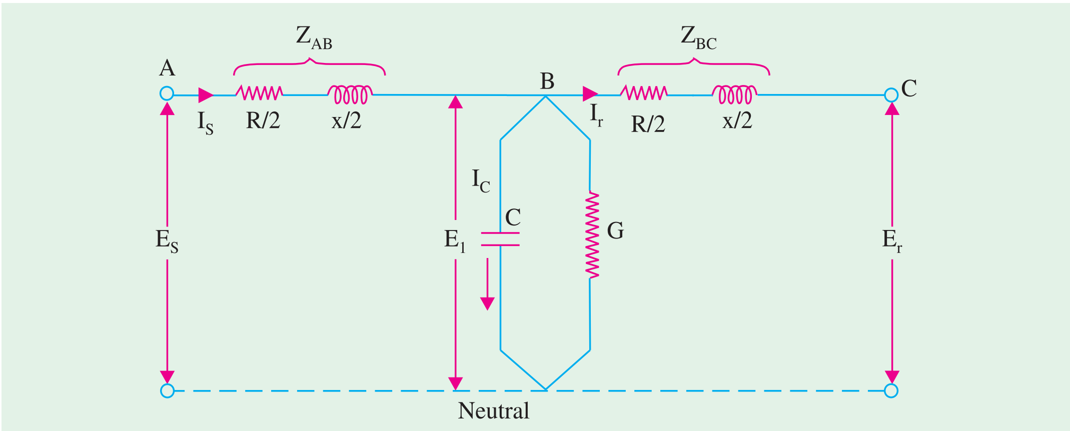


Fig. 41.32

Example 41.24. *A 3-phase, 50-Hz overhead transmission line 100 km long with 132 kV between lines at the receiving end has the following constants :*

resistance/km/phase = 0.15 Ω ; inductance/km/phase = 1.20 mH

capacitance/km/phase = 0.01 mF

Determine, using an approximate method of allowing for capacitance, the voltage, current and p.f. at the sending end when the load at the receiving end is 72 MW at 0.8 p.f. lagging. Draw vector diagram for the circuit assumed. **(Electrical Power System ; Gujarat Univ.)**

Solution. For a 100-km length of the line,

$$R = 0.15 \times 100 = 15\ \Omega\ ;\ X_L = 314 \times 1.2 \times 10^{-3} \times 100 = 37.7\ \Omega$$

$$X_C = 106/314 \times 0.01 \times 100 = 3187\ \Omega$$

Using the nominal *T*-method, the equivalent circuit is shown in Fig. 41.33 (*a*)

$$V_R = 132/\sqrt{3} = 76.23 \text{ kV} = 76{,}230 \text{ V. Hence, } \mathbf{V}_R = 76{,}230 + j\,0$$

Load current, $\mathbf{I}_R = 72 \times 102/\sqrt{3} \times 132 \times 103 \times 0.8 = 394$ A

$\therefore$ $\mathbf{I}_R = 394\,(0.8 - j\,0.6) = 315 - j\,236$ A ; $\mathbf{Z}_{BC} = (7.5 + j\,18.85)$ W

Drop/phase over *BC* $= \mathbf{I}_R\mathbf{Z}_{BC} = (315 - j\,236)\,(7.5 + j\,18.85) = 6802 + j\,4180$

$$\mathbf{V}_1 = \mathbf{VR} + \mathbf{I}_R\mathbf{Z}_{BC} = (76{,}230 + j0) + (\,6802 + j\,4180) = 88{,}030 + j\,4180$$

$$\mathbf{I}_C = \frac{\mathbf{V_1}}{\mathbf{X_C}} = \frac{83{,}030 + j\,4180}{-\,j\,3187} = -1.31 + j\,26$$

$\mathbf{I}_S = \mathbf{I}_C + \mathbf{I}_R = (-1.31 + j\,26) + (315 - j\,236) = (313.7 - j\,210) = 377.3\,\angle{-33.9°}$

Drop/phase over *AB* $= \mathbf{I}_S\mathbf{Z}_{AB} = (313.7 - j\,210)\,(7.5 + j\,18.85) = 6320 + j\,4345$

$\therefore$ $\mathbf{V}_S = \mathbf{V}_1 + \mathbf{I}_S\mathbf{Z}_{AB} = (83{,}030 + j\,4180) + (6320 + j\,4345) = 89{,}350 + j\,8525 = 89{,}750\,\angle\,5.4°$

Line value of sending-end voltage $= \sqrt{3} \times 89{,}750 \times 10^{-3} =$ **155.7 kV**

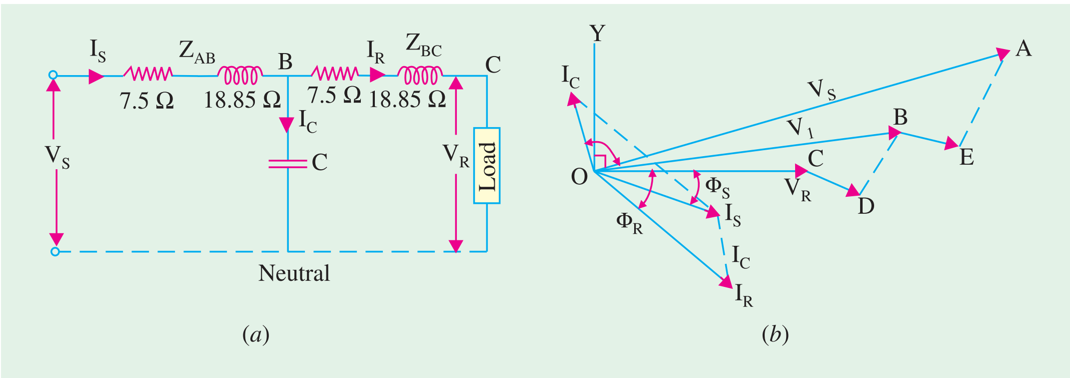


Fig. 41.33

Phase difference between V_S and I_S = 33.9° + 5.4° = 39.3° with current lagging as shown in Fig. 41.33 (*b*) cos ϕ_S = cos 39.3° = **0.774 (lag)**

Example 41.25. *A 3-phase, 50-Hz transmission line, 100 km long delivers 20 MW at 0.9 p.f. lagging and at 110 kV. The resistance and reactance of the line per phase per km are 0.2 Ω and 0.4 Ω respectively while the capacitive admittance is 2.5×10^{-6} S per km. Calculate* **(a)** *the voltage and current at the sending end and* **(b)** *the efficiency of transmission. Use the nominal T-method.*

(Electrical Power-I, M.S. Univ. Baroda)

Solution. Resistance for 100 km $= 0.2 \times 100 = 20\ \Omega$

Reactance for 100 km $= 0.4 \times 100 = 40\ \Omega$

Capacitive admittance for 100 km $= 2.5 \times 10^{-6} \times 100 = 2.5 \times 10^{-4}$ S

Let us take the receiving-end voltage E_R as reference vector.

$$E_R = 110/\sqrt{3} = 63.5 \text{ kV} ; I_R = 20 \times 106/\sqrt{3} \times 110 \times 103 \times 0.9 = 116.6 \text{ A}$$

$$\cos\phi_R = 0.9 ; \sin\phi_R = 0.435 \text{ (from tables)}$$

With reference to Fig. 41.34, we have

$\mathbf{E}_R = (63.5 + j\,0)$ kV ; $\mathbf{I}_R = 116.6(0.9 - j\,0.\,435) = 105 - j\,50.7$; $\mathbf{Z}_{BC} = (10 + j\,20)$

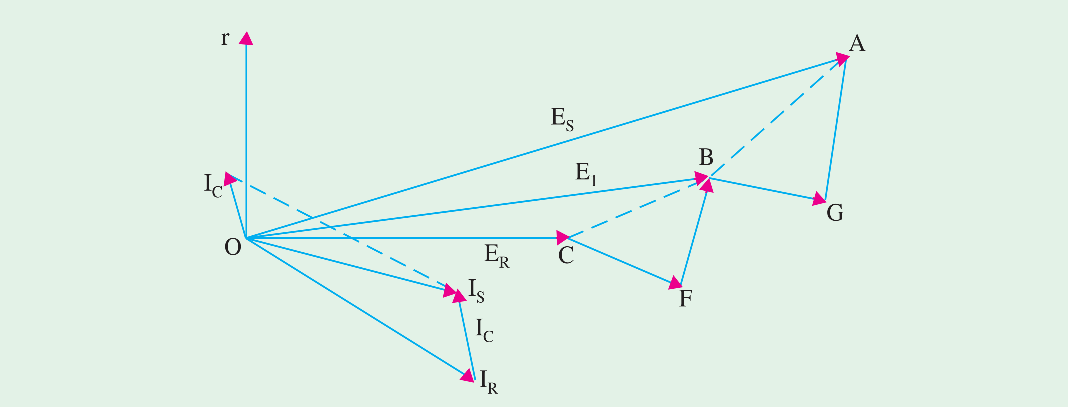


Fig. 41.34

Voltage drop between points *B* and *C* is

$$\mathbf{V}_{BC} = \mathbf{I}_R\mathbf{Z}_{BC} = (105 - j\,50.7)\,(10 + j\,20) = (2{,}064 + j\,1{,}593)\text{ V}$$
$$\mathbf{E}_1 = \mathbf{E}_R + \mathbf{I}_R\mathbf{Z}_{BC} = (63{,}500 + 2{,}064 + j\,1{,}593) = 65{,}564 + j\,1{,}593$$
$$\mathbf{I}_C = \mathbf{E}_1\mathbf{Y} = (65{,}564 + j\,1{,}593) \times j\,2.5 \times 10^{-4} = (-0.4 + j\,16.4)\text{ A}$$
$$\mathbf{I}_S = \mathbf{I}_R + \mathbf{I}_C = (105 - j\,50.7) + (-0.4 + j\,16.4) = (104.6 - j\,34.3) = 110.1\ \angle 18^\circ\ 9$$

Drop between points *A* and *B* is

$$\mathbf{V}_{AB} = \mathbf{I}_S\mathbf{Z}_{AB} = (104.6 - j\,34.3)\,(10 + j\,20) = 1{,}732 + j\,1{,}749$$
$$\mathbf{E}_S = \mathbf{E}_1 + \mathbf{V}_{AB} = (65{,}564 + j\,1{,}593) + (1{,}732 + j\,1{,}749) = 67{,}296 + j\,3{,}342 = 67{,}380\ \angle 2^\circ\ 51$$

(*a*) Sending-end voltage (line value) is $67{,}380 \times \sqrt{3} = 116{,}700$ V = **116.7 kV**

Sending-end current = **110.1 A**

Note. Phase difference between E_S and I_S is $(-18^\circ 9 - 2^\circ 51) = -21^\circ$ with current lagging. Hence, p.f. at sending-end is $\cos\phi_S = \cos 21^\circ = 0.934$ (lag).

(*b*) Copper loss for three phases between points *B* and *C* (Fig. 41.32) is

$$= 3 \times 116.6^2 \times 10 = 0.408\text{ MW}$$

Copper loss for three phases between points *A* and *B* is $= 3 \times 110.1^2 \times 10 = 0.363$ MW

Total Cu loss for 100 km of line length = 0.408 + 0.363 = 0.771 MW

Transmission $\eta = 20 \times 100/20.771 =$ **96.27 %**

Note. Line losses could also be computed by finding the line input which $= \sqrt{3}\,E_S.I_S \cos\phi_S$.

41.22. "Nominal" π-method

In this method, the line-to-neutral capacitance is divided into two halves ; one half being concentrated or localized at the sending-end and the other half at the receiving-end as shown in Fig. 41.35 (*a*). The capacitance at the sending or generating end has no effect on line drop or line regulation but its charging current must be added to the line current in order to obtain the total sending-end current $\mathbf{I}_S$.

It is obvious that I_S is the vector sum of I_{C2} and I_L where I_L is the vector sum of I_{C1} and I_R. The vector diagram is shown in Fig. 41.35 (*b*). E_R is taken as the reference vector. Current I_L is vector sum of I_R and I_{C1} (which is ahead of E_R by 90º). The drop $AB = I_L R$ is in phase with vector for I_L and reactive drop $BC = I_L X$ is in quadrature with I_L. *OC* represents the sending-end voltage. The sending-end current I_S is the vector sum of I_L and I_{C2} (which itself is ahead of E_S by 90º). It may be noted that

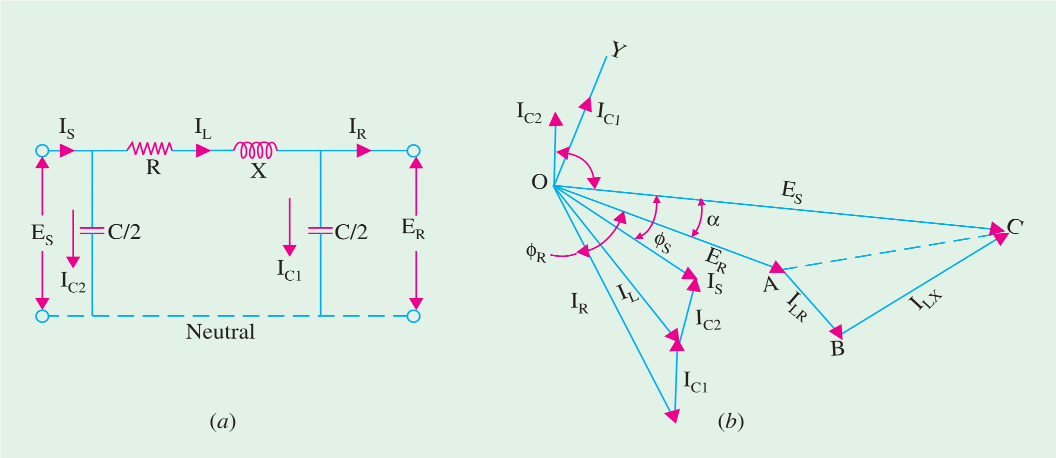


Fig. 41.35

if leakage reactance is not negligible, then leakage conductance G can also be divided into two equal halves and put at both ends in parallel with the capacitors as shown in Fig. 41.36.

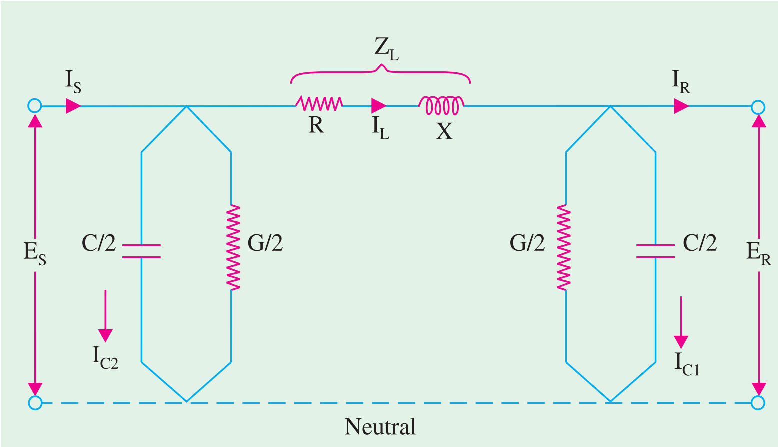


Fig. 41.36

41.23. Ferranti Effect

A long or medium transmission line has considerable capacitance and so draws leading charging current from the generating-end ***even when unloaded***. Moreover, receiving-end voltage V_R under no-load condition is found to be ***greater*** than sending-end voltage V_S. This phenomenon is known as ***Ferranti effect***.

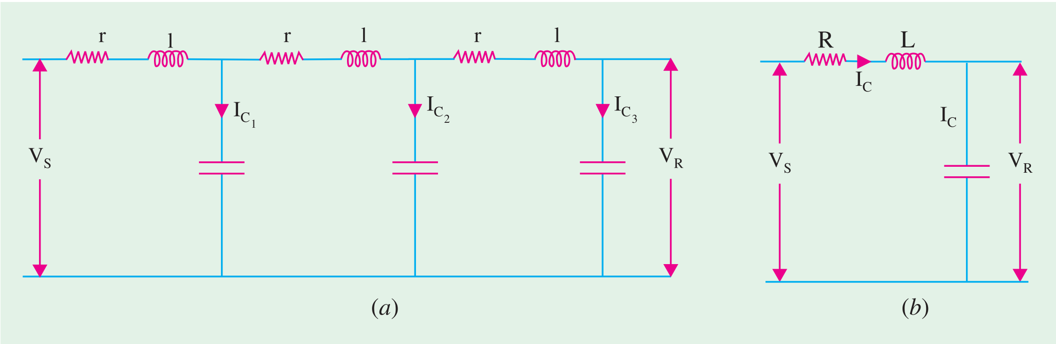


Fig. 41.37

Fig. 41.37 (*a*) shows the distributed parameters of such a line. It may be replaced by the circuit of Fig, 41.37 (*b*) where these distributed parameters have been lumped. As shown in the phasor diagram of Fig. 41.38, the charging current I_C leads V_R by 90° and produces a phase voltage drop = $I_C Z = I_C (R + j X_L)$.

Obviously, $V_S < V_R$ Now, $I_C = V_R \omega C$

As seen from Fig. 41.38

$$V_S = \sqrt{(V_R - I_C X_L)^2 + (I_C R)^2}$$

If R is negligible, then

$$V_S = (V_R - I_C X_L) \text{ or } V_R = V_S + I_C X_L$$

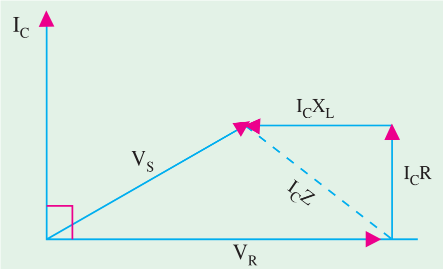


Fig. 41.38

41.24. Charging Current and Line Loss of an Unloaded Transmission Line

Fig. 41.39 shows the distribution of capacitance in a long transmission line of length l. Obviously, charging current I_C has maximum value at the sending end and linearly falls to zero at the receiving end. Accordingly, value of charging current at distance x from the sending end is proportionally equal to $I_C (l - x)/l$. The RMS value I of this current is given by

$$I^2 = \frac{1}{l}\int_0^l \frac{I_C^2 (l-x)^2}{l^2}.dx = \frac{I_C^2}{l^3}\int_0^l (l^2 + x^2 - 2lx)\, dx$$

$$= \frac{I_C^2}{l^3}\left[l^2 x + \frac{x^3}{3} - lx^2\right]_0^l = \frac{I_C^2}{3}$$

$$\therefore \qquad I = I_C/\sqrt{3}$$

If R is the resistance of the line per phase, then total power loss in the line is

$$= 3 I^2 R = 3\left(\frac{I_C}{\sqrt{3}}\right)^2 R = I_C^2 R.$$

Example 41.26. *A 3-phase transmission line, 100 km long has following constants: resistance per km per phase = 0.28 Ω; inductive reactance per km per phase = 0.63 Ω. Capacitive susceptance per km per phase = 4×10^{-6} siemens. If the load at the receiving end is 75 MVA at 0.8 p.f. lagging with 132 kV between lines calculate sending-end voltage, current and p.f. Use nominal-π-method.*

(Power System-I, AMIE, Sec. B. 1994)

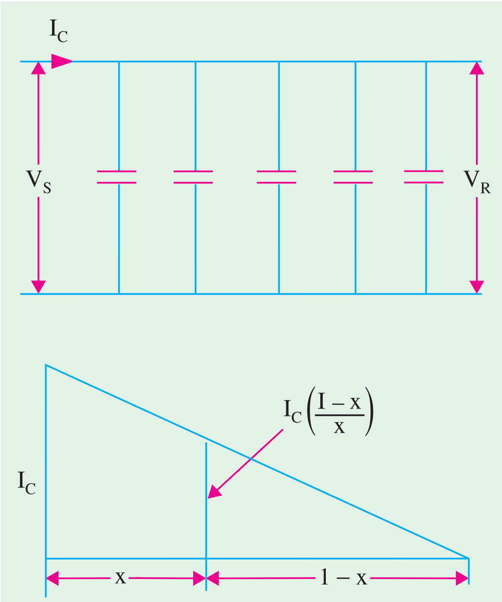


Fig. 41.39

Solution. For a line of length 100 km,

resistance/phase = 0.28 × 100 = 28 Ω ;

inductive reactance/phase = 0.63 × 100 = 63 Ω

Capacitive susceptance/phase = $4 \times 10^{-6} \times 100 = 4 \times 10^{-4}$ S

Capacitive susceptance at each end = 2×10^{-4} S

$\mathbf{V}_R = 132 \times 10^3/\sqrt{3} = 76{,}230$ V ; $\mathbf{V}_R = (76{,}230 + j\,0)$; $\mathbf{I}_R = 75 \times 10^6/\sqrt{3} \times 132 \times 10^3 \times 0.8 = 410$

$\mathbf{I}_R = 410\,(0.8 - j\,0.6) = 328 - j\,246$

$\mathbf{Y}_{C1} = j\,2 \times 10^{-4}$ S ; $\mathbf{I}_{C1} = \mathbf{V}_R \,.\, \mathbf{Y}_{C1}$

$= 76{,}230 \times j\,2 \times 10^{-4} = j\,15.25$ A

$\mathbf{I}_L = \mathbf{I}_R + \mathbf{I}_{C1} = 328 - j\,231$ A

Drop per conductor = $\mathbf{I}_L \,.\, \mathbf{Z}_L$ = (328 – j 231) (28 + j 63) = 23,737 + j 14,196

$\mathbf{V}_S = \mathbf{V}_R + \mathbf{I}_L\,\mathbf{Z}_L$ = 76,230 + 23,737 + j 14,196 = 99,967 + j 14,196 = 100,970 ∠8.1º

Line value of sending-end voltage = $100{,}970 \times 10^{-3} \times \sqrt{3}$ = **174.9 kW**

$\mathbf{I}_C = \mathbf{Y}_S\,\mathbf{Y}_{C2}$ = 100,970 ∠8.1º × 2 × 10^{-4} ∠90º = 20.19 ∠98.1º = –2.84 + j 20

$\mathbf{I}_S = \mathbf{I}_C^{\,2} + \mathbf{I}_L$ = (–2.84 + j 20) + (328 – j 231) = 325.2 – j 211 = 388 ∠–33º

Angle between V_S and I_S = 8.1º + 33º = 41.4º ; cos φ = **0.877 (lag)**

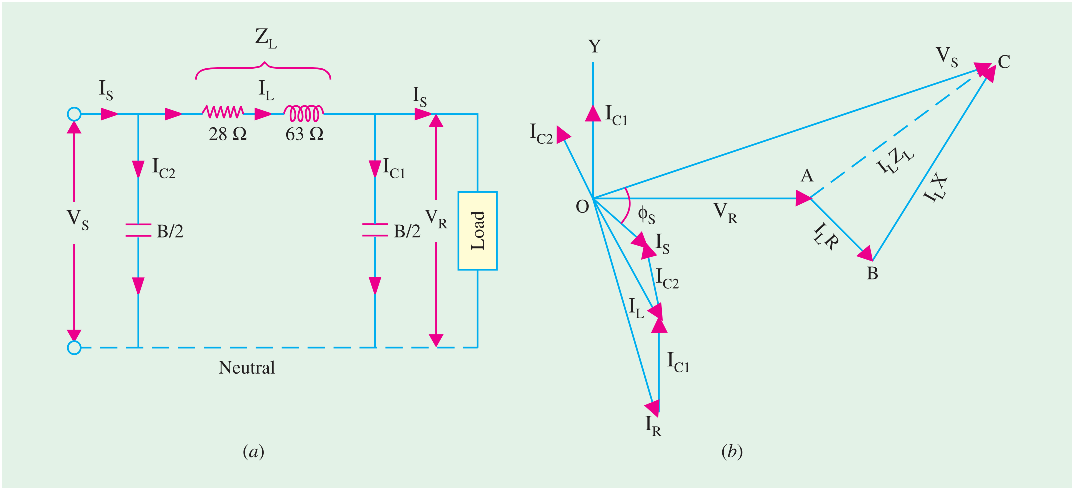


Fig. 41.40

Example 41.27. *A 100-km long, three-phase, 50-Hz transmission line has resistance/phase/km = 0.1 Ω ; reactance/phase/km = 0.5 Ω ; susceptance/phase/km = 10 × 10^{-6} siemens.*

If the line supplies a load of 20 MW at 0.9 p.f. lagging at 66 kV at the receiving end, calculate by nominal 'p' method, the regulation and efficiency of the line. Neglect leakage.

(Electrical Power-I, Bombay Univ. 1991)

Solution. For a 100 km line

resistance/phase = 0.1 × 100 = 10 Ω ; inductive reactance/phase = 0.5 × 100 = 50 Ω

Capacitive susceptance/phase = 10 × 10^{-4} × 100 = 10 × 10^{-4} siemens

Susceptance at each end = 5 × 10^{-4} siemens

$V_R = 66 \times 10^3/\sqrt{3} = 38{,}100$ V ∴ $\mathbf{V}_R$ = (38,100 + j 0)

$I_R = 20 \times 10^6/\sqrt{3} \times 66 \times 10^3 \times 0.9 = 195$ A ; $\mathbf{I}_R$ = 195(0.9 – j 0.435) = 176 – j 85

$\mathbf{I}_{C1}$ = 38100 × j 5 × 10^{-4} = j 19.1 A ; $\mathbf{I}_L = \mathbf{I}_R + \mathbf{I}_{C1}$ = 176 – j 66

drop/conductor = $\mathbf{I}_L\,\mathbf{Z}_L$ = (176 – j 66) (10 + j 50) = 5060 + j 8140

$\mathbf{V}_S = \mathbf{V}_R + \mathbf{I}_L\,\mathbf{Z}_L$ = 38,100 + 5060 + j 8140 = 43,160 + j 8140 = 43,920 ∠10.7º

Line value = $43{,}920 \times 10^{-3} \times \sqrt{3}$ = 76 kV

$\mathbf{I}_{C2} = \mathbf{V}_S.\mathbf{Y}_{C2}$ = 43,920 ∠10.7º × 5 × 10^{-4} ∠ 90º = 21.96 ∠ 100.7º = – 4.1 + j 21.6

$\mathbf{I}_S = \mathbf{I}_{C2} + \mathbf{I}_L$ = 172 – j 44.4 = 177.6 ∠–14.5º ; ϕ_S = 14.5º + 10.7º = 25.2º

cos ϕ_S = 0.905 (lag)

(*i*) % regulation $= \dfrac{76 - 66}{66} \times 100$ = **15.15%**

(*ii*) line input $= \sqrt{3} \times 76 \times 103 \times 177.6 \times 0.905 = 21.5 \times 10^6\ \Omega = 21.15$ MW

$\therefore$ transmission $\eta = 20 \times 100/21.15 =$ **94.56%**

Example 41.28. (*a*) *A 50-Hz, 3-phase, 100-km long line delivers a load of 40 MVA at 110 kV and 0.7 p.f. lag. The line constants (line to neutral) are : resistance of 11 ohms, inductive reactance of 38 ohms and capacitive susceptance of 3×10^{-4} siemens. Find the sending-end voltage, current, power factor and efficiency of power transmission.*

(*b*) *draw the vector diagram.*

(*c*) *If the sending-end voltage is held constant and load is removed, calculate the receiving-end voltage and current.* **(Electrical Power-II, Bangalore Univ.)**

Solution. (*a*) Nominal π-method will be used to solve this problem. Capacitive susceptance (or admittance) at each end $= Y/2 = 3 \times 10^{-4}/2 = 1.5 \times 10^{-4}$ siemens as shown in Fig. 41.41.

$\mathbf{V}_R = 110/\sqrt{3} = 63.5$ kV/phase ; $\mathbf{I}_R = 40 \times 10^6/\sqrt{3} \times 110 \times 10^3 = 210$ A

Let $\mathbf{V}_R = (63{,}500 + j\,0)$; $\mathbf{I}_R = 210\,(0.7 - j\,0.714) = (147 - j\,150)$

$\mathbf{Y}_{C1} = 1.5 \times 10^{-4}$ siemens ; $\mathbf{I}_{C1} = \mathbf{V}_R \mathbf{Y}_{C1} = 63{,}500 \times j1.5 \times 10^{-4} = j\,9.5$ A

As seen from Fig. 41.36

$$\mathbf{I}_L = \mathbf{I}_R + \mathbf{I}_{C1} = (147 - j\,140.5)\text{ A}$$

$$\text{Voltage drop/phase} = \mathbf{I}_L \mathbf{Z}_L = (147 - j\,140.5)\,(11 + j\,38) = 6956 + j\,4041$$

$$\mathbf{V}_S = \mathbf{V}_R + \mathbf{I}_L \mathbf{Z}_L = 63{,}500 + 6{,}956 + j4041 = 70{,}456 + j4041 = 70{,}570\ \angle 3.2^\circ$$

Line value of sending-end voltage

$$= \sqrt{3} \times 70{,}570 = 122.2\text{ kV}$$

$$\mathbf{I}_{C1} = \mathbf{V}_S \mathbf{Y}_{C_2}$$
$$= (70{,}570 + j\,4041) \times j\,1.5 \times 10^{-4}$$
$$= -0.6 + j\,10.6 = 10.6\ \angle 93.2^\circ$$
$$\mathbf{I}_S = \mathbf{I}_{C_2} + \mathbf{I}_L$$
$$= (-0.6 + j\,10.6) + (147 - j\,140.5)$$
$$= 146.4 - j\,129.9 = 195.7\ \angle -41.6^\circ$$

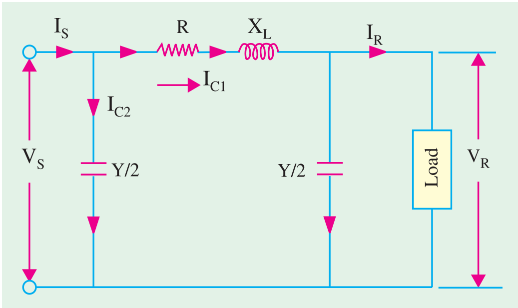


Fig. 41.41

Angle between V_S and $I_S = 41.6^\circ + 3.2^\circ = 44.8^\circ$; $\cos\phi_S = \cos 44.8^\circ = 0.71$ (lag)

$$\text{Power input} = \sqrt{3} \times 122.2 \times 10^3 \times 195.7 \times 0.7 = 29.41\text{ MW}$$

$$\text{Power output} = 40 \times 0.7 = 28\text{ MW}$$

$\therefore$ power transmission $\eta = 28/29.41 = 0.952$ or **95.2%**

(*b*) vector diagram is similar to that shown in Fig. 41.35 (*b*).

(*c*) As seen from Fig. 41.41 under no-load condition, current in the conductor is

$$= \mathbf{I}_{C_1} = j\,9.5\text{ A}$$

$$\text{Drop/phase} = \mathbf{I}_{C_1} \mathbf{Z}_L = j\,9.5(11 + j\,38) = -361 + j\,105$$

$$\mathbf{V}_S = 63{,}500 + (-361 + j\,105) = 63{,}140 + j\,105 = 63{,}140\ \angle 0.1^\circ$$

$$\mathbf{I}_{C2} = (63{,}140 + j\,105) \times j\,1.5 \times 10^{-4} = (-\,0.016 + j\,9.47)\text{ A}$$

$$\mathbf{I}_S = \mathbf{I}_{C_1} + \mathbf{I}_{C_2} = j\,9.5 - 0.016 + j\,9.47 = (-\,0.016 + j\,18.97)\text{ A}$$

Tutorial Problem No. 41.2

1. A 50-Hz, 3-phase line 100 km long delivers a load of 40,000 kVA at 110 kV and a lagging power factor of 0.7. The line constants (line-to-neutral) are : resistance 11 Ω ; inductive reactance 38 Ω; capacitive susceptance 3×10^{-4} S (half at each end), leakage negligible. Find the sending-end voltage, current, power factor and power input. **[122 kV ; 195.8 A ; 0.715 ; 29.675 kW]**

2. A 50-Hz, 3-phase transmission line has the following constants (line-to-neutral). Resistance 11 Ω, reactance 38 Ω; susceptance 3×10^{-4} S, leakage negligible. The capacitance can be assumed located half at each end of the line. Calculate the sending-end voltage, the line current and the efficiency of transmission when the load at the end of the line is 40,000 kVA at 110 kV power factor 0.7 lagging. **[122 kV ; 203.35 A ; 95.5%]**

3. A 3-phase transmission line has the following constants (line-to-neutral) ; R = 10 Ω ; inductive reactance = 20 Ω, capacitive reactance = 2.5 k Ω. Using the nominal T-method, calculate the voltage, line current and power factor at sending-end and the efficiency of transmission when the transmission line supplies a balanced load of 10 MW at 66 kV and power factor 0.8 lagging. **[69.5 kV ; 100 A ; 0.85 lag ; 96.8 %] (*Electrical Technology ; M.S. Univ. Baroda*)**

4. A 3-phase line has a resistance of 5.31 ohms and inductance of 0.0176 henry. Power is transmitted at 33 kV, 50 Hz from one end and the load at the receiving end is 3600 kW at 80 per cent power factor. Find the line current, receiving-end voltage, sending-end power factor and the efficiency of transmission. **[81.5 A, 31.9 kV, 0.796, 97.2%] (*U.P.S.C.*)**

5. Calculate the % regulation of 6.6 kV single-phase A.C. transmission line delivering 40 amps current at 0.8 lagging power factor. The total resistance and reactance of the line are 4.0 ohm and 5.0 ohm per phase respectively. **[3.76%]**

6. A-3 phase load of 2,000 kVA at 0.8 power factor is supplied at 6.6 kV, 50 Hz by means of a 33 kV transmission line, 32 km long in association with a 33/6.6 kV step-down transformer. The resistance and reactance of each conductor per km are 0.4 ohm and 0.5 ohm respectively. The resistance and reactance of the transformer primary are 7.5 ohm and 13.2 ohm while those of secondary are 0.35 ohm and 0.65 Ω respectively. Find the voltage necessary at the sending end of the transmission line when the voltage is maintained at 6.6 kV at the receiving end. Determine also the sending-end power and efficiency of transmission. **[35.4 kV, 1,689.5 kW, 94.7%]**

7. The generalised A and B constants of a transmission line are $0.96 \angle 1^\circ$ and $120 \angle 80^\circ$ respectively. If the line to line voltages at the sending and receiving ends are both 110 kV and the phase angle between them is 30º, find the receiving-end power factor and the current.

   With the sending-end voltage maintained at 110 kV, if the load is suddenly thrown off, find the corresponding receiving-end voltage. **[0.868 (lead), 326 A,114.5 kV]**

8. A 3-phase, 50-Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of 0.1 Ω, 0.5 Ω and 3×10^{-6} siemens/km/phase. If the line delivers 50 MW at 110 kV and 0.8 p.f. lag, determine the sending-end voltage and current. Assume a nominal π-circuit for the line. **[143.7 kV, 306.3 A]**

41.25. Generalised Circuit Constants of a Transmission Line

For any 4-terminal network *i.e.* one having two input and two output terminals (like a transmission line) the sending-end voltage per phase and the currents at the receiving and sending-end can be expressed by the following two equations :

$$\mathbf{V}_S = \mathbf{AV}_R + \mathbf{BI}_R \qquad \text{...(i)}$$

$$\mathbf{I}_S = \mathbf{CV}_R + \mathbf{DI}_R \qquad \text{...(ii)}$$

where **A**, **B**, **C** and **D** are the constants known as '*generalized circuit constants*' of the transmission line. Their values depend on the particular method adopted for solving the transmission network. Let us consider the following cases.

***(i)* Short Line.** In the case of lines up to 50 km, the effect of capacitance on the line performance is negligible. Hence, such a line can be represented as shown in Fig. 41.42 (*a*).

Here, $\mathbf{I}_S = \mathbf{I}_R$ and $\mathbf{V}_S = \mathbf{V}_R + \mathbf{I}_R\mathbf{Z}$

Comparing these with Eq. ***(i)*** and ***(ii)*** above, we get that

$$\mathbf{A} = 1 ; \quad \mathbf{B} = \mathbf{Z}, \quad \mathbf{C} = 0 \quad \text{and} \quad \mathbf{D} = 1$$

Incidentally, it may be noted that **AD – BC** = 1

(*ii*) Medium Line–Nominal-T Method

The circuit is shown in Fig. 41.42 (*b*).

It is seen that, $\mathbf{V}_S = \mathbf{V}_1 + \mathbf{I}_S \mathbf{Z}/2$ ...(*iii*)

Also $\mathbf{V}_1 = \mathbf{V}_R + \mathbf{I}_R\mathbf{Z}/2$ ...(*iv*)

Now, $\mathbf{I}_C = \mathbf{I}_S - \mathbf{I}_R = \mathbf{V}_1\mathbf{Y} = \mathbf{Y}(\mathbf{V}_R + \mathbf{I}_R\mathbf{Z}/2)$

$\therefore$ $\mathbf{I}_S = \mathbf{V}_R\mathbf{Y} + \mathbf{I}_R\left(1+\frac{\mathbf{YZ}}{2}\right)$ ...(*v*)

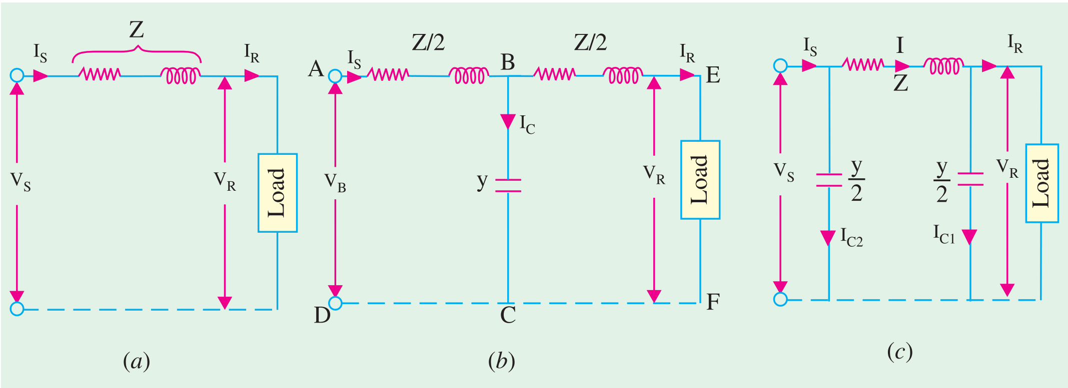


Fig. 41.42

Eliminating $\mathbf{V}_1$ from Eq. (*iii*) and (*iv*) we get

$$\mathbf{V}_S = \mathbf{V}_R + \frac{\mathbf{I}_S\mathbf{Z}}{2} + \frac{\mathbf{I}_R\mathbf{Z}}{2} \quad ...(vi)$$

Substituting the value of I_S, we get

$$\mathbf{V}_S = \left(1+\frac{\mathbf{YZ}}{2}\right)\mathbf{V}_R + \left(\mathbf{Z}+\frac{\mathbf{YZ}^2}{4}\right)\mathbf{I}_R \quad ...(vii)$$

Comparing Eq. (*vii*) and (*v*) with Eq. (*i*) and (*ii*) respectively, it is found that

$$\mathbf{A} = \mathbf{D} = 1+\frac{\mathbf{YZ}}{2}\,;\ \mathbf{B} = \left(1+\frac{\mathbf{YZ}}{4}\right) \text{ and } \mathbf{C} = \mathbf{Y}$$

It can again be proved that **AD – BC** = 1.

(*iii*) Medium Line–Nominal- π Method

The circuit is shown in Fig. 41.42 (*c*). Here, series impedance per phase = (R + j X) and admittance is $\mathbf{Y} = j\omega C$.

As seen, $\mathbf{I}_S = \mathbf{I} + \mathbf{I}_{C2} = \mathbf{I} + \mathbf{V}_S\mathbf{Y}/2$ ...(*viii*)

Also $\mathbf{I} = \mathbf{I}_{C1} + \mathbf{I}_R = \mathbf{I}_R + \mathbf{V}_R\mathbf{Y}/2$ ...(*ix*)

Now, $\mathbf{V}_S = \mathbf{V}_R + \mathbf{IZ} = \mathbf{V}_R + \mathbf{Z}\left(\mathbf{I}_R + \frac{\mathbf{V}_R\mathbf{Y}}{2}\right)$

$$= \left(1+\frac{\mathbf{YZ}}{2}\right)\mathbf{V}_R + \mathbf{ZI}_R \quad ...(x)$$

Eliminating **I** from Eq. (*viii*) and (*ix*), we get

$$\mathbf{I}_S = \mathbf{I}_R + \frac{\mathbf{V}_R\mathbf{Y}}{2} + \frac{\mathbf{V}_S\mathbf{Y}}{2}$$

Now, substituting the value of $\mathbf{V}_S$, we have

$$\mathbf{I}_S = \mathbf{I}_R + \frac{\mathbf{V}_R\mathbf{Y}}{2} + \frac{\mathbf{Y}}{2}\left[\left(1+\frac{\mathbf{YZ}}{2}\right)\mathbf{V}_R + \mathbf{ZI}_R\right]$$

or $$\mathbf{I}_S = \mathbf{Y}\left(1+\frac{\mathbf{YZ}}{4}\right)\mathbf{V}_R + \left(1+\frac{\mathbf{YZ}}{2}\right)\mathbf{I}_R \quad ...(xi)$$

Comparing Eq. (*x*) and (*ii*) with Eq. (*i*) and (*xi*) above, we get,

$$\mathbf{A} = \mathbf{D} = \left(1+\frac{\mathbf{YZ}}{2}\right);\ \mathbf{B} = \mathbf{Z},\ \mathbf{C} = \mathbf{Y}\left(1+\frac{\mathbf{YZ}}{4}\right)$$

Again, it can be shown that **AD – BC = 1**

Example 41.29. *Find the following for a single-circuit transmission line delivering a load of 50 MVA at 110 kV and p.f. 0.8 lagging :*

(i) *sending-end voltage,* ***(ii)*** *sending-end current,* ***(iii)*** *sending-end power, (iv) efficiency of transmission. (Given A = D = 0.98 ∠3° , B = 110 ∠75° ohm, C = 0.0005 ∠80° ohm)*

(Power Systems, AMIE, Sec. B, 1993)

Solution. Receiving-end voltage, $V_R = 110/\sqrt{3} = 63.5$ kV

Taking this voltage as reference voltage, we have

$$\mathbf{V}_R = (63{,}500 + j\,0)$$
$$\mathbf{I}_R = 50 \times 10^6/\sqrt{3} \times 110 \times 10^3 = 262.4 \text{ A}$$
$$\mathbf{I}_R = 262.4\ \angle\cos^{-1} 0.8 = 262.4\ (0.8 - j\,0.6) = (210 - j\,157.5) \text{ A}$$
$$\mathbf{AV}_R = 0.98\ \angle 3^\circ \times 63{,}500\ \angle 0^\circ = 62{,}230\ \angle 3^\circ = (62145 + j\,3260) \text{ V}$$
$$\mathbf{BI}_R = 110\ \angle 75^\circ \times 262.4\ \angle 36.86^\circ = 28{,}865\ \angle 111.8^\circ = (-10{,}720 + j\,26{,}800) \text{ V}$$
(*i*) $$\mathbf{V}_S = \mathbf{AV}_R + \mathbf{BI}_R = 51{,}425 + j\,30{,}060 = 59{,}565, \angle 30.3^\circ \text{ V}$$
$$\mathbf{CV}_R = 0.0005\ \angle 80^\circ \times 63{,}500\ \angle 0^\circ = 31.75\ \angle 80^\circ = (5.5 + j\,31.3) \text{ A}$$

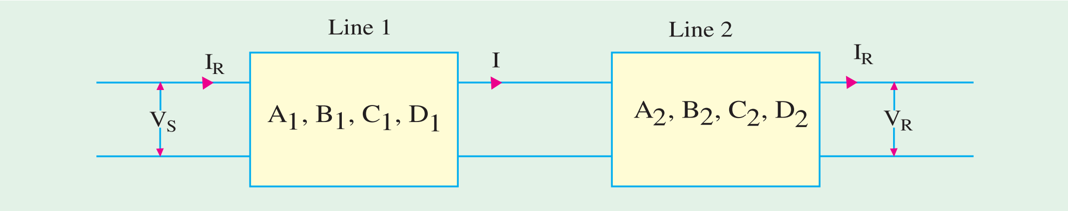


Fig. 41.43

$$\mathbf{DI}_R = 0.98\angle 3^\circ \times 262.4\ \angle 36.86^\circ = 257.75\ \angle 39.86^\circ = 197.4 + j\,164.8$$
(*ii*) $$\mathbf{I}_S = \mathbf{CV}_R + \mathbf{DI}_R = 203 + j\,196 = \mathbf{282\ \angle 44^\circ A}$$

(*iii*) Sending-end power = $3V_S\, I_S \cos\phi_S = 3 \times 59.565 \times 282 \times \cos\angle 44.30^\circ - 30.3^\circ$ = **48.96 MW**

Receiving end power = 50 × 0.8 = 40 MW

(*iv*) Transmission η = 40 × 100/48.96 = **81.7%**

Example 41.30. *A 150 km, 3-φ, 110-V, 50-Hz transmission line transmits a load of 40,000 kW at 0.8 p.f. lag at receiving end.*

resistance/km/phase = 0.15 Ω, reactance/km/phase = 0.6 Ω ; susceptance/km/phase = 10^{-5} S

(a) *determine the A, B, C and D constants of the line* ***(b)*** *find regulation of the line.*

(Power System-I, AMIE, 1993)

Solution. We will use the nominal-π method to solve the problem. For a length of 150 km :

$$\mathbf{R} = 0.15 \times 150 = 22.5\ \Omega\ ;\ \mathbf{X} = 0.6 \times 150 = 90\ \Omega\ ;\ \mathbf{Y} = 150 \times 10^{-5} \text{ S} = 15 \times 10^{-4} \text{ S}$$
$$\mathbf{Z} = (\mathbf{R} + j\,\mathbf{X}) = (22.5 + j\,90) = 92.8\ \angle 75^\circ \text{ ohm}\ ;\ \mathbf{Y} = 15 \times 10^{-4}\ \angle 90^\circ \text{ S}$$

(*a*) $\mathbf{A} = \mathbf{D} = \left(1+\frac{\mathbf{YZ}}{2}\right) = 1 + \frac{j\,15\times10^{-4}}{2}(22.5 + j\,90) = (0.9675 + j\,0.01688)$
$= 0.968 \angle 1.0°$

$\mathbf{B} = \mathbf{Z} = 92.8 \angle 7.5°$; $\mathbf{C} = \mathbf{Y}\left(1+\frac{\mathbf{YZ}}{4}\right) = j\,15\times10^{-4}\left[1+\frac{1}{4}.\,j\,15\times10^{-4}(22.5+j\,90)\right]$
$= -0.00001266 + j\,0.00145 = 0.00145 \angle 90.5°$

(*b*) Now, the regulation at the receiving-end is defined as the change in voltage when full-load is thrown off, the sending-end voltage being held constant.

Now, when load is thrown off, $\mathbf{I}_R = 0$. Hence, putting this value in $\mathbf{V}_S = \mathbf{AV}_R + \mathbf{BI}_R$, we get

$$\mathbf{V}_S = \mathbf{AV}_{RO} \text{ or } \mathbf{V}_{RO} = \frac{\mathbf{V}_S}{\mathbf{A}} \quad \therefore \quad \%\text{ regn.} = \frac{V_{RO}-V_R}{V_R}\times100 = \frac{(V_S/A - V_R)}{V_R}\times100$$

Now, $V_R = 100/\sqrt{3}$ kV = 63,520 V—at load

$I_R = 40\times10^6/\sqrt{3}\times110\times10^3\times0.8 = 263$ A ; $\mathbf{I}_R = 263\,(0.8 - j\,0.6) = (210 - j\,158)$

$\mathbf{V}_S = \mathbf{AV}_R + \mathbf{BI}_R = 63{,}520\,(0.9675 + j\,0.01688) + (22.5 + j\,90)\,(210 - j\,158)$
$= 80{,}450 + j\,16{,}410 = 82{,}110 \angle 11.5°$

$V_S/A = 82{,}110/0.968 = 84{,}800$; regn. $= \frac{84{,}800 - 63{,}520}{63{,}520}\times100 =$ **33.5%**

Example 41.31. *A 132-kV, 50-Hz, 3-phase transmission line delivers a load of 50 MW at 0.8 p.f. lagging at receiving-end.*

The generalised constants of the transmission line are

$A = D = 0.95 \angle 1.4°$; $B = 96 \angle 7.8°$; $C = 0.0015 \angle 90°$

Find the regulation of the line and the charging current. Use nominal T-method.

(Electrical Power-I, Bombay Univ.)

Solution. $I_R = 50\times10^6/\sqrt{3}\times132\times10^3\times0.8 = 273$ A

$\mathbf{I}_R = 273\,(0.8 - j\,0.6) = 218 - j\,164 = 273 \angle -36.9°$

$\mathbf{V}_R = 132{,}000/\sqrt{3} = 76{,}230$; $\mathbf{V}_R = 76{,}230 + j\,0$

$\mathbf{V}_S = \mathbf{AV}_R + \mathbf{BI}_R = 76{,}230 \angle 0° \times 0.95 \angle 1.4° + 96 \angle 78°.\ 273 \angle -36.9°$
$= 72{,}418 \angle 1.4 + 26{,}208 \angle 41.1° = 89{,}000 + j\,21{,}510 = 92{,}150 \angle 13°$

$\mathbf{I}_S = \mathbf{CV}_R + \mathbf{DI}_R = 76{,}230 \times 0.0015 \angle 90° + 0.95 \angle 1.4°.\ 273 \angle -36.9°$
$= j\,114 + 259.3 \angle -35.5° = 211 - j\,36$

$V_{RO} = V_S/A = 92{,}150/0.95 = 97{,}000$; % regn $= \frac{97{,}000 - 76{,}230}{76{,}230} = 27.1$

$\mathbf{I}_C = \mathbf{I}_S - \mathbf{I}_R = (211 - j\,36) - (218 - j\,164) = -7 + j\,128 = 128.2 \angle 93.1°$

Example 41.32. *A 3-phase transmission line consists of two lines 1 and 2 connected in series, line 1 being at the sending end and 2 at the receiving end. The respective auxiliary constants of the two lines are : A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2. Find the A, B, C, D constants of the whole line which is equivalent to two series-connected lines.*

Solution. The two series-connected lines along with their constants as shown in Fig. 41.43.

For line No. 1

$$\mathbf{V}_S = \mathbf{A}_1\mathbf{V} + \mathbf{B}_1\mathbf{I} \quad ; \quad \mathbf{I}_S = \mathbf{C}_1\mathbf{V} + \mathbf{D}_1\mathbf{I} \qquad ...(i)$$

For line No. 2

$$\mathbf{V} = \mathbf{A}_2\mathbf{V}_R + \mathbf{B}_2\mathbf{I}_R \quad ; \quad \mathbf{I} = \mathbf{C}_2\mathbf{V}_R + \mathbf{D}_2\mathbf{I}_2 \qquad ...(ii)$$

Substituting the values of **V** and **I** from Eq. (*ii*) into Eq. (*i*), we get

$$V_S = A_1(A_2V_R + B_2I_R) + B_1(C_2V_R + D_2I_R) = (A_1A_2 + B_1C_2)V_R + (A_1B_2 + B_1D_2)I_R$$

and $$I_S = C_1(A_2V_R + B_2I_R) + D_1(C_2V_R + D_2I_R) = (C_1A_2 + D_1C_2)V_R + (C_1B_2 + D_1D_2)I_R$$

Hence, the two lines connected in series have equivalent auxiliary constants of

$$\mathbf{A} = \mathbf{A_1A_2 + B_1C_2}, \qquad \mathbf{B} = \mathbf{A_1B_2 + B_1D_2}$$

$$\mathbf{C} = \mathbf{C_1A_2 + D_1C_2} \quad \text{and} \quad \mathbf{D} = \mathbf{C_1B_2 + D_1D_2}$$

41.26. Corona

When an alternating potential difference is applied across two conductors whose spacing is large as compared to their diameter, there is no apparent change in the condition of the atmospheric air surrounding the wires if voltage is low. However, when the p.d. is increased, then a point is reached when a faint luminous glow of bluish colour appears along the lengths of conductors and at the same time a hissing sound is heard. This bluish discharge is known as ***corona*****.** Corona is always accompanied by the production of ozone which is readily detected because of its characteristic odour. If the p.d. is further increased, then the glow and hissing both increase in intensity till a spark-over between the conductors takes place due to the break-down of air insulation. If the conductors are smooth and polished, the corona glow is uniform along their length but if there is any roughness, they will be picked up by relatively brighter illumination. In the case of conductors with a spacing shorter as compared to their diameters, sparking may take place without any visible glow. If the p.d. between wires is direct instead of alternating, there is a difference in the appearance of the two wires. The positive wire has a smooth glow about it whereas the glow about the negative wire is spotty.

Corona occurs when the electrostatic stress in the air around the conductors exceeds 30 kVA (maximum)/cm or 21.1 kV (r.m.s.)/cm. The ***effective disruptive critical voltage*** to neutral is given by the relation.

$$V_C = m_0 g_0\, \delta\, r \operatorname{logh} D/r \text{ kV/phase} = 2.3\, m_0 g_0\, \delta\, r \log_{10} D/r \text{ kV/phase}$$

where m_0 = irregularity factor which takes into account the surface conditions of the conductor.

D = distance between conductors in cm.

r = radius of the conductor in cm or the radius of the circumscribing circle in a stranded cable.

g_0 = breakdown strength or disruptive gradient of air at 76 cm of mercury and 25ºC
= 30 kV (max.)/cm = 21.1 kV (r.m.s.)/cm

δ = air density factor

Substituting the value of g_0, we have

$$V_C = 21.1\, m_0\, \delta\, r \operatorname{logh} D/r \text{ kV/phase} = 21.1\, m_0\, \delta\, r \times 2.3 \log_{10} D/r \text{ kV/phase}$$
$$= 48.8\, m_0\, \delta\, r \log_{10} D/r \text{ kV/phase}$$

The value of δ is given by $\delta = \dfrac{3.92\, b}{273 + t}$

where b = barometric pressure in cm of Hg and t = temp. in degrees centigrade.

when b = 76 cm and t = 25º, C δ = 1

The irregularity factor m_0 depends on the shape of cross-section of the wire and the state of its surface. Its value is unity for an absolutely smooth wire of one strand of circular section and less than unity for wires roughened due to weathering as shown below :

Irregularity Factor

Polished wires	...	1.0
Weathered wires	...	0.93 to 0.98
7-strand cables, concentric lay	...	0.83 to 0.87
Cables with more than 7-strands	...	0.80 to 0.85

41.27. Visual Critical Voltage

This voltage is higher than the disruptive critical voltage V_C. It is the voltage at which corona appears all along the line. The *effective* or r.m.s. value of visual critical voltage is given by the following empirical relation.

$$
\begin{aligned}
V_v &= m_v \, g_0 \, \delta \, r \left(1 + \frac{0.3}{\sqrt{\delta \, r}}\right) \text{logh } D/r \text{ kV/phase} \\
&= 21.1 \, m_v \, \delta \, r \left(1 + \frac{0.3}{\sqrt{\delta \, r}}\right) \times 2.3 \log_{10} D/r \text{ kV/phase} \\
&= 48.8 \, m_v \, \delta \, r \left(1 + \frac{0.3}{\sqrt{\delta \, r}}\right) \log_{10} D/r \text{ kV/phase}
\end{aligned}
$$

where m_v is another irregularity factor having a value of 1.0 for smooth conductors and 0.72 to 0.82 for rough conductors.

41.28. Corona Power

Formation of corona is always accompanied by dissipation of energy. This loss will have some effect on efficiency of the line but will not have any appreciable effect on the line regulation. This loss is affected both by atmospheric and line conditions. Soon after the critical voltage is reached, the corona loss increases as the square of the excess voltage. The loss for voltage of V kilovolt to neutral is given by the following empirical relation.

$$
P = 241 \frac{(f + 25)}{\delta} \sqrt{\left(\frac{r}{D}\right)} (V - V_c)^2 \times 10^{-5} \text{ kW/km/phase}
$$

where f is the frequency of the a.c. in Hz.

Obviously, total loss equals 3 times the above.

41.29. Disadvantages of Corona

1. There is a definite dissipation of power although it is not so important except under abnormal weather conditions like storms etc.
2. Corrosion due to ozone formation.
3. The current drawn by the line due to corona losses is non-sinusoidal in character, hence it causes non-sinusoidal drop in the line which may cause some interference with neighbouring communication circuits due to electromagnetic and electrostatic induction. Such a shape of corona current tends to introduce a large third harmonic component.

   However, it has been found that corona works as a safety valve for surges.
4. Particularly intense corona effects are observed at a working voltage of 35 kV or higher. Hence, designs have to be made to avoid any corona on the bus-bars of substations rated for 35 kV and higher voltages during their normal operation. Corona discharge round bus-bars is extremely undesirable because the intense ionization of the air reduces its dielectric strength, makes it easier for the flashover to occur in the insulators and between phases particularly when the surfaces concerned are dirty or soiled with other deposits. The ozone produced due to corona discharge aggressively attacks the metallic components in the substations and switchgear, covering them with oxides. Moreover, the crackling sound of the corona discharge in a substation masks other sounds like light crackling noise due to arcing in a loose contact, the sound of an impending breakdown or creepage discharge in the equipment, the rattling noise due to the loosening of steel in a transformer core etc. The timely detection of such sounds is very important if any serious breakdown is to be avoided.

Example 41.33. *Find the disruptive critical voltage for a transmission line having :*
conductor spacing = 1 m ; conductor (stranded) radius = 1 cm
barometric pressure = 76 cm of Hg ; temperature = 40ºC
Air break-down potential gradient (at 76 cm of Hg and at 25ºC) = 21.1 kV (r.m.s.)/cm.
(Electric Power Systems-I, Gujarat Univ.)

Solution. $V_C = 2.3\, m_0\, g_0\, r \log_{10} D/r$ kV/phase

Here, $g_0 = 21.1$ kV (r.m.s.)/cm ; $m_0 = 0.85$ (assumed)

$\delta = 3.92 \times 76/(273 + 40) = 0.952$; $\log_{10} D/r = \log_{10}100/1 = 2$

$\therefore \quad V_c = 2.3 \times 0.85 \times 21.1 \times 0.952 \times 1 \times 2 = 78.54$ kV (r.m.s.)/phase

Line value $= 78.54 \times \sqrt{3} =$ **136 kV (r.m.s.)/phase**

Example 41.34. *Find the disruptive critical and visual corona voltage of a grid-line operating at 132 kV.*
conductor dia = 1.9 cm ; conductor spacing = 3.81 m
temperature = 44ºC ; barometric pressure = 73.7 cm
conductor surface factor :
fine weather = 0.8 ; rough weather = 0.66.
(Electrical Power Systems-III, Gujarat Univ.)

Solution. $V_c = 48.8\, m_0\, \delta\, r \log_{10} D/r$ kV/phase

Here, $m_0 = 0.8$; $\delta = 3.92 \times 73.7/(273 + 44) = 0.91$

$\log_{10} 381/1.9 = \log_{10} 200.4 = 2.302$

$\therefore \quad V_c = 48.8 \times 0.8 \times 0.91 \times 1.9 \times 2.302 = 155.3$ kV/phase

$$V_v = 48.8\, m_v\, \delta\, r \left(1 + \frac{0.3}{\sqrt{\delta r}}\right) \log_{10} D/r \text{ kV/phase}$$

Here, $m_v = 0.66$; $\delta = 0.91$; $\sqrt{\delta r} = \sqrt{0.91 \times 1.9} = 1.314$

$$\therefore \quad V_v = 48.8 \times 0.66 \times 0.91 \times 1.9 \left(1 + \frac{0.3}{1.314}\right) 2.302 = \textbf{157.5 kV/phase}$$

Example 41.35. *A certain 3-phase equilateral transmission line has a total corona loss of 53 kW at 106 kV and a loss of 98 kW at 110.9 kV. What is the disruptive critical voltage between lines? What is the corona loss at 113 kV?* **(Electrical Power Systems-II, Gujarat Univ.)**

Solution. As seen from Art. 41.28, the total corona loss for three phases is given by

$$P = 3 \times \frac{241\,(f + 25)}{\delta} \cdot \sqrt{\frac{r}{D}}\, (V - V_c)^2 \times 10^{-5} \text{ kW/km}$$

Other things being equal, $P \propto (V - V_c)^2$

$$\therefore \quad 53 \propto \left(\frac{106}{\sqrt{3}} - V_c\right)^2 \propto (61.2 - V_c)^2 \qquad \text{—1st case}$$

$$98 \propto \left(\frac{110.9}{\sqrt{3}} - V_c\right)^2 \propto (64 - V_c)^2 \qquad \text{— 2nd case}$$

$$\therefore \quad \frac{98}{53} = \frac{(64 - V_c)^2}{(61.2 - V_c)^2} \quad \text{or} \quad V_c = \textbf{54.2 kV/km}$$

Similarly, $W \propto \left(\frac{113}{\sqrt{3}} - V_c\right)^2 \propto (65.2 - V_c)^2$

$$\frac{W}{98} = \frac{(65.2 - V_c)^2}{(64 - V_c)^2} = \frac{(65.2 - 54.2)^2}{(64 - 54.2)^2} \quad \therefore \quad W = \mathbf{123.4\ kW}$$

Example 41.36. *A 3-phase, 50-Hz, 220-kV transmission line consists of conductors of 1.2 cm radius spaced 2 metres at the corners of an equilateral triangle. Calculate the corona power loss per km of the line at a temperature of 20ºC and barometric pressure of 72.2 cm. Take the surface factors of the conductor as 0.96.* **(Electrical Power-II, Bangalore Univ.)**

Solution. As seen from Art. 41.28, corona loss per phase is

$$P = 241\frac{(f+25)}{\delta}\sqrt{(r/D)}\,.\,(V - V_c)^2 \times 10^{-5}\ \text{kW/km/phase}$$

Here,
$$\delta = \frac{3.92\,b}{273+t} = \frac{3.92 \times 72.2}{273+20} = 0.966$$

$$V_c = 48.8\, m_0\, \delta\, r \log_{10} D/r = 48.8 \times 0.96 \times 0.966 \times 1.2 \times \log_{10}\frac{200}{1.2}$$

$$= 120.66 \text{ kV/phase}$$

$$V = 220/\sqrt{3} = 127 \text{ kV/phase}$$

$$\therefore \quad P = 241 \times \frac{75}{0.966} \times \sqrt{\left(\frac{1.2}{200}\right)} \times (127 - 120.66)^2 \times 10^{-5} = 0.579 \text{ kW/km/phase}$$

Total loss for 3 phase = 3 × 0.579 = **1.737 kW/km**

41.30. Underground Cables

Underground cables are used where overhead transmission is difficult

Underground cables are used where overhead lines are not possible as in large cities despite the fact that in their case, cost per kW per km is much more as compared to overhead transmission lines. Another advantage of overhead system for distributors is that tapping can be made at any time without any disturbance, which is of great importance in rapidly developing areas. However, underground cables are more advantageous for feeders which are not likely to be disturbed for tapping purposes because, being less liable to damage through storms or lighting or even wilful damage, they offer a safer guarantee of supply. But this advantage may be offset by the cost of trenching and expensive jointing necessary in case of repairs.

However, cables score over overhead lines in cases where voltage regulation is more important, because, due to very small spacing of their conductors, they have a very low inductance and hence low inductive drops.

Cables may be classified in two ways according to *(i)* the type of insulating material used in their manufacture or *(ii)* the voltage at which they transmit power. The latter method of classification is, however, more generally used according to which cables are divided into three groups :

1. Low-tension cables–up to 1000 V
2. High-tension cables–up to 23,000 V
3. Super-tension cables–from 66 kV to 132 kV

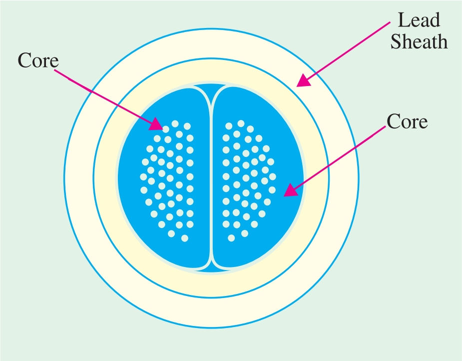


Fig. 41.44

For all cables, the conductor is tinned stranded copper of high conductivity. Stranding is done to secure flexibility and the number of conductors in a core is generally 3, 7, 19 and 37 etc. Except for 3-strand, all numbers have a centrally-disposed conductor with all others surrounding it. A cable may have one or more than one core depending on the type of service for which it is intended. It may be *(i)* single-core *(ii)* two-core *(iii)* three-core and *(iv)* four-core etc.

In Fig. 41.44 is shown a section through a twin-cored, high-tension lead-covered underground cable sheathed with a continuous tube of pure lead whereas Fig. 41.45 shows a section of a typical concentric type 2-core cable used for single-phase distribution. The cores are arranged concentrically. The outer core is arranged in the form of hollow tubing. Both cores are of stranded copper and paper-insulated and are protected by a lead-sheath.

In Fig. 41.46 is shown a section through a 3-core extra-high-tension paper-insulated lead-covered and steel-wire armoured cable.

The cores are surrounded by insulation or impregnated paper, varnished cambric or vulcanised bitumen.

The insulation is, in turn, surrounded by a metal sheath made of lead or a lead alloy and prevents the entry of moisture into the inner parts. On the sheath is applied the bedding which consists of two compounded paper tapes along with suitable compounded fibrous materials.

Next comes 'armouring' which is placed over the bedding and consists of either galvanized steel wires or two layers of steel tape. Armouring may not be done in the case of some cables.

Next comes 'serving' which consists of compounded fibrous material (jute etc.) placed over the armouring in the case of armoured cables or over the metal sheath in the case of unarmoured cables-in which case serving consists of two layers of compounded paper tapes and a final covering of compounded fibrous material.

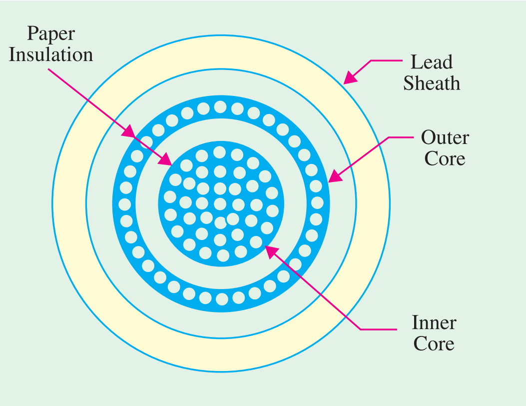


Fig. 41.45

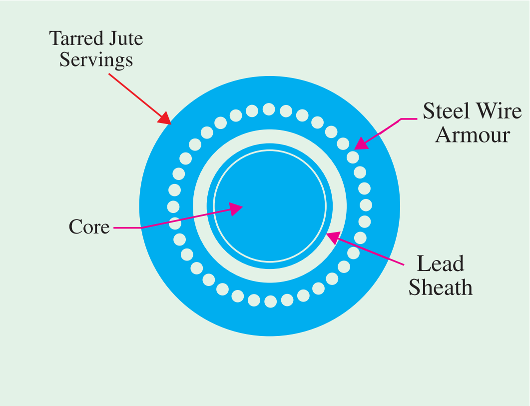


Fig. 41.46

41.31. Insulation Resistance of a Single-core Cable

The expression for the insulation resistance of such a cable, as derived in Art. 5.14.

$$= \frac{2.3\,\rho}{2\,\pi l} \log_{10} \frac{r_2}{r_1} \text{ ohm}$$

The value of specific resistance for paper is approximately 500 Ω-m.

41.32. Capacitance and Dielectric Stress

This has already been dealt with in Art. 5.9 and 5.13.

41.33. Capacitance of 3-core Belted Cables

The capacitance of a cable is much more important than of the overhead wires of the same length because in cables (*i*) conductors are nearer to each other and the sheath and (*ii*) they are separated by a dielectric medium of higher permittivity as compared to air. Fig. 41.47 shows a system of capacitances

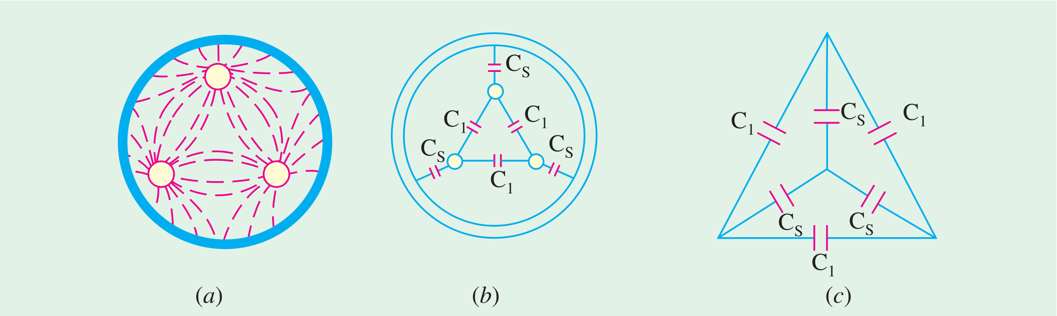


Fig. 41.47

in a belted 3-core cable used for 3-phase system. It can be regarded as equivalent to three-phase cables having a common sheath. Since there is a p.d. between pairs of conductors and between each conductor and sheath, there exist electrostatic fields as shown in Fig. 41.47 (*a*) which shows average distribution of electrostatic flux, though, actually, the distribution would be continually changing because of changing potential difference between conductors themselves and between conductors and sheath. Because of the existence of this electrostatic coupling, there exist six capacitances as shown in Fig. 41.47 (*b*). The three capacitances between three cores are delta-connected whereas the other three between each core and the sheath are star-connected, the sheath forming the star-point [Fig. 41.47 (*c*)].

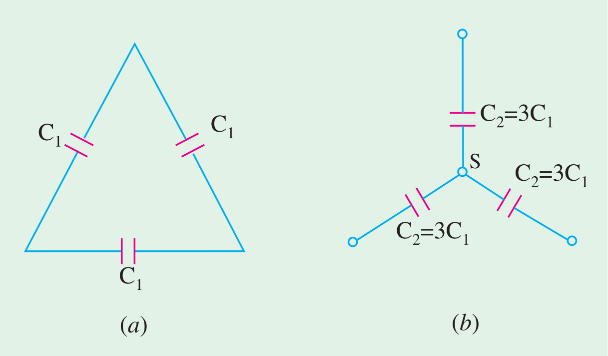


Fig. 41.48

The three delta-connected capacitances each of value C_1 can be converted into equivalent star-capacitance C_2 which will be three times the delta-capacitances C_1 as shown in Fig. 41.48.

The two star capacitances can now be combined as shown in Fig. 41.49 (*a*). In this way, the whole cable is equivalent to three star-connected capacitors each of capacitance $C_n = 3C_1 + C_S$ as shown in Fig. 41.49 (*b*).

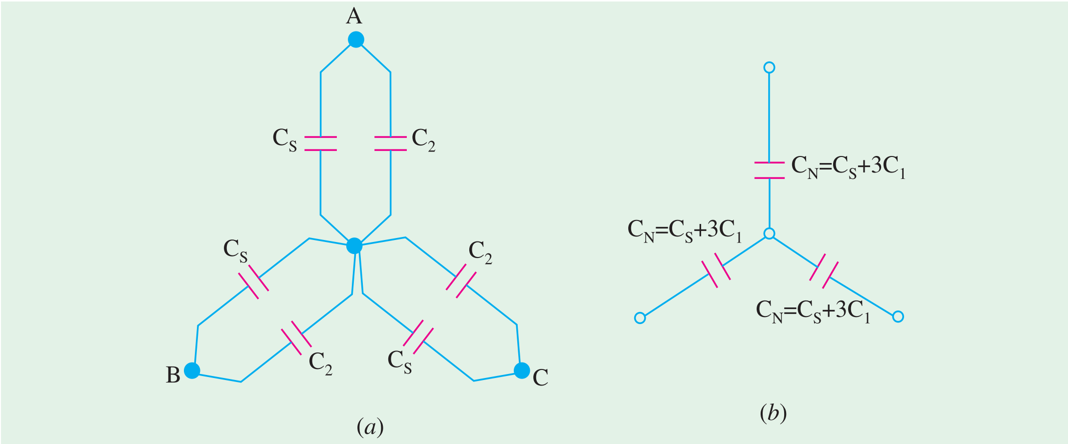


Fig. 41.49

If Vp is the phase voltage, then the charging current is given by

$$I_C = V_P(C_S + 3C_1)\omega = V_P\,\omega C_n$$

41.34. Tests for Three-phase Cable Capacitance

The following three tests may be used for the measurement of C_1 and C_S :

(i) In the first test, all the three cores are bunched together and then capacitance is measured by usual methods between these bunched cores and the earthed sheath. This gives $3\,C_S$ because the three capacitances are in parallel.

(ii) In the second method, two cores are bunched with the sheath and capacitance is measured between these and the third core. It gives $2C_1 + C_S$. From this value, C_1 and C_S can be found.

(iii) In the third method, the capacitance C_L between two cores or lines is measured with the third core free or shorted to earth. This gives

$$\left(C + \frac{C_1}{2} + \frac{C_S}{2}\right) = \frac{1}{2}(3\,C_1 + C_S) = \frac{C_n}{2}$$

Hence, C_n is twice the measured value *i.e.* $C_n = 2C_L$

Therefore, charging current $I_C = \omega V_P C_n = \dfrac{2}{\sqrt{3}}\,.\,\omega V_L C_L$

where V_L is the line voltage (not phase voltage).

Example 41.37. *A single-core lead-covered cable is to be designed for 66-kV to earth. Its conductor radius is 1.0 cm and its three insulating materials A, B, C have permittivities of 5.4 and 3 respectively with corresponding maximum safe working stress of 38 kV per cm (r.m.s. value), 26-kV per cm and 20-kV per cm respectively. Find the minimum diameter of the lead sheath.*

Solution.

$$g_{1max} = \frac{Q}{2\pi\epsilon_0\epsilon_{r1}\,r} \quad \text{or} \quad 38 = \frac{Q}{2\pi\epsilon_0 \times 5 \times 1} \qquad \ldots(i)$$

$$g_{2max} = \frac{Q}{2\pi\epsilon_0\epsilon_{r2}\,r_1} \quad \text{or} \quad 26 = \frac{Q}{2\pi\epsilon_0 \times 4 \times r_1} \qquad \ldots(ii)$$

$$g_{3max} = \frac{Q}{2\pi\epsilon_0\epsilon_{r3}\,r_2} \quad \text{or} \quad 20 = \frac{Q}{2\pi\epsilon_0 \times 3 \times r_2} \qquad \ldots(iii)$$

From ***(i)*** and ***(ii)***, we get, $38/26 = 4r_1/5$, $r_1 = 1.83$ cm

Similarly, from ***(i)*** and ***(ii)***,

$$38/20 = 3r_2/5\ ;\ r_2 = 3.17 \text{ cm}$$
$$V_1 = g_{1max} \times r \times 2.3 \log_{10} r_1/r$$
$$= 38 \times 1 \times 2.3 \log_{10} \frac{1.83}{1} = 22.9 \text{ kV}$$
$$V_2 = g_{2max} \times r_1 \times 2.3 \log_{10} \frac{r_2}{r_1}$$
$$= 26 \times 1.83 \times 2.3 \log_{10} \frac{3.17}{1.83}$$
$$= 26.1 \text{ kV}$$
$$V_3 = g_{3max} \times r_2 \times 2.3 \log_{10} \frac{r_3}{r_2}$$
$$= 20 \times 3.17 \times 2.3 \log_{10} \frac{r_3}{3.17}$$
$$= 145.82 \log_{10} \frac{r_3}{3.17}$$

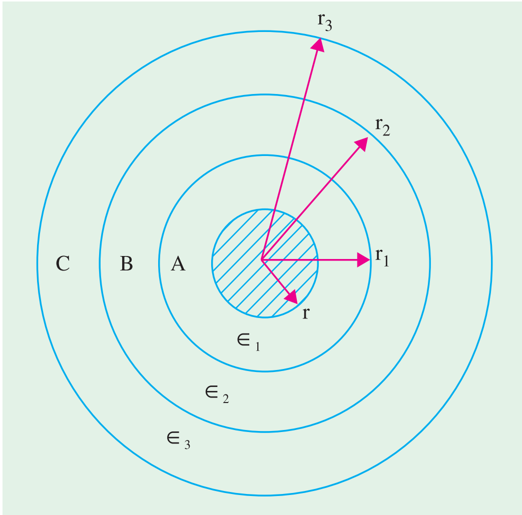


Fig. 41.50

Now, $\quad V = V_1 + V_2 + V_3$

$\therefore \quad 66 = 22.9 + 26.1 + 145.82 \log_{10} \frac{r_3}{3.17}$

$\therefore \quad r_3 = 4.15$ cm

$\therefore \quad$ diameter of the sheath $= 2r_3 = 2 \times 4.15 =$ **8.3 cm**

Example 41.38. *The capacitances per kilometer of a 3-phase cable are 0.63 µF between the three cores bunched and the sheath and 0.37 µF between one core and the other two connected to sheath. Calculate the charging current taken by eight kilometres of this cable when connected to a 3-phase, 50-Hz, 6,600-V supply.*

Solution. As shown in Art. 40.33, $0.63 = 3C_S \quad \therefore \quad C_S = 0.21\ \mu$ F/km

From the second test,

$$0.37 = 2C_1 + C_2 = 2C_1 + 0.21 \quad \therefore \quad C_1 = 0.08\ \mu \text{ F/km}$$

$\therefore \quad C_S$ for 8 km $= 0.21 \times 8 = 1.68\ \mu$ F ; C_1 for 8 km $= 0.08 \times 8 = 0.64\ \mu$ F

$$C_n = C_S + 3C_1 = 1.68 + (3 \times 0.64) = 3.6\ \mu \text{ F}$$

Now, $\quad V_p = (6{,}600/\sqrt{3})$; $\quad \omega = 314$ rad/s

$I_C = \omega V_p C_n = (6{,}600/\sqrt{3}) \times 3.6 \times 10^{-6} \times 314 =$ **4.31 ampere**

Example 41.39. *A 3-core, 3-phase belted cable tested for capacitance between a pair of cores on single phase with the third core earthed, gave a capacitance of 0.4 mF per km. Calculate the charging current for 1.5 km length of this cable when connected to 22 kV, 3-phase, 50-Hz supply.*

Solution. $\quad C_L = 0.4\ \mu$F, $\quad V_L = 22{,}000$ V, $\quad \omega = 314$ rad/s

$$I_C = \frac{2}{\sqrt{3}} \quad \omega V_L C_L = \frac{2}{\sqrt{3}} \times 22{,}000 \times 0.4 \times 10^{-6} \times 314 = 3.2\ A$$

Charging current for 15 km $= 3.2 \times 15 =$ **48 A**

Example 41.40. *A 3-core, 3-phase metal-sheathed cable has* ***(i)*** *capacitance of 1 µF between shorted conductors and sheath and* ***(ii)*** *capacitance between two conductors shorted with the sheath and the third conductor 0.6 µF. Find the capacitance* ***(a)*** *between any two conductors* ***(b)*** *between any two shorted conductors and the third conductor.* **(Power Systems-I, AMIE, Sec. B, 1993)**

Solution. ***(a)*** The capacitance between two cores or lines when the third core is free or shorted to earth is given by $1/2\ (3C_1 + C_S)$

Now, ***(i)*** we have $3C_S = 1\mu$ F or $C_s = 1/3\ \mu$ F $= 0.333\ \mu$ F

From **(ii)** we get, $2C_1 + C_S = 0.6\ \mu\text{ F}, 2C_1 = 0.6 - 0.333 = 0.267\ \mu\text{ F},$

$\therefore \quad C_1 = 0.133\ \mu\text{ F}$

(b) The capacitance between two shorted conductors and the other is given by

$$2C_1 + \frac{2C_S \times C_S}{3\,C_S} = 2C_1 + \frac{2}{3}C_S = 2 \times 0.133 + \frac{2}{3} \times 0.333 = \mathbf{0.488\ mF}$$

41.35. A.C. Distributor Calculations

These calculations are similar to those for d.c. distributor but with the following differences :

1. The loads tapped off will be at different power factors. Each power factor is taken with respect to the voltage at the feeding point which is regarded as a reference vector.
2. The currents in the sections of the distributor will be given by the vector sum of load currents and not by their arithmetic sum as in a d.c. distributor. Currents can be added algebraically only when they are expressed in the complex notation.
3. The voltage drop, in the case of a.c. circuits, is not only due to ohmic resistance but due to inductive reactance as well (neglecting capacitive reactance if any).

   It has already been shown that voltage drop in an inductive circuit is given by

$$I\,(R\cos\phi + X\sin\phi)$$

   The total drop will be given by $\Sigma I\,(R\cos\phi + X\sin\phi)$.

Questions on a.c. distributors may be solved in the following three ways :

1. Express voltages, currents and impedances in complex notation and then proceed exactly as in d.c. distributors.
2. Split the various currents into their active and reactive components. Now, the drop in the case of active components will be due to resistance only and in the case of reactive components due to reactance only. Find out these two drops and then add the two to find the total drop.
3. In cases where approximate solutions are sufficient, quick results can be obtained by finding the "distribution centre" or centre of gravity of the load.

All these three methods are illustrated by Ex. 41.42 given on the next page :

Example 41.41. *A 2-wire a.c. feeder 1 km long supplies a load of 100 A at 0.8 p.f. lag 200 volts at its far end and a load of 60 A at 0.9 p.f. lag at its mid-point. The resistance and reactance per km (lead and return) are 0.06 ohm and 0.08 ohm respectively. Calculate the voltage drop along the distributor from sending end to mid-point and from mid-point to far end.*

(Power Systems-I, AMIE, Sec. B, 1993)

Solution. Fig. 41.51 shows the feeder AC 1 km long having B as its mid-point and A as its sending-end point.

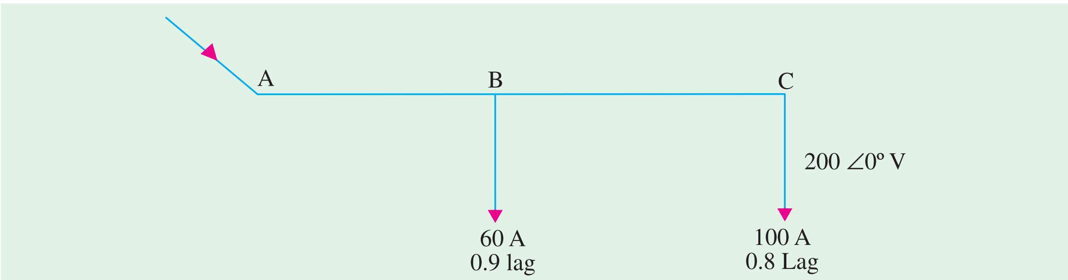


Fig. 41.51

Let the voltage of point C be taken as reference voltage.

$V_C = 200 + j_0$, $I_C = 100(0.8 - j\,0.6) = (80 - j\,60)$ A

Loop impedance of feeder BC (lead and return) $= (0.06 + j\,0.08)/2 = (0.03 + j\,0.04)$ ohm

Voltage drop in $BC = (80 - j\,60)(0.03 + j\,0.04) =$ **$(4.8 + j1.4)$ V**

$\therefore\quad V_B = 200 + 4.8 + j\,1.4 = (204.8 + j\,1.4)$ V

Current $I_B = 60\,(0.9 - j\,0.4357) = (54 - j\,26.14)$ A

Current in feeder AB *i.e.* $I_{AB} = I_C + I_B = (80 - j\,60) + (54 - j\,26.14) = (134 - j\,86.14)$ A

Drop in section $AB = (134 - j\,86.14)(0.03 + j\,0.04) = (7.46 + j\,2.78)$ V

Voltage drop from point A to point B = **$(7.46 + j\,2.78)$ V**

Example 41.42. *A single-phase a.c. distributor 500 m long has a total impedance of (0.02 + j 0.04) Ω and is fed from one end at 250V. It is loaded as under :*

(i) *50 A at unity power factor 200 m from feeding point.*

(ii) *100 A at 0.8 p.f. lagging 300 m from feeding point.*

(iii) *50 A at 0.6 p.f. lagging at the far end.*

Calculate the total voltage drop and voltage at the far end. **(Power System-I, AMIE, 1994)**

Solution. First Method

Current in section AD (Fig. 41.52) is the vector sum of the three load currents.

$\therefore$ current in AD

$= 50 + 100\,(0.8 - j\,0.6) + 50\,(0.6 - j\,0.8)$

$= 160 - j100$

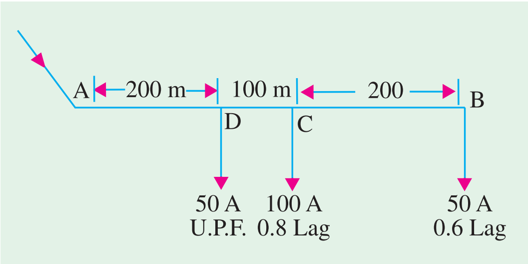


Fig. 41.52

Impedance of section AD

$= (200/500)\,(0.02 + j\,0.04)$

$= (0.008 + j\,0.016)$ W

Voltage drop in section $AD = (160 - j\,100) \times (0.008 + j\,0.016) = (2.88 + j\,1.76)$V

Current in section $DC = (160 - j100) - 50 = (110 - j100)$ A

Impedance of $DC = (0.004 + \text{j}\ 0.008)\ \Omega$

Drop in $CD = (100 - j100)\,(0.004 + j\,0.008) = (1.24 + j\,0.48)$ V

Current in $CB = 50\,(0.6 - j0.8) = (30 - j\,40)$ A

Impedence of $CB = (0.008 + j\,0.016)\ \Omega$

$\therefore$ drop in $CB = (30 - j40)\,(0.008 + j0.016) = (0.88 + j0.16)$ V

Total drop $= (2.88 + j\,1.76) + (1.24 + j\,0.48) + (0.88 + j\,0.16) = (5 + j\,2.4)$ V

Voltage at far end $= (250 + j0) - (5 + j2.4) = 245 - j\,2.4$ volt

Its magnitude is $= \sqrt{245^2 + 2.4^2} = 245$ V (approx)

Second Method

We will split the currents into their active and reactive components as under :

$50 \times 1 = 50$ A ; $\quad 100 \times 0.8 = 80$ A; $\quad 50 \times 0.6 = 30$ A

These are shown in Fig. 41.51 (*a*). The reactive or wattless components are

$50 \times 0 = 0$; $\quad 100 \times 0.6 = 60$ A ; $\quad 50 \times 0.8 = 40$ A

These are shown in Fig. 41.53 (*b*). The resistances and reactances are shown in their respective figures.

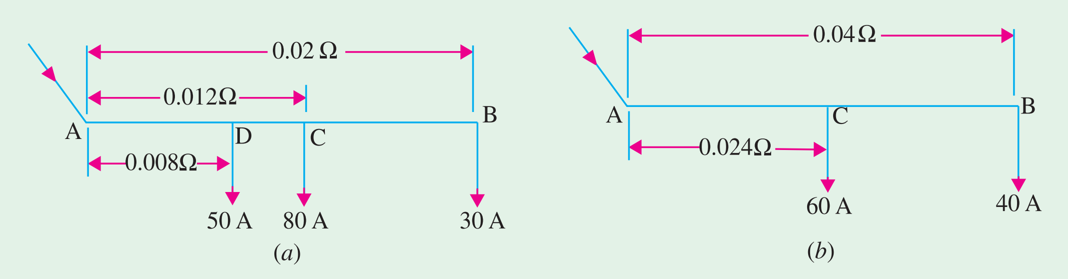


Fig. 41.53

Drops due to active components of currents are given by taking moments

$$= 50 \times 0.008 + 80 \times 0.012 + 30 \times 0.02 = 1.96 \text{ V}$$

Drops due to reactive components $= 60 \times 0.024 + 40 \times 0.04 = 3.04$

Total drop $= 1.96 + 3.04 = 5$ V

This is approximately the same as before.

Third Method

The centre of gravity (C.G.) of the load is at the following distance from the feeding end

$$\frac{50 \times 200 + 100 \times 300 + 50 \times 500}{200} = 325 \text{ m}$$

Value of resistance upto C.G. $= 325 \times 0.02/500 = 0.013\ \Omega$.

Value of reactance upto C.G. $= 325 \times 0.04/500 = 0.026\ \Omega$.

Average p.f. $= \dfrac{50 \times 1 + 100 \times 0.8 + 50 \times 0.6}{200} = 0.8$

$\cos \phi_{av} = 0.8$; $\sin \phi_{av} = 0.6$

Drop $= 200(0.013 \times 0.8 + 0.026 \times 0.6) = 5.2$ V

This is approximately the same as before.

Example 41.43. *A single-phase distributor, one km long has resistance and reactance per conductor of 0.2 Ω and 0.3 Ω respectively. At the far end, the voltage V_B = 240 V and the current is 100 A at a power factor of 0.8 lag. At the mid-point A of the distributor current of 100 A is tapped at a power factor of 0.6 lag with reference to the voltage V_A at the mid-point. Calculate the supply voltage V_S for the distributor and the phase angle between V_S and V_B.*

Solution. As shown in Fig. 41.54 (*a*), let *SB* be the distributor with *A* as the mid point. Total impedance of the distributor is = (0.4 + *j* 0.6) Ω.

Let the voltage V_B at point *B* be taken as the reference voltage.

$\therefore \quad \mathbf{V}_B = (240 + j_0)\text{V}$; $\mathbf{I}_B = 100\,(0.8 - j\,0.6) = 80 - j\,60$ A

Drop in section $AB = (80 - j60)\,(0.2 + j0.3) = (34 + j12)$ V

$\mathbf{V}_A = \mathbf{V}_B$ + drop over $\boldsymbol{AB} = (240 + j\,0) + (34 + j12) = (274 + j12)$ V

The phase difference between V_A and V_B is $= \tan^{-1}(12/274) = 2°30'$

The load current I_A has lagging power factor of 0.6 with respect to V_A. It lags V_A by an angle $\phi = \cos^{-1}(0.6) = 53°8'$.

Hence, it lags behind V_B by an angle of $(53°8' - 2°30') = 50°38'$ as shown in the vector diagram of Fig. 41.54 (*b*).

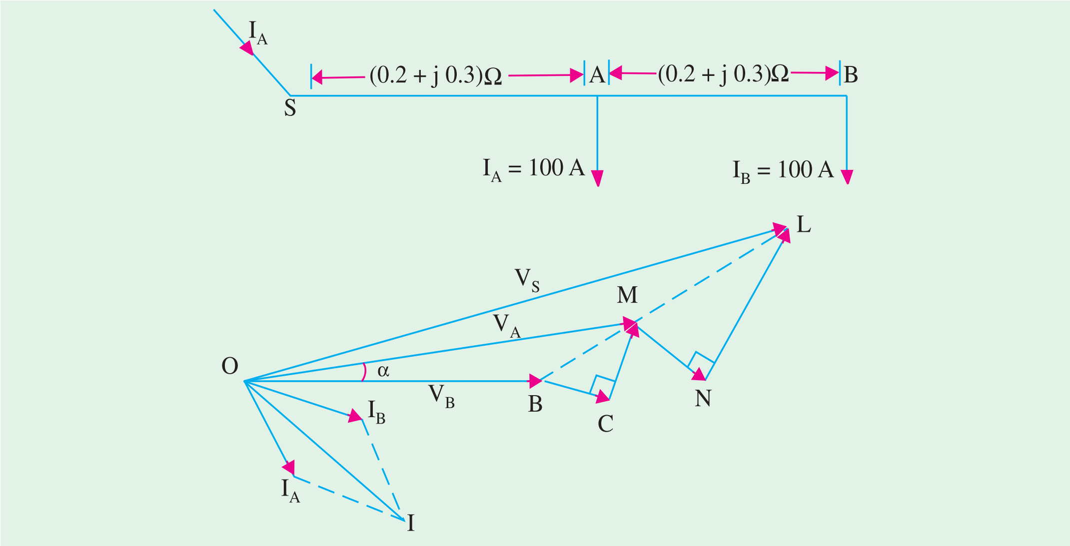


Fig. 41.54

$$\mathbf{I}_A = 100\,(\cos 50°38' - j\sin 50°38') = (63.4 - j77.3)\text{ A}$$

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = (80 - j\,60) + (63.4 - j77.3) = (143.4 - j137.3)\text{ A}$$

Drop in section $S_A = (143.4 - j137.3)\,(0.2 + j\,0.3) = (69.87 + j\,15.56)$

$$\mathbf{V}_S = \mathbf{V}_A + \text{drop in section } S_A$$

$$= (274 + j12) + (69.87 + j15.56) = 343.9 + j\,27.6 = 345\,\angle 5°28'\text{ V}$$

Hence, supply voltage is 345 V and lead V_B by 5°28′

Example 41.44. *A 1-phase ring distributor ABC is fed at A. The loads at B and C are 20 A at 0.8 p.f. lagging and 15 A at 0.6 p.f. lagging respectively, both expressed with reference to voltage at A. The total impedances of the sections AB, BC and CA are (1 + j1), (1 + j2) and (1 + j3) ohm respectively. Find the total current fed at A and the current in each section.*

(Transmission and Distribution-II, Madras Univ.)

Solution. Thevenin's theorem will be used to solve this problem. The ring distributor is shown in Fig. 41.55 (*a*). Imagine feeder *BC* to be removed [Fig. 41.55 (*b*)].

Current in $AB = 20(0.8 - j0.6) = (16 - j12)$ A Current in section $AC = 15(0.6 - j0.8) = (9 - j12)$A
Drop over $AB = (16 - j12)(1 + j1) = (28 + j4)$ V Drop over $AC = (9 - j12)(1 + j3) = (45 + j15)$V

Obviously, point *C* is at a lower potential as compared to point *B*.

p.d. between *B* and *C* = $(45 + j15) - (28 + j4) = (17 + j11)$ V

Impedance of the network as looked into from points *B* and *C* is $= (1 + j1) + (1 + j3) = (2 + j4)\Omega$.. The equivalent Thevenin's source is shown in Fig. 41.55 (*c*) with feeder *BC* connected across it.

$$\text{Current in } BC = \frac{17 + j\,11}{(2 + j4) + (1 + j2)} = (2.6 - j1.53)\text{ A}$$

Current in AB $= (16 - j12) + (2.6 - j1.53) = 18.6 - j\,13.53 = 23\,\angle -36°$ A

Current in BC $= (9 - j12) - (2.6 - j1.53) = 6.4 - j11.5 = 13.2\,\angle -60.9°$ A

Total current fed at point A $= (16 - j12) + (9 - j12) = 25 - j24 = 34.6\,\angle -43.8°$ A.

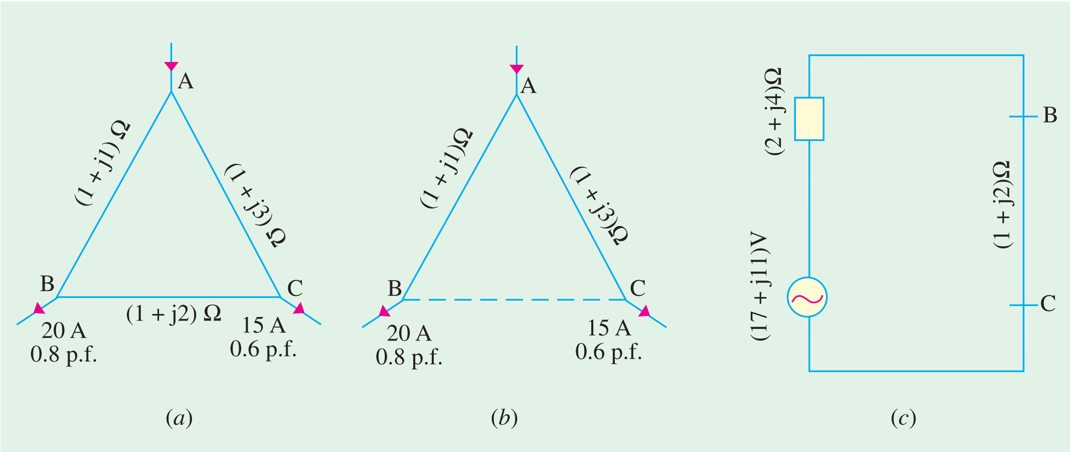


Fig. 41.55

Example 41.45. *A 2-wire ring distributor ABC is supplied at A at 400 V. Point loads of 20 A at a p.f. of 0.8 lagging and 30 A at a p.f. 0.6 lagging are tapped off at B and C respectively. Both the power factors refer to the voltage at A. The respective go-and-return impedances of sections AB,BC and CA are (1 + j2) ohm, (2 + j3) ohm and (1 + j3) ohm. Calculate the current flowing through each section and the potentials at B and C. Use Superposition theorem.*

(Power Systems-I, M.S. Univ. Baroda 1992)

Solution. The distributor circuit is shown in Fig. 41.56 (*a*). Currents in various sections are as shown. First, consider the load at point *B* acting alone as in Fig. 41.56 (*b*). The input current at point A divides in the inverse ratio of the impedances of the two paths *A B* and *ACB*.

Let the currents be $\mathbf{I}_1'$ and $\mathbf{I}_2'$

$$\mathbf{I}_1' = (16 - j12) \times \frac{(1 + j3) + (2 + j3)}{(1 + j3)\,(2 + j3) + (1 + j3)} = (16 - j12) \times \frac{(3 + j6)}{4 + j8} = (12 - j9)\ A$$

$$\mathbf{I}_2' = (16 - j12) - (12 - j9) = (4 - j3)\ \text{A}$$

Now, consider the load at point *C* to act alone as shown in Fig. 40.56 (c). Let the currents now be $\mathbf{I}_1''$ and $\mathbf{I}_2''$.

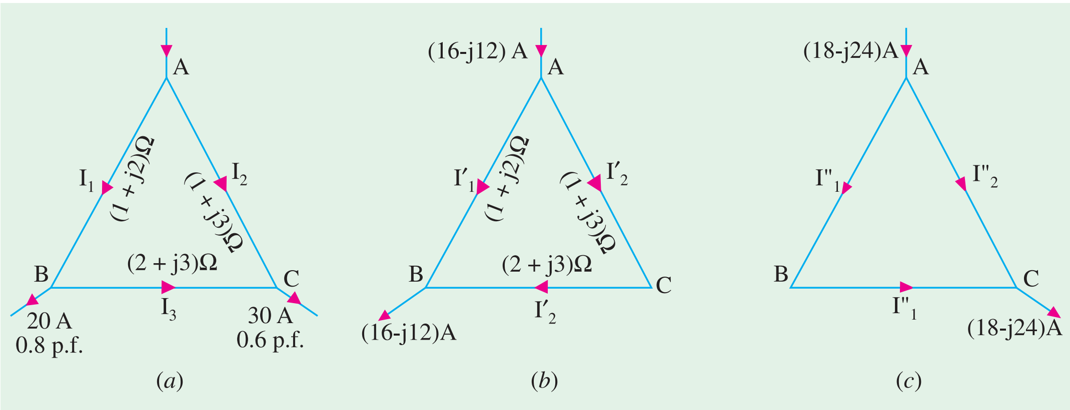


Fig. 41.56

$$\mathbf{I}_1'' = (18 - j24) \times \frac{1 + j3}{4 + j8} = 7.5 - j7.5;\ \mathbf{I}_2'' = (18 - j24) - (7.5 - j7.5) = (10.5 - j16.5)\ \text{A}$$

As per Superposition theorem, the current $\mathbf{I}_1$ in section AB is the vector sum of $\mathbf{I}_1'$ and $\mathbf{I}_1''$.

$\therefore \quad \mathbf{I}_1 = \mathbf{I}_1' + \mathbf{I}_1'' = (12 - j9) + (7.5 - j7.5) = (19.5 - j16.5)$ A

$\mathbf{I}_2 = \mathbf{I}_2' + \mathbf{I}_2'' = (4 - j3) + (10.5 - j16.5) = (14.5 - j19.5)$ A

$\mathbf{I}_3 = \mathbf{I}_1'' - \mathbf{I}_2' = (7.5 - j7.5) - (4 - j3) = (3.5 - j4.5)$ A

Potential of $B = 400 -$ drop over AB

$= 400 - (19.5 - j16.5)(1 + j2) = 347.5 - j22.5 = 348 \angle -3.7°$V

Potential of $C = 400 - (14.5 - j19.5)(1 + j3) = 327 - j24 = 328 \angle -4.2°$V

Example 41.46. *A 3-phase ring main ABCD, fed from end A, supplies balanced loads of 50 A at 0.8 p.f. lagging at B, 120 A at u.p.f. at C and 70 A at 0.866 p.f. lagging at D, the load currents being referred to the voltage at point A.*

The impedance per phase of the various line sections are :

section AB = (1 + j0.6) Ω ; section BC = (1.2 + j0.9) Ω

section CD = (0.8 + j0.5) Ω ; section DA = (3 + j2) Ω

Determine the currents in the various sections. **(Electrical Power-I, Bombay Univ.)**

Solution. One phase of the ring main is shown in Fig. 41.57. Let the current in section $AB = (x + jy)$ A.

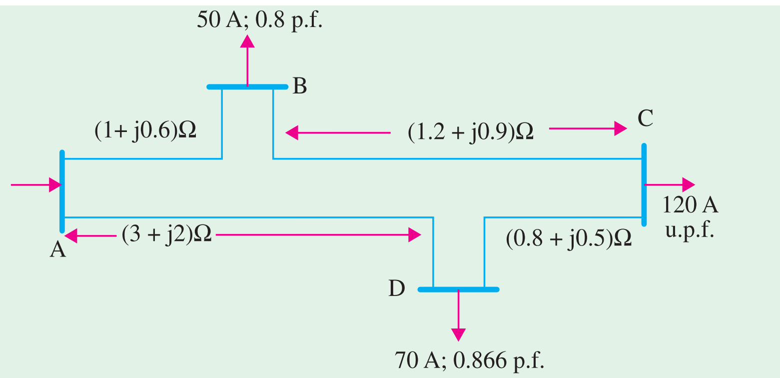


Fig. 41.57

Current in $BC = (x + jy) - 50(0.8 - j0.6) = (x - 40) + j(y + 30)$

Current in $CD = (x - 40) + j(y + 30) - (120 + j0) = (x - 160) + j(y + 30)$

Current in $DA = (x - 160) + j(y + 30) - 70(0.866 - j0.5) = (x - 220.6) + j(y + 65)$

Applying Kirchhoff's voltage law to the closed loop $ABCDA$, we have

$(1 + j0.6)(x + jy) + (1.2 + j0.9)$

$[(x - 40) + j(y + 30)] + (0.8 + j0.5)$

$[(x - 160) + j(y + 30)] + (3 + j2)$

$[(x - 220.6) + j(y + 65)] = 0$

or $\quad (6x - 4y + 1009.8) + j(4x + 6y - 302.2) = 0$

Since the real (or active) and imaginary (or reactive) parts have to be separately zero.

$\therefore \quad 6x - 4y + 1009.8 = 0 \quad$ and $\quad 4x + 6y - 302.2 = 0$

Solving for x and y, we get

$x = 139.7$ and $y = -42.8$

$\therefore$ Current in section $AB = (139.7 - j42.8)$ A

Current in section $BC = (139.7 - 40) + j(-42.8 + 30) = (99.7 - j12.8)$ A

Current in section CD = $(139.7 - 160) + j(-42.8 + 30) = (-20.3 - j\,12.8)$ A

Current in section DA = $(139.7 - 220.6) + j(-42.8 + 65) = (-80.9 + j\,22.2)$ A

41.36. Load Division Between Parallel Lines

It is common practice to work two or more cables or overhead lines in parallel when continuity of supply is essential. In the case of a fault developing in one line of cable, the other lines or cables carry the total load till the fault is rectified."

Let us take the case of two lines in parallel and having impedances of $\mathbf{Z}_1$ and $\mathbf{Z}_2$. Their combined impedance is

$$\mathbf{Z} = \frac{\mathbf{Z}_1 \times \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

If $\mathbf{I}$ is the current delivered to both lines, then total drop = $\mathbf{IZ} = \mathbf{I} \times \dfrac{\mathbf{Z}_1\,\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$

If $\mathbf{I}_1$ and $\mathbf{I}_2$ are the respective currents flowing in the two lines, then

$$\mathbf{I}_1 = \frac{\text{voltage drop}}{\mathbf{Z}_1} = \frac{\mathbf{IZ}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\,; \quad \text{Similarly, } \mathbf{I}_2 = \frac{\mathbf{IZ}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

It may be noted that in the case of two impedances in parallel, it is convenient to take voltage vector as the reference vector.

Example 41.47. *A total load of 12,000 kW at a power factor of 0.8 lagging is transmitted to a substation by two overhead three-phase lines connected in parallel. One line has a conductor resistance of 2 Ω per conductor and reactance (line to neutral) of 1.5 Ω, the corresponding values for the other line being 1.5 and 1.2 Ω respectively. Calculate the power transmitted by each overhead line.* **(London Univ.)**

Solution. Let us assume a line voltage of 1000 kV for convenience.

$\mathbf{Z}_1 = (2 + j1.5)$; $\mathbf{Z}_2 = (1.5 + j1.2)\ \Omega$

Total load current $\mathbf{I} = 12{,}000/\sqrt{3} \times 1000 \times 0.8 = 8.66$ A

Taking voltage along reference vector, we have $\mathbf{I} = 8.66\,(0.8 - j0.6)$

$$\mathbf{I}_1 = \frac{8.66\,(0.8 - j\,0.6)\,(1.5 + j\,1.2)}{(2 + j\,1.5)\,(1.5 + j\,1.2)} = 4.437(6.882 - j5.39) \text{ A}$$

∴ power transmitted by 1st line is

$$W_1 = 10^6 \times \frac{\sqrt{3}}{1000} \times 4.437 \times 6.882 = \mathbf{5{,}280\ kW}$$

Similarly
$$\mathbf{I}_2 = \frac{8.66\,(0.8 - j\,0.6)\,(2 + j\,1.5)}{(2 + j\,1.5)\,(1.5 + j\,1.2)} = 4.437(8.75 - j6.75) \text{ A}$$

$$W_2 = 10^6 \times \frac{\sqrt{3}}{1000} \times 4.437 \times 8.75 = \mathbf{6{,}720\ kW}$$

As a check, total power $= 5{,}280 + 6{,}720 = \mathbf{12{,}000\ kW}$

41.37. Suspension Insulators

Suspension insulators are used when transmission voltage is high. A number of them are connected in series to form a chain and the line conductor is carried by the bottom most insulator.

As shown in Fig. 41.58 a 'cap' type suspension insulator consists of a single disc-shaped piece of porcelain grooved on the under surface to increase the surface leakage path. A galvanized cast iron cap is cemented at the top of the insulator. In the hollow cavity of the insulator is cemented a galvanized forged steel pin, the lower enlarged end of which fits into the cavity of the steel cap of the lower suspension insulator and forms a ball and socket connection.

A string of suspension insulators consists of many units, the number of units depending on the value of the transmission voltage. There exists mutual capacitance between different units and, in addition, there is capacitance to ground of each unit because of the nearness of the tower, the cross-arm and the line. Due to this capacitance to ground, the total system voltage is not equally distributed over the different units of the string. The unit nearest to the line conductor carries the maximum percentage of voltage, the figure progressively decreasing as the unit nearest to the tower is approached. The inequality of voltage distribution between individual units becomes more pronounced as the number of insulators increases and it also depends on the ratio (capacitance of insulator/capacitance of earth).

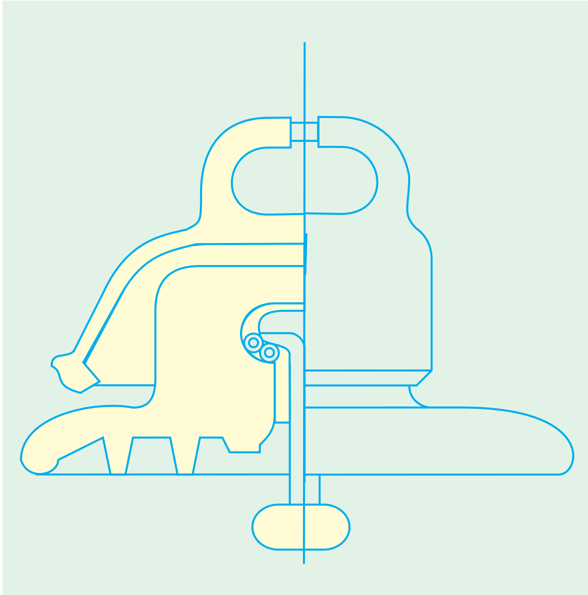


Fig. 41.58

String Efficiency

If there are n units in the string, then its efficiency is given by

$$\%\text{ string } \eta = \frac{\text{total voltage across the string}}{n \times \text{voltage across unit adjacent the line}} \times 100$$

41.38. Calculation of Voltage Distribution along Different Units

Let, C = capacitance to ground

kC = mutual capacitance between units

The current and voltage distribution is as shown in Fig. 41.59. It is seen that

$$I_1 = \frac{V_1}{1/\omega kC} = \omega kCV_1\text{; Similarly } i_1 = \frac{V_1}{1/\omega C} = \omega CV_1$$

Now, $$I_2 = I_1 + i_1 = \omega CV_1(1+k) \text{ and } V_2 = \frac{I_2}{\omega kC} = \frac{V_1(1+k)}{k}$$

The current i_2 is produced by the voltage combination of $(V_1 + V_2)$

$$\text{Now, } V_1 + V_2 = V_1\left(1+\frac{1+k}{k}\right) = V_1\left(1+\frac{1+2k}{k}\right); \therefore i_2 = \omega CV_1\left(1+\frac{1+2k}{k}\right)$$

At junction B, we have

$$I_3 = I_2 + i_2 = \omega CV_1\left\{1+k\,\frac{(1+2k)}{k}\right\}$$

$$= \omega CV_1 \frac{(1+3k+k^2)}{k}$$

However, $I_3 = \omega kCV_3$; $\therefore$ $$V_3 = \frac{V_1(1+3k+k^2)}{k^2} = V_1\left(1+\frac{3}{k}+\frac{1}{k^2}\right)$$

The current i_3 is produced by the voltage combination of $(V_1 + V_2 + V_3)$

Now, $V_1 + V_2 + V_3$

$$= V_1\left[1 + \frac{(1+k)}{k} + \frac{(1+3k+k^2)}{k^2}\right]$$

$$\therefore \quad i_3 = \omega C V_1 \left(\frac{1+4k+3k^2}{k^2}\right)$$

At junction C, we have

$$I_4 = I_3 + i_3 = \omega C V_1$$

$$\left[\frac{(1+3k+k^2)}{k^2} + \frac{(1+4k+3k^2)}{k^2}\right]$$

Now, $\quad I_4 = \omega k C V_4,$

$$\therefore \quad V_4 = V_1\left[\frac{(1+3k+k^2)}{k^2} + \frac{(1+4k+3k^2)}{k^2}\right]$$

$$= V_1\left(1 + \frac{6}{k} + \frac{5}{k^2} + \frac{1}{k^2}\right)$$

For the fifth insulator from the top, we have

$$V_5 = \left(1 + \frac{10}{k} + \frac{15}{k^2} + \frac{7}{k^3} + \frac{1}{k^4}\right)$$

and so on.

If the string has n units, the total voltage is given by

$$V = (V_1 + V_2 + V_3 + V_4 + \ldots\ldots\ldots\ldots\ldots + V_n)$$

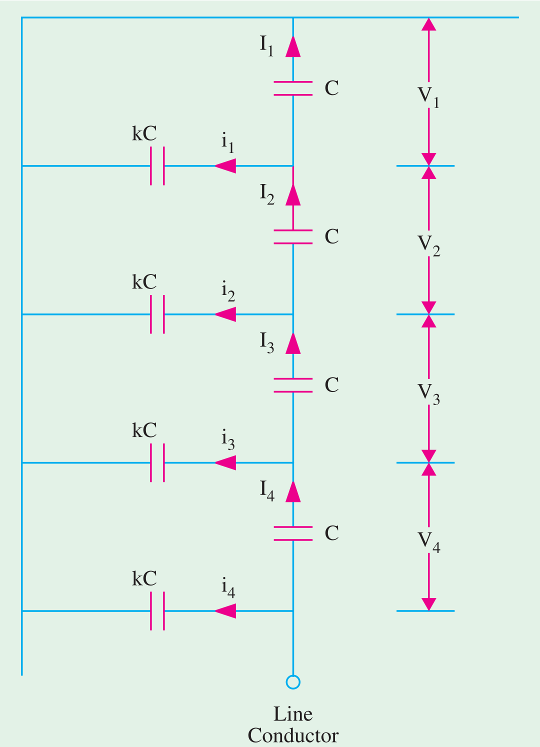


Fig. 41.59

Example 41.48. *For a string insulator with four discs, the capacitance of the disc is ten times the capacitance between the pin and earth. Calculate the voltage across each disc when used on a 66-kV line. Also, calculate the string efficiency.* **(Power Systems-I, AMIE Sec. B, 1994)**

Solution. Let C be the self-capacitance of each disc and kC the capacitance between each link pin and earth. We are given that $k = 10$

As seen from Art. 41.38,

$$V_2 = \frac{(1+k)}{k}V_1 = \frac{11}{10}V_1$$

$$V_3 = V_1\left(1 + \frac{3}{k} + \frac{1}{k^2}\right) = \frac{131}{100}V_1$$

$$V_4 = V_1\left(1 + \frac{6}{k} + \frac{5}{k^2} + \frac{1}{k^3}\right) = \frac{1561}{100}V_1$$

$$V = V_1 + V_2 + V_3 + V_4 = V_1 + \frac{11}{10}V_1 + \frac{131}{100}V_1 + \frac{1561}{1000}V_1 = \frac{4971}{1000}V_1$$

$$\therefore \quad V_1 = V\frac{1000}{4971} = \frac{66}{\sqrt{3}} \times \frac{1000}{4971} = \mathbf{7.66\ kV}$$

$$V_2 = \frac{11}{10}V_1 = \frac{11}{10} \times 7.66 = \mathbf{8.426\ kV}$$

$$V_3 = \frac{131}{100}V_1 = \frac{131}{100} \times 7.66 = \mathbf{10.03\ kV}$$

$$V_4 = \frac{1561}{1000} V_1 = \frac{1561}{1000} \times 7.66 = \mathbf{11.96\ kV}$$

$$\text{String } \eta = \frac{V}{4 \times V_4} \times 100 = \frac{66 \times 100}{\sqrt{3} \times 4 \times 11.96} = \mathbf{79.65\ \%}$$

Example 41.49. *Explain what is meant by string efficiency and how it can be improved. Each line of a 3-phase, 33-kV system is suspended by a string of 3 identical insulator discs. The capacitance of each disc is 9 times the capacitance to ground. Find voltage distribution across each insulator and the string efficiency. Suggest a method for improving the string efficiency.*

(Power Systems-I, AMIE, Sec. B, 1993)

Solution. Let C be the self-capacitance of each disc and kC the mutual capacitance between the units. We are given that $k = 9$.

$$V_2 = V_1 \frac{(1+k)}{k} = \frac{10}{9} V_1;$$

$$V_3 = V_1 \frac{(1+3k+k^2)}{k^2} = \frac{109}{81} V_1$$

Total voltage, $$V = V_1 + V_2 + V_3 = V_1 + \frac{10}{9} V_1 + \frac{109}{81} V_1 = \frac{280}{81} V_1$$

$\therefore$ $$\frac{33}{\sqrt{3}} = \frac{280}{81} V_1; \quad \therefore V_1 = \frac{33 \times 81}{\sqrt{3} \times 280} = \mathbf{5.51\ kV}$$

$$V_2 = (10/9)\, V_1 = \mathbf{6.12\ kV}\ ;\ V_3 = (109/81) V_1 = \mathbf{7.41\ kV}$$

String efficiency $$= \frac{V}{3 \times V_3} \times 100 = \frac{33/\sqrt{3}}{3 \times 7.41} \times 100 = \mathbf{85\%}$$

The string efficiency can be improved by providing a guard ring surrounding the lower-most unit which is connected to the metal work at the bottom. This ring increases the capacitance between the metal work and the line and helps in equalising the voltage distribution along the different units. The efficiency can also be improved by grading the insulators and by making the ratio (capacitance to earth/capacitance per insulator) as small as possible.

41.39. Interconnectors

An interconnector is a tieline which enables two generating stations to operate in parallel. It facilitates the flow of electric power in either direction between the two stations.

41.40. Voltage Drop Over the Interconnector

Let station 1 supply a current of 1 to station 2 along an interconnector having a resistance of RW and reactance of XTW per phase as shown in Fig. 41.60 (*a*). If the receiving end p.f. is cos ϕ lagging, then the vector diagram will be as shown in Fig. 41.60 (*b*).

Voltage drop over the interconnector

$$= I(\cos\phi - j\sin\phi)(R + jX)$$

$$= I(R\cos\phi + X\sin\phi) + jI(X\cos\phi - R\sin\phi)$$

% voltage drop $$= \left\{\frac{I(R\cos\phi + X\sin\phi) + jI(X\cos\phi - R\sin\phi)}{E_2}\right\} \times 100$$

Let $I\cos\phi = I_d$ and $I\sin\phi = I_q$, where I_d and I_q are the in-phase and quadrature components.

$\therefore$ voltage drop $$= \left\{\frac{(I_d R + I_q X) + j(I_d X - I_q R)}{E_2}\right\} \times 100$$

$\therefore$ in-phase voltage drop $= \dfrac{(I_d R + I_q X)}{E_2} \times 100$

and quadrature voltage drop $= \dfrac{(I_d X^2 - I_q R)}{E_2} \times 100$

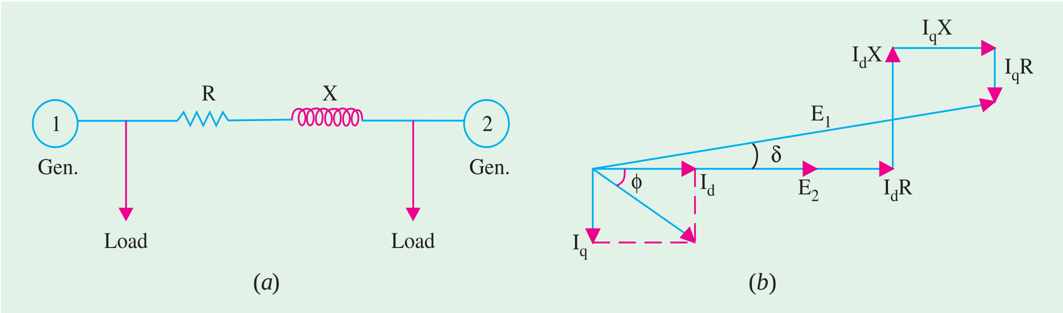


Fig. 41.60

Example 41.50. *The bus-bar voltages of two stations A and B are 33 kV and are in phase. If station A sends 8.5 MW power at u.p.f. to station B through an interconnector having an impedance of (3 + j4)Ω, determine the bus-bar voltage of station A and the phase angle shift between the bus-bar voltages.*

Solution. With reference to Fig. 41.61,

Voltage of station B, $V_B = 33{,}000/\sqrt{3} = 19{,}050$ V/phase

Since power transferred to station B is 8.5 MW, current in the interconnector is

$$I = 8.5 \times 10^3/\sqrt{3} = 148.7 \text{ A}$$

Taking V_B as the reference vector,

$$V_A = V_B + I(\cos\phi - j\sin\phi)(R + jX)$$
$$= 19{,}050 + 148.7\,(1 - j0)\,(3 + j4)$$
$$= 19{,}496 + j595 = 19{,}502\ \angle 1.75°$$

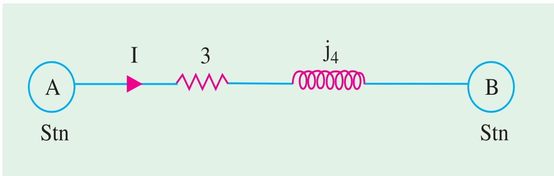


Fig. 41.61

The line-to-line bus-bar voltage of station A is

$$= 19{,}502 \times \sqrt{3} = \mathbf{33.78\ kV}$$

The phase-shift angle between bus-bar voltages is **1.75°**

41.41. Sag and Stress Analysis

The conductors of a transmission line are attached to suitable insulators carried on supports of wood, iron, steel or reinforced concrete. Obviously, the supports must be strong enough to withstand not only the dead weight of the conductors themselves but also the loads due to ice and sleet that may adhere to them and to wind pressure. Moreover, the minimum factor of safety for the conductors should be 2.0 based on ultimate strength.

Sag and stresses vary with temperature on account of thermal expansion and contraction of the line conductors. The value of sag as well as tension of a conductor would now be calculated when ***(i)*** supports are at equal levels and ***(ii)*** supports are at unequal levels.

41.42. Sag and Tension with Supports at Equal Levels

Fig. 41.62 shows a span of a wire with the two supports at the same elevation and separated by a horizontal distance 2*l*. It can be proved that the conductor AB forms a catenary with the lowest point O forming the mid-point (where the curve is straight).

Let W be the weight of the wire per unit length and let point O be chosen as the reference point for measuring the co-ordinates of different points on the wire. Consider a point P having co-ordinates of x and y. The tension T at point P can be resolved into two rectangular components. T_x and T_y so that $T = \sqrt{T_x^2 + T_y^2}$.

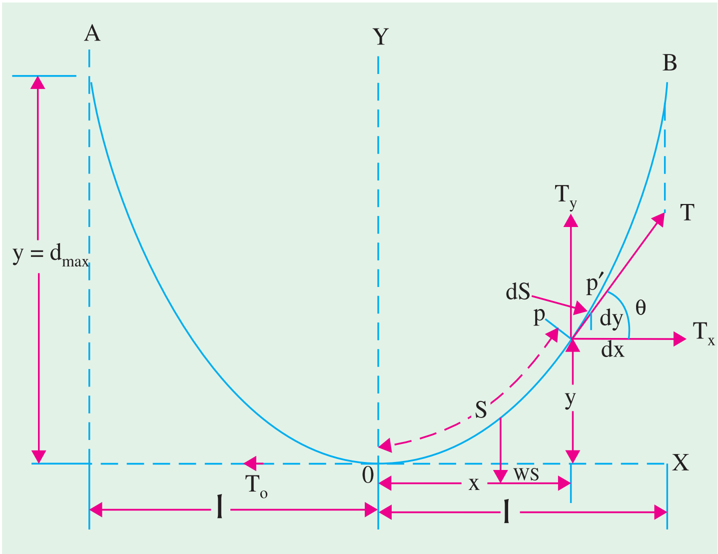


Fig. 41.62

If S is the length of the arc OP, then its weight is WS which acts vertically downward through the centre of gravity of OP. There are four forces acting on OP—two vertical and two horizontal. Since OP is in equilibrium, the net force is zero. Equating the horizontal and vertical components, we have,

$$T_0 = T_x \text{ and } T_y = W_S.$$

It may be noted that the horizontal component of tension is constant throughout the length of the wire :

Since line PT is tangential to the curve OB at point P, $\tan\theta = {}^{T_y}\!/_{T_x}$

It is also seen from the elementary piece PP' of the line that $\tan\theta = dy/dx$

$$\therefore \qquad dy/dx = \tan\theta = \frac{T_y}{T_x} \text{ or } \quad dy/dx = WS/T_0 \qquad \text{...}(i)$$

$$\text{If} \qquad PP' = dS, \quad \text{then } dS = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1 + (dy/dx)^2}$$

$$\text{or} \qquad \frac{dS}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \left(\frac{WS}{T_0}\right)^2} \text{ or } dx = \frac{dS}{\sqrt{1 + \left(\frac{W^2 S^2}{T_0^2}\right)}}$$

Integrating both sides, we have $x = \left(\frac{T_0}{W}\right)\sinh^{-1}\left(\frac{WS}{T_0}\right) + C$

where C is the constant of integration.

Now, when $x = 0$, $S = 0$. Putting these values above, we find that $C = 0$.

$$\therefore \qquad x = \left(\frac{T_0}{W}\right)\sinh^{-1}\left(\frac{WS}{T_0}\right) \text{or } S = \frac{T_0}{W}\sinh\left(\frac{Wx}{T_0}\right) \qquad \text{...}(ii)$$

Substituting this value of S in Eq. (*i*), we get

$$\frac{dy}{dx} = \sinh\left(\frac{Wx}{T_0}\right) \text{ or } dy = \sinh\left(\frac{Wx}{T_0}\right)dx$$

$$\therefore \qquad y = \int \sinh\left(\frac{Ex}{T_0}\right)dx = \left(\frac{T_0}{W}\right)\cosh\left(\frac{Wx}{T_0}\right) + D$$

where D is also the constant of integration. At the origin point O, $x = 0$ and $y = 0$. Hence, the above equation becomes

$$0 = \left(\frac{T_0}{W}\right)\cosh 0 + D + \left(\frac{T_0}{W}\right) + D \qquad \therefore \; D = -\frac{T_0}{W}$$

Substituting this value of D in the above equation, we get

$$y = \left(\frac{T_0}{W}\right)\cosh\left(\frac{Wx}{T_0}\right) - \frac{T_0}{W} = \frac{T_0}{W}\left[\cosh\left(\frac{Wx}{T_0}\right) - 1\right] \qquad ...(iii)$$

This is the equation of the curve known as catenary. Hence, when a wire is hung between two supports, it forms a catenary

(*a*) The tension at point P (x, y) is given by

$$T^2 = T_{x2} + T_{y2} = T_{02} + W^2S^2 = T_{02} + T_{02}\sinh^2\left(\frac{Wx}{T_0}\right) \qquad \text{—from Eq. } (iii)$$

$$= T_{02}\left[1 + \sinh^2\left(\frac{Wx}{T_0}\right)\right] = T - 2\cosh^2\left(\frac{Wx}{T_0}\right) \quad \therefore\; T = T_0\cosh\left(\frac{Wx}{T_0}\right) \qquad ...(iv)$$

(*b*) Tension at points A and B where $x = \pm\, l$ is given by $T = T_0 \cosh (W_1/T_0)$ ...(*v*)

(*c*) The maximum sag is represented by the value of y at either of the two points A and B for which $x = +\, l$ and $x = -\, l$ respectively. Writing $y = d_{max}$ and putting $x = \pm\, l$ in Eq. (*iii*), we get,

$$d_{max} = \frac{T_0}{W}\left[\cosh\left(\frac{Wl}{T_0}\right) - 1\right] \qquad ...(vi)$$

(*d*) The length of the wire or conductor in a half span is as seen from Eq. (*ii*) above,

$$S = \frac{T_0}{W}\sinh\left(\frac{Wl}{T_0}\right)$$

Approximate Formulae

The hyperbolic sine and cosine functions can be expanded into the following series

$$\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \frac{z^7}{7!} + \ldots \text{ and } \cosh z = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \frac{z^6}{6!} + \ldots$$

Using the above, the approximate values of T d and S points at A and B may be found as follows :

(*i*)
$$T = T_0\cosh\left(\frac{Wl}{T_0}\right) = T_0\left(1 + \frac{W^2l^2}{2T_0^2} + \ldots\right) \quad \text{—neglecting higher powers}$$

$$= T_0 + \frac{W^2l^2}{2T_0} \cong T_0 \qquad \text{—if } \frac{W^2l^2}{2T_0} \text{ is negligible}$$

$\therefore$ $T = T_0$ — *i.e.* tension at the supports is very approximately equal to the horizontal tension acting at any point on the wire.

(*ii*)
$$d = \frac{T_0}{W}\left[\cosh\left(\frac{Wl}{T_0}\right) - 1\right] = \frac{T_0}{W}\left[\left(1 + \frac{W^2l^2}{2T_0^2} + \ldots\right) - 1\right]$$

$$= \frac{Wl^2}{2T_0} = \frac{Wl^2}{2T}\text{ (approx)} \qquad \therefore\; d = \frac{Wl^2}{2T}$$

It should be noted that W and T should be in the same units *i.e.* kg-wt or newton.

(iii) $S = \frac{T_0}{W}\sinh + \left(\frac{Wl}{T_0}\right) = \frac{T_0}{W}\left(\frac{Wl}{T_0} + \frac{W^3l^3}{6T_0^3} + ...\right) = l + \frac{W^2l^3}{6T_0^2}$ —neglecting higher terms.

The total length of the wire along the curve is $L = 2S = 2l + \frac{W^2l^2}{3T^2}$

This length consists of the unstertched length L_u and the stretch or extension.

$\therefore$ unstretched length $L_u = L$ —extension ΔL

Now, $E = \frac{T/A}{\Delta l/2l}$ or $\Delta l = \frac{2l.T}{EA}$

Substituting the value of T from the relation $d = Wl^2/2T$, we have

$$\Delta l = \frac{2l}{EA} \cdot \frac{Wl^2}{2d} = \frac{Wl^3}{EAd} \qquad \therefore \quad L_u = L - \frac{Wl^3}{EAd}$$

(iv)
$$y = \frac{T_0}{W}\left[\cosh\left(\frac{Wx}{T_0}\right) - 1\right] = \frac{T_0}{W}\left[\left(1 + \frac{W^2x^2}{2T_0^2} + ...\right) - 1\right]$$

or $y = \frac{Wx^2}{2T}$ —**the equation of a parabola**

It shows that the catenary curve formed by the sagging is very approximately like a parabola in shape.

The above formulae are sufficiently accurate for all practical purposes provided sag is less than 10% of the span.

Overhead cables are economical and easy to maintain, which are usually made of bare copper or aluminum wires

41.43. Sag and Tension with Supports at Unequal Levels

Fig. 41.63 shows a span between two supports A and B whose elevations differ by η, their horizontal spacing being $2l$ as before. Such spans are generally met with in a hilly country. Let O be the lowest point of the catenary AOB. Obviously, OA is a catenary of half-span x_1 and OB of half-span x_2.

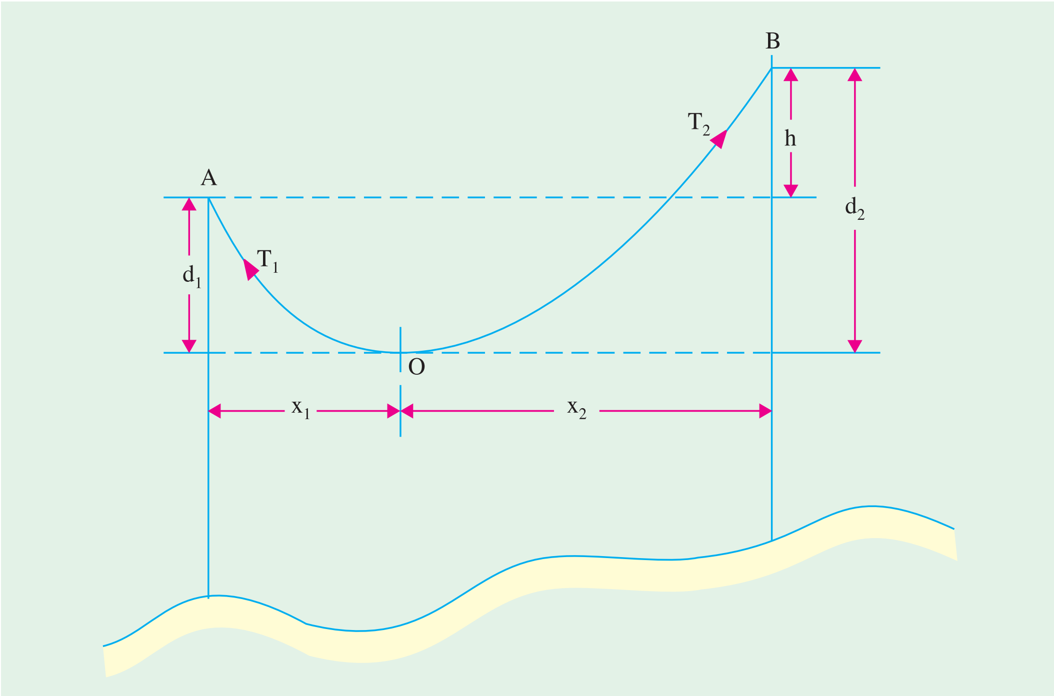


Fig. 41.63

The equations derived in Art. 41.42 also apply to this case.

(*i*) As seen from Eq. (*vi*) of Art. 41.42.

$$d_1 = \frac{T_0}{W}\left[\cosh\left(\frac{Wx_1}{T_0}\right) - 1\right] \qquad \text{—putting } l = x_1$$

$$d_2 = \frac{T_0}{W}\left[\cosh\left(\frac{Wx_2}{T_0}\right) - 1\right] \qquad \text{—putting } l = x_2$$

(*ii*) $$T_1 = T_0 \cosh\left(\frac{Wx_1}{T_0}\right) \text{ and } T_2 = T_0 \cosh\left(\frac{Wx_2}{T_0}\right)$$

It is obvious that maximum tension occurs at the higher support *B*.

$\therefore$ max. permissible tension $= T_0 \cosh\left(\dfrac{Wx_2}{T_0}\right)$

Approximate Formulae

Is is obvious that

$$d_2 - d_1 = \frac{T_0}{W}\left[\cosh\left(\frac{Wx_2}{T_0}\right) - \cosh\left(\frac{Wx_2}{T_0}\right)\right] = h \text{ and } x_1 + x_2 = 2l$$

Using approximations similar to those in Art. 41.42, we get

$$d_1 = \frac{Wx_1^2}{2T_0} = \frac{Wx_1^2}{2T} \quad \text{and} \quad d_2 = \frac{Wx_2^2}{2T_0} = \frac{Wx_2^2}{2T}$$

(it has been assumed, as before, that $T = T_0$)

$$\therefore \quad d_2 - d_1 = h = \frac{W}{2T}(x_2^2 - x_1^2) = \frac{W}{2T}(x_2 + x_1)(x_2 - x_1) = \frac{Wl}{T}(x_2 - x_1)$$

$$\therefore \qquad x_2 - x_1 = \frac{hT}{Wl}; \quad \text{Now, } x_2 - x_1 = hT/Wl \text{ and } x_2 + x_1 = 2l$$

$$\therefore \qquad x_1 = 1 - \frac{hT}{2Wl} \quad \text{and} \quad x_2 = 1 + \frac{hT}{2Wl} \,*$$

Having found x_1 and x_2, values of d_1 and d_2 can be easily calculated. It is worth noting that in some cases, x_1 may be negative which means that there may be no horizontal point (like point O) in the span. Such a thing is very likely to happen when the line runs up a steep mountain side.

41.44. Effect of Wind and Ice

In the formulae derived so far, effect of ice and wind loading has not been taken into account. It is found that under favourable atmospheric conditions, quite an appreciable thickness of ice is formed on transmission lines. The weight of ice acts vertically downwards *i.e.* in the same direction as the weight of the conductor itself as shown in Fig. 41.64. In addition, there may be high wind which exerts considerable force on the conductor. This force is supposed to act in a horizontal direction. If W_i is the weight of ice per unit length of the conductor and W_u the force per unit length exerted by the wind, then total weight of the conductor per unit length is

Effects of wind, ice and snow are important considerations while designing electric cables

$$W_t = \sqrt{(W + W_i)^2 + W_w^2}$$

It is obvious that in all equations derived in Art. 41.42 and 41.43, W should be replaced by W_t. As seen from Fig. 41.64 (*b*), the conductor sets itself in a plane at an angle of $\theta = \tan^{-1} [W_\omega/(W + W_i)]$ to the vertical but keeps the shape of a catenary (or a parabola approximately).

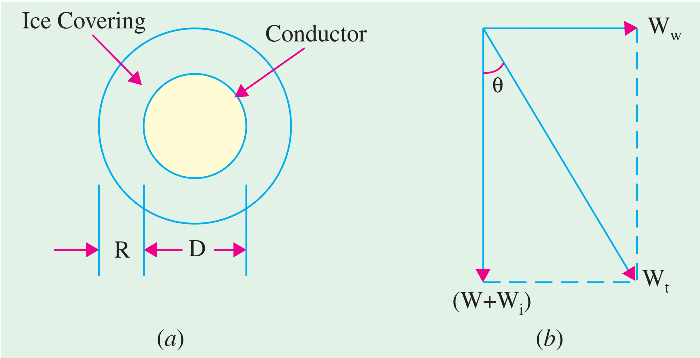


Fig. 41.64

Note. (*i*) If P is the wind pressure per unit projected area of the conductor, then wind load or force per unit length of the ice-covered conductor is

$$W_\omega = P \times (D + 2R) \times l = P \times D \qquad \text{—if conductor is without ice}$$

* If we put $h = 0$ *i.e.* assume supports at the same level, then as expected $x_1 = x_2 = l$

(*ii*) Ice is in the form of a hollow cylinder of inner diameter = D and outer diameter = $(D + 2R)$. Hence, volume per unit length of such a cylinder is

$$= \frac{\pi}{4}[(D + 2R)^2 - D^2] \times 1 = \frac{\pi}{4}(4DR + 4R^2) = \pi R(D + R)$$

If ρ is the ice density, then weight of ice per unit length of the conductor is

$$W_i = \rho \times \text{volume} = \pi\rho\, R(D + R)$$

When ice and wind loads are taken into account, then the approximate formulae derived in Art. 41.42 become

$$d = \frac{W_t l^2}{2T};\quad S = 1 + \frac{W_t^2 l^3}{6T^2} \quad \text{and} \quad y = \frac{W_t x^2}{2T}$$

Here 'd' represents the slant sag in a direction making an angle of θ with the vertical. The vertical sag would be = $d \cos\theta$.

Similarly, formulae given in Art. 41.43 become

$$d_1 = \frac{W_t \cdot x_1^2}{2T};\quad d_2 = \frac{W_t \cdot x_2^2}{2T};\quad x_1 = 1 - \frac{hT}{2W_t \cdot l} \text{ and } x_2 = 1 + \frac{hT}{2W_t \cdot l}$$

Example 41.51. *A transmission line conductor at a river crossing is supported from two towers at heights of 70 m above water level. The horizontal distance between towers is 300 m. If the tension in conductor is 1,500 kg, find the clearance at a point midway between the towers. The size of conductor is 0.9 cm². Density of conductor material is 8.9 g/cm³ and suspension length of the string is 2 metres.*

Solution. Sag, $d = Wl^2/2T$

$l = 150$ m, $W = lA\rho = 2 \times (0.9 \times 10^{-4}) \times (8.9 \times 10^3) = 0.8$ kg wt; $T = 1500$ kg wt

$d = Wl^2/2T = 0.8 \times 150^2/2 \times 1500 = 6$ m

Clearance between conductor and water at mid-way between the towers

$$= 70 - 6 - 2 = 62 \text{ m}$$

Example 41.52. *The effective diameter of a line is 1.96 cm and it weighs 90 kg per 100 metre length. What would be the additional loading due to ice of radial thickness 1.25 cm and a horizontal wind pressure of 30 kg/m² of projected area? Also, find the total weight per metre run of the line. Density of ice is 920 kg/m³.*

Solution. It should be noted that weights of the conductor and ice act vertically downwards whereas wind pressure is supposed to act horizontally. Hence, the total force on one metre length of the conductor is found by adding the horizontal and vertical forces vectorially.

∴ total weight $W_t = \sqrt{(W + W_i)^2 + W_w^2},\quad W_i = \pi\rho R\,(D + R)$

Here $\rho = 920$ kg/m³, $R = 0.0125$ m; $D = 0.0196$ m

$W_w = P(D + 2R) = 30(0.0196 + 2 \times 0.0125) = 1.34$ kg wt/m; $W = 90/100 = 0.9$ kg wt/m

∴ $W_i = \pi \times 920 \times 0.0125(0.0196 + 0.0125) = 1.16$ kg wt/m

∴ $W_t = \sqrt{(W + W_i)^2 + W_w^2} = \sqrt{(0.9 + 1.16)^2 + 1.34^2}$ = **2.46 kg wt/m**

Example 41.53. *A transmission line has a span of 150 metres between supports, the supports being at the same level. The conductor has a cross-sectional area of 2 cm². The ultimate strength is 5,000 kg/cm². The specific gravity of the material is 8.9. If the wind pressure is 1.5 kg/m length of the conductor, calculate the sag at the centre of the conductor if factor of safety is 5.*

(Electrical Technology-I, Bombay Univ.)

Solution. Safety factor $= \dfrac{\text{breaking or ultimate stress}}{\text{working stress}}$

$\therefore$ working stress = 5,000/5 $= 10^3$ kg/cm^2; Working tension, $T = 10^2 \times 2 = 2{,}000$ kg wt.

Vol. of one metre length of conductor $= 2 \times 100 = 200$ cm^3

Wt. of 1 m of material $= 8.9 \times 200$ g or $W = 8.9 \times 200/1{,}000 = 1.98$ kg-wt.

Total Wt. per metre $= \sqrt{W^2 + W_w^2} = \sqrt{1.98^2 + 1.5^2} = 2.48$ kg-wt

Now $d = Wl^2/2T = 2.48 \times (150/2)^2/2 \times 2{,}000 =$ **3.5 metre**

Example 41.54. *A transmission line has a span of 200 metres between level supports. The conductor has a cross-sectional area of 1.29 cm^2, weighs 1,170 kg/km and has a breaking stress of 4,218 kg/cm^2. Calculate the sag for a factor of safety of 5 allowing a wind pressure of 122 kg per m^2 of projected area. What is the vertical sag?*

(Transmission and Distribution-II, Kerala Univ.)

Solution. Safety factor $= \dfrac{\text{breaking or ultimate stress}}{\text{working stress}}$

working stress $= 4{,}218/5 = 843.6$ kg/cm^2

Working tension $T = 843.6 \times 1.29 = 1{,}088$ kg.wt

$\therefore$ $W = 1{,}170$ kg/km $= 1.17$ kg.wt/metre

Let us now find diameter of the conductor from the equation

$\pi d^2/4 = 1.29$ or $d = 1.28$ cm

$\therefore$ Projected area of the conductor per metre length is $= 1.28 \times 10^{-2} \times 1 = 1.28 \times 10^{-2}$ m^2

$W_w = 122 \times 1.28 \times 10^{-2} = 1.56$ kg-wt/m

$\therefore$ $W_t = \sqrt{W^2 + W_w^2} = \sqrt{1.17^2 + 1.56^2} = 1.95$ kg-wt/m

Slant sag $d = W_t l^2/2T = 1.95 \times 100^2/2 \times 1088 =$ **8.96 m**

Now, $\tan\theta = W_w/W = 1.56/1.17 = 1.333$; $\theta = 53.2°$

$\therefore$ vertical sag $= d \cos 53.2° = 8.96 \times 0.599 =$ **5.3 m**

Example 41.55. *A transmission line has a span of 214 metres. The line conductor has a cross-section of 3.225 cm^2 and has an ultimate breaking strengh of 2,540 kg/cm^2. Assuming that the line is covered with ice and provides a combined copper and ice load of 1.125 kg/m while the wind pressure is 1.5 kg/m run* ***(i)*** *calculate the maximum sag produced. Take a factor of safety of 3* ***(ii)*** *also determine the vertical sag.* **(Electrical Power Systems, Gujarat Univ.)**

Solution. ***(i)*** Maximum sag in a direction making an angle of θ (Fig. 41.64) is $d = W_t l^2/2T$

Here, $W + Wi = 1.125$ kg-wt/m ; $Ww = 1.5$ kg-wt/m

$W_t = \sqrt{1.125^2 + 1.5^2} = 1.874$ kg-wt/m; $l = 214/2 = 107$ m

Safety factor $= \dfrac{\text{ultimate breaking stress}}{\text{working stress}}$

$\therefore$ working stress $= 2{,}540/3 = 847$ kg-wt/cm^2

Permissible tension $T = 847 \times 3.225 = 2{,}720$ kg-wt

$\therefore$ $d = 1.874 \times 107^2/2 \times 2{,}720 =$ **3.95 m**

(ii) Now $\tan\theta = W_w/(W_i + W) = 1.5/1.125 = 0.126$; $\theta = 53.2°$; $\cos\theta = 0.599$

$\therefore$ vertical sag $= d \cos\theta = 3.95 \times 0.599 =$ **2.35 m**

Example 41.56. *Two towers of height 30 and 90 m respectively support a transmission line conductor at water crossing. The horizontal distance between the towers is 500 m. If the tension in the conductor is 1,600 kg, find the minimum clearance of the conductor and the clearance of the conductor mid-way between the supports. Weight of the conductor is 1.5 kg/m. Bases of the towers can be considered to be at the water level.* **(Electrical Power-I, Bombay Univ.)**

Solution. $x_1 = l - \dfrac{hT}{2Wl}$

Here, $l = 500/2 = 250$ m

$h = 90 - 30 = 60$ m

$T = 1{,}600$ kg-wt ;

$W = 1.5$ kg-wt/m

$$\therefore \quad x = 250 - \frac{60 \times 1{,}600}{2 \times 1.5 \times 250} = 250 - 128 = 122 \text{ m}$$

$$x_1 = 250 + 128 = 378 \text{ m}$$

$$d_1 = \frac{W_{x1}^2}{2T} = \frac{1.5 \times 122^2}{2 \times 1600} = 7 \text{ m}$$

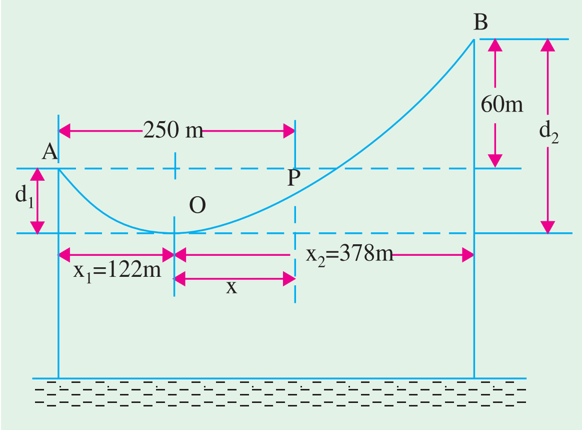


Fig. 41.65

As seen from Fig. 41.65, clearance of the lowest point O from the water level

$$= 30 - 7 = 23 \text{ m}$$

The horizontal distance of mid-point P from the reference point O is $x = (250 - 122) = 128$m*. The height of the point P above O is

$$d_{mid} = \frac{Wx_1^2}{2T} = \frac{1.5 \times 128^2}{2 \times 1{,}600} = 7.68 \text{ m}$$

Hence, clearance of mid-point above water level is 23 + 7.68 = **30.68 m**

Example 41.57. *An overhead transmission line at a river crossing is supported from two towers at heights of 50 m and 100 m above the water level, the horizontal distance between the towers being 400 m. If the maximum allowable tension is 1,800 kg and the conductor weighs 1 kg/m, find the clearance between the conductor and water at a point mid-way between the towers.*

(Power System-I, AMIE, Sec. B, 1994)

Solution. Here, $h = 100 - 50 = 50$ m ;

$l = 400/2 = 200$ m

$T = 1800$ kg-wt

$W = 1$ kg-wt/m

$$x_1 = l - \frac{hT}{2Wl} = 200 - \frac{50 \times 1800}{2 \times 1 \times 200} = -25 \text{ m}$$

$$x_2 = l + \frac{hT}{2Wl} \quad 200 + 225 = 425 \text{ m}$$

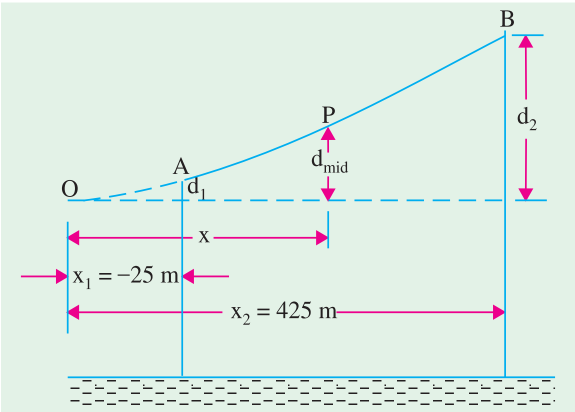


Fig. 41.66

* Or $x = \frac{1}{2}(x_2 - x_1)$. In this case, $x = \frac{1}{2}(378 - 122) = 128$ m

Since x_1 turns out to be negative, point A lies on the same side of O as B (Fig. 41.66). Distance of mid-point P from O is x = (425 + 25) = 225 m. Hence, height of P above O is

$$d_{mid} = \frac{Wx^2}{T} = \frac{1 \times 225^2}{2 \times 1{,}800} = 14 \text{ m}$$

Now, $$d_2 = \frac{Wx_2^2}{2T} = \frac{1 \times 425^2}{2 \times 1{,}800} = 50.2 \text{ m}$$

Hence, P is (50.2–14) = 36.2 m below point B. It means that mid-point P is (100 – 36.2) = 63.8 m above water level.

Example 41.58. *A conductor is strung across a river, being supported at the two ends at heights of 20 m and 16 m respectively, from the bed of the river. The distance between the supports is 375 m and the weight of the conductor = 1.2 kg/m.*

If the clearance of the conductor from the river bed be 9 m, find the horizontal tension in the conductor. Assume a parabolic configuration and that there is no wind or ice loading.

(Electrical Power-I, Bombay Univ.)

Solution. Here, l = 375/2 = 187.5 m; h = 20 – 16 = 4 m; W = 1.2 kg-wt/m; T = ?

$$x_1 = l - \frac{hT}{2Wl} = 187.5 - \frac{4 \times T}{2 \times 1.2 \times 187.5} = 187.5 - \frac{T}{112.5}$$

or $x_1 = (187.5 - m)$ where $m = T/112.5$

Also, $d_1 + 9 = 16$ or $d_1 = 7 = \dfrac{Wx_1^2}{2T}$

$$\therefore \quad 7 = \frac{1.2\,(187.5 - m)^2}{225\,m} \qquad (\because\ 2T = 2m \times 112.5 = 225\ m)$$

or $1.2\,m^2 - 2{,}025\,m + 42{,}180 = 0$ or m = 1,677 or 10.83

Rejecting the bigger value which is absurd, we have

m = 10.83 or $T/112.5$ = 10.83 or T = **1,215 kg-wt/m**

Tutorial Problem No. 41.3

1. Show diagrammatically the distribution of electrostatic capacitance in a 3-core, 3-phase lead-sheathed cable.

   The capacitance of such a cable measured between any two of the conductors, the sheathing being earthed, is 0.3 μF per km. Find the equivalent star-connected capacitance and the kVA required to keep 10 km of the cable charged when connected to 20,000-V, 50 Hz bus-bars. **[0-6 μ F ; 754 kVA]**

2. The 3-phase output from a hydro-electric station is transmitted to a distributing centre by two over-head lines connected in parallel but following different routes. Find how a total load of 5,000 kW at a p.f. of 0.8 lagging would divide between the two routes if the respective line resistances are 1.5 and 1.0 Ω and their reactances at 25 Hz are 1.25 and 1.2 Ω.

   [2,612 kW ; 2,388 kW](*City & Guilds, London*)

3. Two 3-phase cables connected in parallel supply a 6,600-V, 1,000-kW load at a lagging power factor of 0.8. The current in one of the cables is 70 A and it delivers 600 kW. Calculate its reactance and resistance, given that the other cable has reactance of 2.6 Ω and a resistance of 2 Ω.

   [R = .795 Ω ; X = 1.7 Ω] (*London Univ.*)

4. A concentric cable has a conductor diameter of 1 cm and an insulation thickness of 1.5 cm. Find the maximum field strength when the cable is subjected to a test pressure of 33 kV.

   [47.6 kV/cm (r.m.s.) or 67.2 kV/cm (peak)] (*London Univ.*)

5. A single-phase ring distributor XYZ is fed at X. The loads at Y and Z are 20 A at 0.8 p.f. lagging and 15 A at 0.6 p.f. lagging respectively, both expressed with reference to voltage at X. The total impedances of the three sections X Y, YZ and ZX are $(1 + j1)$, $(1 + j2)$ and $(1 + j3)$ ohms respectively. Find the total current fed at X and the current in each section with respect to suply voltage at X. ***(Allahabad Univ.)***

[34.6 ∠ – 43.8° A ; AXY = 23.1 ∠– 32° A, AXZ = 13.1 ∠– 60.8°. YZ = 3.01 ∠– 30.5°A]

6. A single-phase distributor has a resistance of 0.2 ohm and reactance 0.3 ohm. At the far end, the voltage V_B = 240 volts, the current is 100 A and the power factor is 0.8. At the mid point A, current of 100 A is supplied at a power factor 0.6 with reference to voltage V_A at point A. Find supply voltage V_S and the phase angle between V_S and V_B. **[292 V, 2.6º]**

7. Estimate the corona loss for a 3-phase 110-kV, 50-Hz, 150-km long transmission line consisting of three conductors, each of 10 mm diameter and spaced 2.5 metre apart in an equilateral triangle formation. The temperature of air is 30ºC and the atmospheric pressure is 750 mm of mercury. Take the irregularity factor as 0.85. **[385 kW]*(AMIE)***

8. A transmission line conductor at a river crossing is supported from two towers at heights of 20 m and 60 m above water level. The horizontal distance between the towers is 300 m. If the tension in the conductor is 1800 kg and the conductor weighs 1.0 kg per metre, find the clearance between the conductor and the water level at a point mid-way between the towers. Use approximate method. **(18.75 m) *(AMIE)***

9. Show that the positive and negative sequence impedances of transmission lines are same where as its zero sequence impedance is higher than positive sequence impedance. ***(Nagpur University, Summer 2004)***

10. A 132 kV, 3 phase, 50 Hz overhead line of 100 km length has a capacitance to earth of each line of 0.01 μF/km. Determine inductance and kVA rating of the arc suppression coil suitable for this system. ***(Nagpur University, Summer 2004)***

OBJECTIVE TESTS – 41

1. With same maximum voltage between conductors, the ratio of copper volumes in 3-phase, 3-wire system and 1-phase, 2-wire system is
   (*a*) 4/3 (*b*) 3/4
   (*c*) 5/3 (*d*) 3/5
2. The volume of copper required for an a.c. transmission line is inversely proportional to
   (*a*) current (*b*) voltage
   (*c*) power factor (*d*) both (*b*) and (*c*)
   (*e*) both (*a*) and (*c*).
3. For a.c. transmission lines less than 80 km in length, it is usual to lump the line capacitance at
   (*a*) the receiving end
   (*b*) the sending end
   (*c*) the mid-point
   (*d*) any convenient point.
4. Corona occurs between two transmission wires when they
   (*a*) are closely-spaced
   (*b*) are widely-spaced
   (*c*) have high potential difference
   (*d*) carry d.c. power.
5. The only advantage of corona is that it
   (*a*) makes line current non-sinusoidal
   (*b*) works as a safety-valve for surges
   (*c*) betrays its presence by hissing sound
   (*d*) produces a pleasing luminous glow.
6. The sag produced in the conductor of a transmission wire depends on
   (*a*) weight of the conductor per unit length
   (*b*) tension in the conductor
   (*c*) length of the conductor
   (*d*) all of the above
   (*e*) none of the above.
7. Suspension insulators are used when transmission voltage is
   (*a*) high (*b*) low
   (*c*) fluctuating (*d*) steady
8. The string efficiency of suspension insulators can be increased by
   (*a*) providing a guard ring
   (*b*) grading the insulators

(*c*) using identical insulator disc
(*d*) both (*a*) & (*b*).

9. An interconnector between two generating stations facilitates to
(*a*) keep their voltage constant
(*b*) run them in parallel
(*c*) transfer power in either direction
(*d*) both (*b*) & (*c*)

10. The effective disruptive critical voltage of a transmission line does NOT depend on
(*a*) irregularity factor
(*b*) conductor radius
(*c*) distance between conductors
(*d*) material of the conductors.

11. By which of the following systems electric power may be transmitted?
(*a*) Overhead system
(*b*) Underground system
(*c*) Both (a) and (b)
(*d*) None of the above

12. .... are the conductors, which cannect the consumer's terminals to the distribution
(*a*) Distributors
(*b*) Service mains
(*c*) Feeders
(*d*) None of the above

13. The underground system cannot be operated above
(*a*) 440 V (*b*) 11 kV
(*c*) 33 kV (*d*) 66 kV

14. Overhead system can be designed for operation upto
(*a*) 11 kV (*b*) 33 kV
(*c*) 66 kV (*d*) 400 kV

15. If variable part of annual cost on account of interest and depreciation on the capital outlay is equal to the annual cost of electrical energy wasted in the conductors, the total annual cost will be minimum and the corresponding size of conductor will be most economical. This statement is known as
(*a*) Kelvin's law (*b*) Ohm's law
(*c*) Kirchhoff's law (*d*) Faraday's law
(*e*) none of the above

16. The wooden poles well impregnated with creosite oil or any preservative compound have life
(*a*) from 2 to 5 years
(*b*) 10 to 15 years
(*c*) 25 to 30 years
(*d*) 60 to 70 years

17. Which of the following materials is not used for transmission and distribution of electrical power?
(*a*) Copper
(*b*) Aluminium
(*c*) Steel
(*d*) Tungsten

18. Galvanised steel wire is generally used as
(*a*) stay wire
(*b*) earth wire
(*c*) structural components
(*d*) all of the above

19. The usual spans with R.C.C. poles are
(*a*) 40–50 metres
(*b*) 60–100 metres
(*c*) 80–100 metres
(*d*) 300–500 metres

20. The corona is considerably affected by which of the following?
(*a*) Size of the conductor
(*b*) Shape of the conductor
(*c*) Surface condition of the conductor
(*d*) All of the above

21. Which of the following are the constants of the transmission lines?
(*a*) Resistance
(*b*) Inductance
(*c*) Capacitance
(*d*) All of the above

22. %age regulation of a transmission line is given by

(*a*) $\dfrac{V_R - V_S}{V_R{}^2} \times 100$

(*b*) $\dfrac{V_S - V_R}{V_R} \times 100$

(*c*) $\dfrac{V_S - V_R}{V_S} \times 100$

(*d*) $\dfrac{V_S - V_R}{V_R{}^2} \times 100$

where V_S and V_R are the voltages at the sending end and receiving and respectively.

23. The phenomenon of rise in voltage at the receiving end of the open-circuited or lightly loaded line is called the
(*a*) Seeback effect
(*b*) Ferranti effect
(*c*) Raman effect
(*d*) none of the above

24. The square root of the ratio of line impedance and shunt admittance is called the
(*a*) surge impedance of the line
(*b*) conductance of the line
(*c*) regulation of the line
(*d*) none of the above

25. Which of the following is the demerit of a 'constant voltage transmission system'?
(*a*) Increase of short-circuit current of the system
(*b*) Availability of steady voltage at all loads at the line terminals
(*c*) Possibility of better protection for the line due to possible use of higher terminal reactances
(*d*) Improvement of power factor at times of moderate and heavy loads
(*e*) Possibility of carrying increased power for a given conductor size in case of long-distance heavy power transmission

26. Low voltage cables are meant for use up to
(*a*) 1.1 kV
(*b*) 3.3 kV
(*c*) 6.6 kV
(*d*) 11 kV

27. The operating voltage of high voltage cables is upto
(*a*) 1.1 kV
(*b*) 3.3 kV
(*c*) 6.6 kV
(*d*) 11 kV

28. The operating voltage of supertension cables is upto
(*a*) 3.3 kV
(*b*) 6.6 kV
(*c*) 11 kV
(*d*) 33 kV

29. The operating voltage of extra high tension cables is upto
(*a*) 6.6 kV
(*b*) 11 kV
(*c*) 33 kV
(*d*) 66 kV
(*e*) 132 kV

30. Which of the following methods is used for laying of underground cables?
(*a*) Direct laying
(*b*) Draw-in-system
(*c*) Solid system
(*d*) All of the above

31. Which of the following is the source of heat generation in the cables?
(*a*) Dielectric losses in cable insulation
(*b*) I^2R losses in the conductor
(*c*) Losses in the metallic sheathings and armourings
(*d*) All of the above

32. Due to which of the following reasons the cables should not be operated too hot?
(*a*) The oil may loose its viscosity and it may start drawing off from higher levels
(*b*) Expansion of the oil may cause the sheath to burst
(*c*) Unequal expansion may create voids in the insulation which will lead to ionization
(*d*) The thermal instability may rise due to the rapid increase of dielectric losses with temperature

33. Which of the following D.C. distribution system is the simplest and lowest in first cost?
(*a*) Radial system
(*b*) Ring system
(*c*) Inter-connected system
(d) Non of the above

34. A booster is a
(*a*) series wound generator
(*b*) shunt wound generator
(*c*) synchronous generator
(*d*) none of the above

35. Besides a method of trial and error, which of the following methods is employed for solution of network problems in interconnected system?
(*a*) Circulating current method
(*b*) Thevenin's theorem
(*c*) Superposition of currents
(*d*) direct application of Kirchhoff's laws
(*e*) All of the above

36. Which of the following faults is most likely to occur in cables?
(*a*) Cross or short-circuit fault
(*b*) Open circuit fault
(*c*) Breakdown of cable insulation
(*d*) all of the above

37. The cause of damage to the lead sheath of a cable is
(*a*) crystallisation of the lead through vibration
(*b*) chemical action on the lead when
(*c*) mechanical damage
(*d*) all of the above

38. The voltage of the single phase supply to residential consumers is
(*a*) 110 V
(*b*) 210 V
(*c*) 230 V
(*d*) 400 V

39. Most of the high voltage transmission lines in India are
(*a*) underground
(*b*) overhead

(*c*) either of the above
(*d*) none of the above

40. The distributors for residential areas are
(*a*) single phase
(*b*) three-phase three wire
(*c*) three-phase four wire
(*d*) non of the above

41. The conductors of the overhead lines are
(*a*) solid
(*b*) stranded
(*c*) both solid and stranded
(*d*) none of the above

42. High voltage transmission lines use
(*a*) suspension insulators
(*b*) pin insulators
(*c*) both (*a*) and (*b*)
(*d*) none of the above

43. Multicore cables generally use
(*a*) square conductors
(b) circular conductors
(c) rectangular conductors
(d) sector-shaped conductors
(e) none of the above

44. Distributio lines in India generally use
(*a*) wooden poles
(*b*) R.C.C. poles
(*c*) steel towers
(*d*) none of the above

45. The material commonly used for insulation in high voltage cables is
(*a*) lead
(*b*) paper
(*c*) rubber
(*d*) none of the above

46. The loads on distributors systems are generally
(*a*) balanced
(*b*) unbalanced
(*c*) either of the above
(*d*) none of the above

47. The power factor of industrial loads is generally
(*a*) unity
(*b*) lagging
(*c*) leading
(*d*) zero

48. Overhead lines generally use
(*a*) copper conductors
(*b*) all aluminium conductors
(*c*) A.C.S.R. conductors
(*d*) none of these

49. In transmission lines the cross-arms are made of
(*a*) copper
(*b*) wood
(*c*) R.C.C.
(*d*) steel

50. The material generally used for armour of high voltage cables is
(*a*) aluminium
(*b*) steel
(*c*) brass
(*d*) copper

51. Transmission line insulators are made of
(*a*) glass
(*b*) porcelain
(*c*) iron
(*d*) P.V.C.

52. The material commonly used for sheaths of underground cables is
(*a*) lead
(*b*) rubber
(c) copper
(d) iron

53. The minimum clearance between the ground and a 220 kV line is about
(*a*) 4.3 m
(*b*) 5.5 m
(*c*) 7.0 m
(*d*) 10.5 m

54. The spacing between phase conductors of a 220 kV line is approximately equal to
(*a*) 2 m
(*b*) 3.5 m
(*c*) 6 m
(*d*) 8.5 m

55. Large industrial consumers are supplied electrical energy at
(*a*) 400 V
(*b*) 11 kV
(*c*) 66 kV
(*d*) 400 kV

56. In a D.C. 3-wire distribution system, balancer fields are cross-connected in order to
(*a*) boost the generated voltage
(*b*) balance loads on both sides of the neutral
(*c*) make both machines run as unloaded motors
(*d*) equalize voltages on the positive and negative outers

57. In a D.C. 3-wire distributor using balancers and having unequal loads on the two sides
(*a*) both balancers run as generators
(*b*) both balancers run as motors
(*c*) balancer connected to lightly-loaded side runs as a motor
(*d*) balancer connected to heavily-loaded side runs as a motor

58. Transmitted power remaining the same, if supply voltae of a D.C. 2-wire feeder is increased 100 percent, saving in copper is

(*a*) 25 percent
(*b*) 50 percent
(*c*) 75 percent
(*d*) 100 percent

59. A uniformly-loaded D.C. distributor is fed at both ends with equal voltages. As compared to a similar distributor fed at one end only, the drop at a middle point is
(*a*) one-fourth
(*b*) one-third
(*c*) one-half
(*d*) twice
(*e*) none of the above

60. As compared to a 2-wire D.C. distributor, a 3-wire distributor with same maximum voltage to earth uses only
(*a*) 31.25 percent of copper
(*b*) 33.3 percent of copper
(*c*) 66.7 percent of copper
(*d*) 125 percent of copper

61. Which of the following is usually not the generating voltage?
(*a*) 6.6 kV
(*b*) 8.8 kV
(*c*) 11 kV
(*d*) 13.2 kV

62. For an overhead line, the surge impedance is taken as
(*a*) 20–40 ohms
(*b*) 70–80 ohms
(*c*) 100–200 ohms
(*d*) 500–1000 ohms
(*e*) none of the above

63. The presence of ozone due to corona is harmful because it
(*a*) reduces power factor
(*b*) corrodes the material
(*c*) gives odour
(*d*) transfer energy to the ground
(*e*) none of the above

64. A feeder, in a transmission system, feeds power to
(*a*) distributors
(*b*) generating stations
(*c*) service mains
(*d*) all of the above

65. The power transmitted will be maximum when
(*a*) corona losses are minimum
(*b*) reactance is high
(*c*) sending end voltage is more
(*d*) receiving end voltage is more

66. A 3-phase 4 wire system is commonly used on
(*a*) primary transmission
(*b*) secondary transmission
(*c*) primary distribution
(*d*) secondary distribution

67. Which of the following materials is used for overhead transmission lines?
(*a*) Steel cored aluminium
(*b*) Galvanised steel
(*c*) Cadmium copper
(*d*) Any of the above

68. Which of the following is not a constituent for making porcelain insulators?
(*a*) Quartz
(*b*) Kaolin
(*c*) Felspar
(*d*) Silica

69. There is a greater possibility of occurence of corona during
(*a*) dry weather
(*b*) winter
(*c*) summer heat
(*d*) humid weather
(*e*) none of the above

70. Which of the following relays is used on long transmission lines?
(*a*) Impedance realy
(*b*) Mho's relay
(*c*) Reactance relay
(*d*) None of the above

71. The steel used in steel cored conductors is usually
(*a*) alloy steel
(*b*) stainless steel
(*c*) mild steel
(*d*) high speed steel
(*e*) all of the above

72. Which of the following distribution system is more reliable?
(*a*) Radial system
(*b*) Tree system
(*c*) Ring main system
(*d*) All are equally reliable

73. Which of the followin characteristics should the line supports for transmission lines possess?
(*a*) Low cost
(*b*) High mechanical strength
(*c*) Longer life
(*d*) All of the above

74. Transmission voltage of 11 kV is normally used for distance upto
(*a*) 20–25 km
(*b*) 40–50 km
(*c*) 60–70 km
(*d*) 80–100 km

75. which of the following regulations is considered best?
(*a*) 50%
(*b*) 20%

(*c*) 10%
(*d*) 2%

76. Skin effect is proportional to
(*a*) (conductor diameter)^4
(*b*) (conductor diameter)^3
(*c*) (conductor diameter)^2
(*d*) (conductor diameter)$^{1/2}$
(*e*) none of the above

77. A conductor, due to sag between two supports, takes the form of
(*a*) semi-circle
(*b*) triangle
(*c*) ellipse
(*d*) catenary

78. In A.C.S.R. conductors, the insulation between aluminium and steel conductors is
(*a*) insulin
(*b*) bitumen
(*c*) varnish
(*d*) no insulation is required

79. Which of the following bus-bar schemes has the lowest cost?
(*a*) Ring bus-bar scheme
(*b*) Single bus-bar scheme
(*c*) Breaker and a hlaf scheme
(*d*) Main and transfer scheme

80. Owing to skin effect
(*a*) current flows through the half cross-section of the conductor
(*b*) portion of the conductor near the surface carries more current and core of the conductor carries less current
(*c*) portion of the conductor near the surface carries less current and core of the conductor carries more current
(*d*) none of the above

81. By which of the following methods string efficiency can be improved?
(*a*) Using a guard ring
(*b*) Grading the insulator
(*c*) Using long cross arm
(*d*) Any of the above
(*e*) None of the above

82. In aluminium conductors, steel core is provided to
(*a*) compensate for skin effect
(*b*) neutralise proximity effect
(*c*) reduce line inductance
(*d*) increase the tensile strength

83. By which of the following a bus-bar is rated?
(*a*) Current only
(*b*) Current and voltage
(*c*) Current, voltage and frequency
(*d*) Current, voltage, frequency and short time current

84. A circuit is disconnected by isolators when
(*a*) line is energize
(*b*) there is no current in the line
(*c*) line is on full load
(*d*) circuit breaker is not open

85. For which of the following equipment current rating is not necessary?
(*a*) Circuit breakers
(*b*) Isolators
(*c*) Load break switch
(*d*) Circuit breakers and load break switches

86. In a substation the following equipment is not installed
(*a*) exciters
(*b*) series capacitors
(*c*) shunt reactors
(*d*) voltage transformers

87. Corona usuall occurs when the electrostatic stress in air around the conductor exceeds
(*a*) 6.6 kV (r.m.s. value)/cm
(*b*) 11 kV (r.m.s. value)/cm
(*c*) 22 kV (maximum value)/cm
(*d*) 30 kV (maximum value)/cm

88. The voltage drop, for constant voltage transmission is compensated by installing
(*a*) inductors
(*b*) capacitors
(*c*) synchronous motors
(*d*) all of above
(*e*) none of the above

89. The use of strain type insulators is made where the conductors are
(*a*) dead ended
(*b*) at intermediate anchor towers
(*c*) any of the above
(*d*) none of the above

90. The current drawn by the line due to corona losses is
(*a*) non-sinusoidal
(*b*) sinusoidal
(*c*) triangular
(*d*) square

91. Pin type insulators are generally not used for voltages beyond
(*a*) 1 kV
(*b*) 11 kV
(*c*) 22 kV
(*d*) 33 kV

92. Aluminium has a specific gravity of
(*a*) 1.5
(*b*) 2.7
(*c*) 4.2
(*d*) 7.8

93. For transmission of power over a distance of 200 km, the transmission voltage should be

(a) 132 kV
(b) 66 kV
(c) 33 kV
(d) 11 kV

94. For aluminium, as compared to copper, all the following factors have higher values except
(a) specific volume
(b) electrical conductivity
(c) co-efficient of linear expansion
(d) resistance per unit length for same cross-section

95. Which of the following equipment, for regulating the voltage in distribution feeder, will be most economical?
(a) Static condenser
(b) Synchronous condenser
(c) The changing transformer
(d) Booster transformer

96. In a tap changing transformer, the tappings are provided on
(a) primary winding
(b) secondary winding
(c) high voltage winding
(d) any of the above

97. Constant voltage transmission entails the following disadvantage
(a) large conductor area is required for same power transmission
(b) short-circuit current of the system is increased
(c) either of the above
(d) none of the above

98. On which of the following factors skin effect depends?
(a) Frequency of the current
(b) Size of the conductor
(c) Resistivity of the conductor material
(d) All of the above

99. The effect of corona can be detected by
(a) presence of zone detected by odour
(b) hissing sound
(c) faint luminous glow of bluish colour
(d) all of the above

100. for transmission of power over a distance of 500 km, the transmission voltage should be in the range
(a) 150 to 220 kV
(b) 100 to 120 kV
(c) 60 to 100 kV
(d) 20 to 50 kV

101. In the analysis of which of the following lines shunt capacitance is neglected?
(a) Short transmission lines
(b) Medium transmission lines
(c) Long transmission lines
(d) Modium as well as long transmission lines

102. When the interconnector between two stations has large reactance
(a) the transfer of power will take place with voltage fluctuation and noise
(b) the transfer of power will take place with least loss
(c) the stations will fall out of step because of large angular displacement between the stations
(d) none of the above

103. The frequency of voltage generated, in case of generators, can be increased by
(a) using reactors
(b) increasing the load
(c) adjusting the governor
(d) reducing the terminal voltage
(e) none of the above

104. When an alternator connected to the bus-bar is shut down the bus-bar voltage will
(a) fall
(b) rise
(c) remain uncharged
(d) none of the above

105. The angular displacement between two interconnected stations is mainly due to
(a) armature reactance of both alternators
(b) reactance of the interconnector
(c) synchronous reactance of both the alternators
(d) all of the above

106. Electro-mechanical voltage regulators are generally used in
(a) reactors
(b) generators
(c) transformers
(d) all of the above

107. Series capacitors on transmission lines are of little use when the load VAR requirement is
(a) large
(b) small
(c) fluctuating
(d) any of the above

108. The voltage regulation in magnetic amplifier type voltage regulator is effected by
(a) electromagnetic induction
(b) varying the resistance
(c) varying the reactance
(d) variable transformer

109. when a conductor carries more current on the surface as compared to core, it is due to
(a) permeability variation
(b) corona
(c) skin effect
(d) unsymmetrical fault

(*e*) none of the above

110. The following system is not generally used
(*a*) 1-phase 3 wire
(*b*) 1-phase 4 wire
(*c*) 3-phase 3 wire
(*d*) 3-phase 4 wire

111. The skin effect of a conductor will reduce as the
(*a*) resistivity of conductor material increases
(*b*) permeability of conductor material increases
(*c*) diameter increases
(*d*) frequency increases

112. When a live conductor of public electric wupply breaks down and touches the earth which of the following will happen?
(*a*) Current will flow to earth
(*b*) Supply voltage will drop
(*c*) Supply voltage will increase
(*d*) No current will flow in the conductor

113. 310 km line is considered as
(*a*) a long line
(*b*) a medium line
(*c*) a short line
(*d*) any of the above

114. The conductors are bundled primarily to
(*a*) increase reactance
(*b*) reduce reactance
(*c*) reduce ratio interference
(*d*) none of the above

115. The surge impedance in a transmission line having negligible resistance is given as
(*a*) $\sqrt{LC}$
(*b*) $\sqrt{L/C}$
(*c*) $\sqrt{1/LC}$
(*d*) $\sqrt{L+C}$
(*e*) none of the above

116. The top most conductor in a high transmission line is
(*a*) earth conductor
(*b*) R-phase conductor
(*c*) Y-phase conductor
(*d*) B-phase conductor

117. In A.C.S.R. conductor the function of steel is to
(*a*) provide additional mechanical strength
(*b*) prevent corona
(*c*) take care of surges
(*d*) reduce inductance and subsequently improve power factor

118. In transmission and distribution system the permissible voltage variation is
(*a*) ±1 percent
(*b*) ± 10 percent
(*c*) ± 20 percent
(*d*) ± 30 percent
(*e*) none of the above

119. By which of the following methods voltage of transmission can be regulated?
(*a*) use of series capacitors to neutralies the effect of series reactance
(*b*) Switching in shunt capacitors at the receiving end during have loads
(*c*) Use of tap changing transformers
(*d*) Any of the above methods

120. Which of the following distribution systems is the most economical?
(*a*) A.C. 1-phase system
(*b*) A.C. 3-phase 3 wire system
(*c*) A.C. 3-phase 4 wire system
(*d*) Direct current system

121. Which of the following is the main advantage of A.C. transmission system over D.C. transmission system?
(*a*) Less instability problem
(*b*) Less insulation problems
(*c*) Easy transformation
(*d*) Less losses in transmission over long distances

122. A tap changing transformer is used to
(*a*) supply low voltage current for instruments
(*b*) step up the voltage
(*c*) step down the voltage
(*d*) step up as well as step down the voltage

123. Which of the following bar schemes is the most expensive?
(*a*) Double bus-bar double breaker
(*b*) Ringbus-bar scheme
(*c*) Single bus-bar scheme
(*d*) Main and transfer scheme

124. By which of the following methods the protection agains direct lightning strokes and high voltage sweep waves is provided?
(*a*) Lightening arresters
(*b*) Ground wire
(*c*) Lightening arresters and ground wires
(*d*) Earthing of neutral
(*d*) None of the above

125. In which of the following voltage regulators the effect of dead zero is found?
(*a*) Electromagnetic type
(*b*) Magnetic amplifier
(*c*) Electronic type using integrated circuits
(*e*) all of the above

126. Corona results in

(*a*) radio interference
(*b*) power factor improvement
(*c*) better regulation
(*d*) none of the above

127. Which of the following has least effect on corona?
(*a*) Atmospheric temperature
(*b*) Number of ions
(*c*) Size and charge per ion
(*d*) Mean free path

128. In context of corona, if the conductors are polished and smooth, which of the following statements is correct?
(*a*) Hissing sound will be more intense
(*b*) Power loss will be least
(*c*) Corona glow will be uniform along the length of the conductor
(*d*) Corona glow will not occur

129. Power loss due to corona is not directly proportional to
(*a*) spacing between conductors
(*b*) supply voltage frequency
(*c*) phase-neutral voltage
(*d*) all of the above

130. Poles which carry transformers are usually
(*a*) circular
(*b*) I-type
(*c*) A-type
(*d*) H-type
(*e*) none of the above

131. Out of the following which type of poles are bulky?
(*a*) Transmission towers
(*b*) Concrete poles
(*c*) Tubular steel poles
(*d*) Wooden poles

132. The effect of ice on transmission line conductors is to increase the
(*a*) transmission losses
(*b*) weight of the conductor
(*c*) tendency for corona
(*d*) resistance to flow of current

133. If the height of transmission tower is increased
(*a*) the line capacitance will decrease but line inductance will remain uncharged
(*b*) the line capacitance and inductance will not change
(*c*) the line capacitance will increase but line inductance will decrease
(*d*) the line capacitance will decrease and line inductance will increase

134. If staring efficiency is 100 percent it means that
(*a*) potential across each disc is zero
(*b*) potential across each disc is same
(*c*) one of the insulator discs is shorted
(*d*) none of the above

135. In a 70/6 A.C.S.R. conductor there are
(*a*) 35 aluminium conductors and 3 steel conductors
(*b*) 70 aluminium conductors and 6 steel conductors
(*c*) 70 steel conductors and 6 aluminium conductors
(*d*) none of the above

136. On which of the following does the size of a feeder depend?
(*a*) Voltage drop
(*b*) Voltage
(*c*) Frequency
(*d*) Current carrying capacity

137. Which of the following are connected by the service mains?
(*a*) Transformer and earth
(*b*) Distributor and relay system
(*c*) Distributor and consumer terminals
(*d*) Distributor and transformer

138. In the design of a distributor which of the following is the major consideration?
(*a*) Voltage drop
(*b*) Current carrying capacity
(*c*) Frequency
(*d*) kVA of system
(*e*) None of the above

139. In a distribution system major cost is that of
(*a*) earthing system
(*b*) distribution transformer
(*c*) conductors
(*d*) meters

140. A booster is connected in
(*a*) parallel with earth connection
(*b*) parallel with the feeder
(*c*) series with the feeder
(*d*) series with earth connection

141. With which of the following are step-up substations associated?
(*a*) Concentrated load
(*b*) Consumer location
(*c*) Distributors
(*d*) Generating stations
(*e*) None of the above

142. Which of the following equipment should be installed by the consumers having low power factor?
(*a*) Synchronous condensers
(*b*) Capacitor bank
(*c*) Tap changing transformer
(*d*) Any of the above
(*e*) None of the above

143. Which of the following equipment is used to

limit short-circuit current level in a substation?
(*a*) Isolator
(*b*) Lightning switch
(*c*) Coupling capacitor
(*d*) Series reactor

144. Steepness of the travelling waves is alternated by .... of the line
(*a*) capacitance
(*b*) inductance
(*c*) resistance
(*d*) all of the above

145. The limit of distance of transmission line may be increased by the use of
(*a*) series resistances
(*b*) shunt capacitors and series reactors
(*c*) series capacitors and shunt reactors
(*d*) synchronous condensers
(*e*) none of the above

146. By which of the following feactors is the sag of a transmission line least affected?
(*a*) Current through the conductor
(*b*) Ice deposited on the conductor
(*c*) Self weight of conductor
(*d*) Temperature of surrounding air
(*e*) None of the above

147. Which of the following cause transient disturbances?
(*a*) Faults
(*b*) Load variations
(*c*) Switching operations
(*d*) Any of the above

148. A gay wire
(*a*) protects conductors against shortcircuiting
(*b*) provides emergency earth route
(*c*) provides protection against surges
(*d*) supports the pole

149. Which of the following is neglected in the analysis of short transmission lines?
(*a*) Series impedance
(*b*) Shunt admittance
(*c*) I^2R loss
(*d*) None of the above
(*e*) All of the above

150. Basically the boosters are
(*a*) synchronous motors
(*b*) capacitors
(*c*) inductors
(*d*) transformers

151. Which of the following is a static exciter?
(*a*) Rectifier
(*b*) Rotorol
(*c*) Amplidyne
(*d*) D.C. separately excited generator

152. For exact compensation of voltage drop in the feeder the booster
(*a*) must be earthed
(*b*) must work on line voltage
(*c*) must work on linear portion of its V-I characteristics
(*d*) must work on non-linear portion of its V-I characteristics

153. The purpose of using a booster is to
(*a*) increase current
(*b*) reduce current
(*c*) reduce voltage drop
(*d*) compensate for voltage drop
(e) none of the above

154. Induction regulators are used for voltage control in
(*a*) alternators
(*b*) primary distribution
(*c*) secondary distribution
(*d*) none of the above

155. A synchronous condenser is generally installed at the .... of the transmission line
(*a*) receiving end
(*b*) sending end
(*c*) middle
(*d*) none of the above

156. The area of cross-section of the neutral in a 3-wire D.C. system is generally .... the area of cross-section of main conductor
(*a*) same as
(*b*) one-fourth
(*c*) one half
(*d*) double

157. For which of the following, the excitation control method is satisfactory?
(*a*) Low voltage lines
(*b*) High voltage lines
(*c*) Short lines
(*d*) Long lines

158. In which of the following cases shunt capacitance is negligible?
(*a*) Short transmission lines
(*b*) Medium transmission lines
(*c*) Long transmission lines
(*d*) All transmission lines

159. A lightning arrester is usually located nearer to
(*a*) transformer
(*b*) isolator
(*c*) busbar
(*d*) circuit breaker
(*e*) none of the above

160. The material used for the manufacture of grounding wires is
(*a*) cast iron
(*b*) aluminium

(*c*) stainlesssteel
(*d*) galvanised steel

161. Surge absorbers protect against .... oscillations
(*a*) high voltage high frequency
(*b*) high voltage low frequency
(*c*) low voltage high frequency
(*d*) low voltage low frequency

162. Skin effect is noticeable only at .... frequencies
(*a*) audio
(*b*) low
(*c*) high
(*d*) all

163. Per system stability is least affected by
(*a*) reactance of generator
(*b*) input torque
(*c*) losses
(*d*) reactance of transmission line

164. When the load at the receiving end of a long transmission line is removed, the sending end voltage is less than the receiving end voltage. This effect is known as
(*a*) Ferranti effect
(*b*) Proximity effect
(*c*) Kelvin effect
(*d*) Faraday effect
(*e*) Skin effect

165. In medium transmission lines the shunt capacitance is taken into account in
(*a*) T-method
(*b*) π-method
(*c*) steinmetz method
(*d*) all of the above

166. System grounding is done so that
(*a*) inductive interference between power and communication circuits can be controlled
(*b*) the floating potential on the lower voltage winding for a transformer is brought down to an insignificant value
(*c*) the arcing faults to earth would not set up dangerously high voltage on healthy phases
(*d*) for all above reasons

167. Which of the following can be used for bus-bars?
(*a*) Tubes
(*b*) Rods
(*c*) Bars
(*d*) Any of the above

168. If the height of transmission tower is increased, which of the following parameters is likely to change?
(*a*) Capacitance
(*b*) Inductance
(*c*) Resistance
(*d*) All of the above
(*e*) None of the above

169. A.C.S.R. conductor having 7 steel stands surrounded by 25 aluminium conductors will be specified as
(*a*) 25/7
(*b*) 50/15
(*c*) 7/25
(*d*) 15/50

170. Impedance relay is used on .... transmission lines
(*a*) short
(*b*) medium
(*c*) long
(*d*) all

171. Corona is likely to occur maximum in
(*a*) transmission lines
(*b*) distribution lines
(*c*) domestic wiring
(*d*) all of the above

172. The effect of wind pressure is more predominant on
(*a*) supporting towers
(*b*) neutral wires
(*c*) transmission lines
(*d*) insulators

173. As compared to cables, the disadvantages of transmission lines is
(*a*) inductive interference between power and communication circuits
(*b*) exposure to lightning
(*c*) exposure to atmospheric hazards like smoke, ice, etc.
(*d*) all of the above

174. In overhead transmission lines the effect of capacitance cand be neglected when the length of line is less than
(*a*) 80 km
(*b*) 110km
(*c*) 150 km
(*d*) 210 km

175. The effective resistance of a conductor will be the same as 'ohmic resistance' when
(*a*) power factor is unity
(*b*) current is uniformly distributed in the conductor cross-section
(*c*) voltage is low
(*d*) current is in true sine wave from

176. Conductors for high voltage transmission lines are suspended from towers to
(*a*) increase clearance from ground
(*b*) reduce clearance from ground
(*c*) take care of extension in length during summer
(*d*) reduce wind and snow loads

(*e*) none of the above

177. To increase the capacity of a transmission line for transmitting power which of the following must be decreased?
(*a*) Capacitance
(*b*) Line inductance
(*c*) Voltage
(*d*) All of the above

178. By using bundled conductors which of the following is reduced?
(*a*) Power loss due to corona
(*b*) Capacitance of the circuit
(*c*) Inductance of the circuit
(*d*) None of the above
(*e*) All of the above

179. Which of the following short-circuits is most dangerous?
(*a*) Dead short-circuit
(*b*) Line to ground short-circuit
(*c*) Line to line short-circuit
(*d*) Line to line and ground short-circuit
(*e*) all of the above

180. Due to which of the following reasons aluminium is being favoured as busbar material?
(*a*) Low density
(*b*) Low cost
(c) Ease of fabrication
(*d*) None of the above

181. In case of transmission line conductors with the increase in atmospheric temperature
(*a*) length decreases but stress increases
(*b*) length increases but stress decreases
(*c*) both the length and stress increases
(*d*) both the length and stress decrease

182. Skin effect exists only in
(*a*) a.c. transmission
(*b*) high voltage d.c. overhead transmission
(*c*) low voltage d.c. overhead transmission
(*d*) cables carrying d.c. current

183. Floating neutral, in 3-phase supply, is undesirable because it causes
(*a*) low voltage across the load
(*b*) high voltage across the load
(*c*) unequal line voltages across the load
(*d*) none of the above

184. The surge resistance of cables is
(*a*) 20 ohms
(*b*) 50 ohms
(*c*) 200 ohms
(*d*) 300 ohms

185. The electrostatic stress in underground cables is
(*a*) zero at the conductor as well as on the sheath
(*b*) same at the conductor and sheath
(*c*) minimum at the conductor and minimum at the sheath
(*d*) maximum at the conductor and minimum at the sheath

186. The ground ring transmission lines are used to
(*a*) reduce the transmission losses
(*b*) reduce the earth capacitance of the lowest unit
(*c*) increase the earth capacitance of the lowest unit
(*d*) none of the above

187. The string efficiency of an insulator can be increased by
(*a*) correct grading of insulators of various capacitances
(*b*) reducing the number of strings
(*c*) increasing the number of strings in the insulator
(*d*) none of the above

188. High voltages for transmitting power is economically available from
(*a*) d.c. currents
(*b*) a.c. currents
(*c*) carrier currents
(*d*) none of the above

189. High voltage is primarily used, for long distance power transmission, to
(*a*) reduce the time of transmission
(*b*) reduce the transmission losses
(*c*) make the system reliable
(*d*) none of the above

190. By using bundle conductors, the critical voltage for the formation of corona will
(*a*) remain same
(*b*) decrease
(*c*) increase
(*d*) not occur

191. If the voltage is increased x times, the size of the conductor would be
(*a*) reduced to $1/x^2$ times
(*b*) reduced to 1/x times
(*c*) increased x times
(*d*) increased to x^2 times
(*e*) none of the above

192. The colour of the neutral of three-core flexible cable is
(*a*) blue
(*b*) brown
(*c*) red
(*d*) black

193. In the cablessheaths are used to
(*a*) prevent the moisture from entering the cable

(*b*) provide strength to the cable
(*c*) provide proper insulation
(*d*) none of the above

194. The charging current in the cables
(*a*) leads the voltage by 180^o
(*b*) leads the voltageby 90^o
(*c*) lags the voltage by 90^o
(*d*) lags the voltage by 180^o

195. Ground wire is used to
(*a*) avoid overloading
(*b*) give the support to the tower
(*c*) give good regulation
(*d*) connect a circuit conductor or other device to an earth-plate

196. Earthing is necessary to give protection against
(*a*) danger of electric shock
(*b*) voltage fluctuation
(*c*) overloading
(*d*) high temperature of the conductors

197. Resistance grounding isused for voltage between
(*a*) 3.3kV to 11 kV
(*b*) 11 kV to 33 kV
(*c*) 33 kV to 66 kV
(*d*) none of the above

198. Solid grounding is adopted for voltages below
(*a*) 100 V
(*b*) 200 V
(*c*) 400 V
(*d*) 660 V

199. The size of the earth wire is determined by
(*a*) the atmospheric conditions
(*b*) the voltage of the service wires
(*c*) the ampere capacity of the service wires
(*d*) none of the above

200. Transmission lines link
(*a*) generating station to receiving and station
(*b*) receiving and station to distribution transformer
(*c*) distribution transformer to consumer premises
(*d*) service points to consumer premises
(*e*) none of the above

ANSWERS

1. (*b*)	**2.** (*d*)	**3.** (*a*)	**4.** (*c*)	**5.** (*b*)	**6.** (*d*)	**7.** (*a*)	**8.** (*d*)	**9.** (*d*)	**10.** (*d*)
11. (*c*)	**12.** (*b*)	**13.** (*d*)	**14.** (*d*)	**15.** (*a*)	**16.** (*c*)	**17.** (*d*)	**18.** (*d*)	**19.** (*c*)	**20.** (*d*)
21. (*d*)	**22.** (*b*)	**23.** (*b*)	**24.** (*a*)	**25.** (*a*)	**26.** (*c*)	**27.** (*d*)	**28.** (*d*)	**29.** (*d*)	**30.** (*d*)
31. (*d*)	**32.** (*e*)	**33.** (*a*)	**34.** (*a*)	**35.** (*e*)	**36.** (*d*)	**37.** (*d*)	**38.** (*c*)	**39.** (*b*)	**40.** (*c*)
41. (*b*)	**42.** (*a*)	**43.** (*d*)	**44.** (*b*)	**45.** (*b*)	**46.** (*b*)	**47.** (*b*)	**48.** (*c*)	**49.** (*d*)	**50.** (*b*)
51. (*b*)	**52.** (*a*)	**53.** (*c*)	**54.** (*c*)	**55.** (*c*)	**56.** (*d*)	**57.** (*c*)	**58.** (*b*)	**59.** (*a*)	**60.** (*a*)
61. (*b*)	**62.** (*c*)	**63.** (*b*)	**64.** (*a*)	**65.** (*c*)	**66.** (*d*)	**67.** (*d*)	**68.** (*d*)	**69.** (*d*)	**70.** (*b*)
71. (*c*)	**72.** (*c*)	**73.** (*d*)	**74.** (*a*)	**75.** (*b*)	**76.** (*c*)	**77.** (*d*)	**78.** (*d*)	**79.** (*b*)	**80.** (*b*)
81. (*d*)	**82.** (*d*)	**83.** (*d*)	**84.** (*b*)	**85.** (*b*)	**86.** (*a*)	**87.** (*d*)	**88.** (*c*)	**89.** (*c*)	**90.** (*a*)
91. (*d*)	**92.** (*b*)	**93.** (*a*)	**94.** (*b*)	**95.** (*d*)	**96.** (*b*)	**97.** (*b*)	**98.** (*d*)	**99.** (*d*)	**100.** (*a*)
101. (*a*)	**102.** (*c*)	**103.** (*c*)	**104.** (*c*)	**105.** (*b*)	**106.** (*b*)	**107.** (*b*)	**108.** (*c*)	**109.** (*c*)	**110.** (*b*)
111. (*a*)	**112.** (*a*)	**113.** (*a*)	**114.** (*b*)	**115.** (*b*)	**116.** (*a*)	**117.** (*a*)	**118.** (*b*)	**119.** (*d*)	**120.** (*d*)
121. (*d*)	**122.** (*d*)	**123.** (*a*)	**124.** (*c*)	**125.** (*a*)	**126.** (*a*)	**127.** (*a*)	**128.** (*c*)	**129.** (*a*)	**130.** (*d*)
131. (*b*)	**132.** (*b*)	**133.** (*a*)	**134.** (*b*)	**135.** (*b*)	**136.** (*d*)	**137.** (*c*)	**138.** (*a*)	**139.** (*b*)	**140.** (*c*)
141. (*d*)	**142.** (*b*)	**143.** (*d*)	**144.** (*c*)	**145.** (*c*)	**146.** (*a*)	**147.** (*d*)	**148.** (*d*)	**149.** (*b*)	**150.** (*d*)
151. (*a*)	**152.** (*c*)	**153.** (*d*)	**154.** (*b*)	**155.** (*a*)	**156.** (*c*)	**157.** (*c*)	**158.** (*a*)	**159.** (*a*)	**160.** (*d*)
161. (*c*)	**162.** (*c*)	**163.** (*c*)	**164.** (*a*)	**165.** (*d*)	**166.** (*d*)	**167.** (*d*)	**168.** (*a*)	**169.** (*a*)	**170.** (*b*)
171. (*a*)	**172.** (*a*)	**173.** (*d*)	**174.** (*a*)	**175.** (*b*)	**176.** (*a*)	**177.** (*b*)	**178.** (*a*)	**179.** (*a*)	**180.** (*b*)
181. (*b*)	**182.** (*a*)	**183.** (*c*)	**184.** (*b*)	**185.** (*d*)	**186.** (*b*)	**187.** (*a*)	**188.** (*b*)	**189.** (*b*)	**190.** (*c*)
191. (*a*)	**192.** (*a*)	**193.** (*a*)	**194.** (*b*)	**195.** (*d*)	**196.** (*a*)	**197.** (*a*)	**198.** (*d*)	**199.** (*c*)	**200.** (*a*)

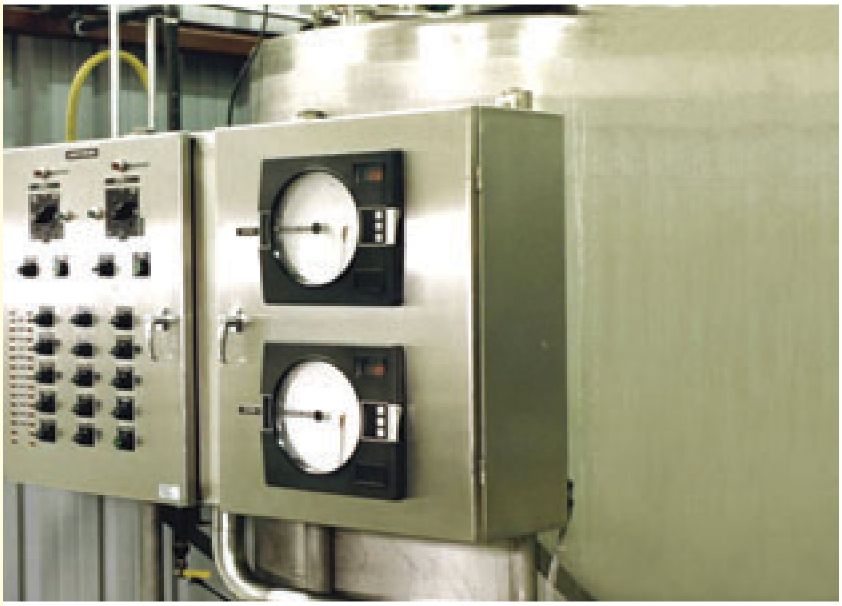
CONTENTS

CHAPTER 42

DISTRIBUTION AUTOMATION

Learning Objectives

- Introduction
- Need Based Energy Management (NBEM)
- Advatanges of NBEM
- Automated System
- Sectionalizing Switches
- Remote Terminal Units (RTU's)
- Data Acquisition System (DAS)
- Communication Interface
- Radio Communication
- Machine Interface
- A Typical SCADA System
- Load Management in DMS Automated Distribution System
- Substation Automation
- Control System
- Protective System
- Feeder Automation
- Distribution Equipment
- Automation Equipment
- Consumer Side Automation
- Energy Auditing
- Reduced Line Loss
- Power Quality
- Deferred Capital Expenses
- Energy Cost Reduction
- Optimal Energy Use



An automated electric distribution system ensures that the power is transmitted efficiently and without interruptions. Faults in transmission are identified and rectified with minimum human intervention

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42.1. Introduction

The power industry may seem to lack competition. This thought arises because each power company operates in a geographic region not served by other companies. Favorable electric rates are a compelling factor in the location of an industry, although this factor is much less important in times when costs are rising rapidly and rates charged for power are uncertain than in periods of stable economic conditions. Regulation of rates by State Electricity Boards, however, places constant pressure on companies to achieve maximum economy and earn a reasonable profit in the face of advancing costs of production.

Power shortage in India is endemic. Despite heavy investments in the power sector and additional power generation every year, there appears to be no chance of the shortage syndrome easing-up in the near future. Barring one or two small States with hydro-potential, all other States and Union Territories have perpetual power deficits. There seems to be no respite from these crippling deficits as seen from the figures that are being published from time to time.

In power sector, it has almost become a ritual to insistently clamor for more generation. This school of thought which is predominant majority, believes in classical concept of load management which emphasizes continuous and copious supply of energy to all sectors of consumers at all times. This ideal proposition could be achieved, only if unlimited resources at our command are available. But in actual practice, these resources are not only limited but scarce. These scarce resources are to be distributed throughout the country uniformly and called for immediate attention. A demand based energy management would therefore only result in shortage syndrome repeating itself endlessly since the ever-growing 'demand' could never be met fully and satisfactorily. A better alternative would be to replace the Demand Based Energy Management with a Need Based one.

42.2. Need Based Energy Management (NBEM)

In power sector, there is a distinct difference between 'demand' and 'need'. Consumers of electric power could be classified into five broad categories: the industrial users, agricultural sector, commercial organizations, domestic consumers and essential services. Industrial users could be further sub-divided as shift based industries and continuous process industries. Agricultural sector would include irrigation tubewells and rural industries. Out of these several groups and sub-groups, only three - viz - continuous process industries, domestic consumers and essential services need power round the clock, others may demand power for 24 hours of the day, but they don't need it.

A Need Based Energy Management would :

(*i*) Identify the needs of various consumers
(*ii*) Forecast the generation requirement based on the need
(*iii*) Plan power generation as per forecast
(*iv*) Laydown a suitable transmission and distribution network
(*v*) Regulate distribution as per need
(*vi*) Monitor matching of need with supply

The greatest bugbear of NBEM is distribution network. Consumers cannot be supplied power as per their need since the distribution network is not designed for that purpose. Present day power distribution network is full of constraints and is clumsy to the core. The ills of the system are many – poor reliability, high line losses, low voltage profiles, overloading of transformers, poor maintenance, absence of conservation measures, stealing of power, haphazard layouts, whimsical load connections, inadequate clearances etc. With a single feeder connected to all types of consumers, there is no load discipline and the distribution network is exposed to several malpractices and distortions. Generation of power also suffers and as a result, engineers incharge of power generation in state electricity boards are not enthusiastic about optimizing power generation because they feel that their efforts are wasted due to the chaotic and sub-optimal distribution system. So if power problem is to be solved, distribution holds the key.

Figure 42.1 shows utility system with SCADA (Supervisory Control and Data Acquisition) and Distribution Automation.

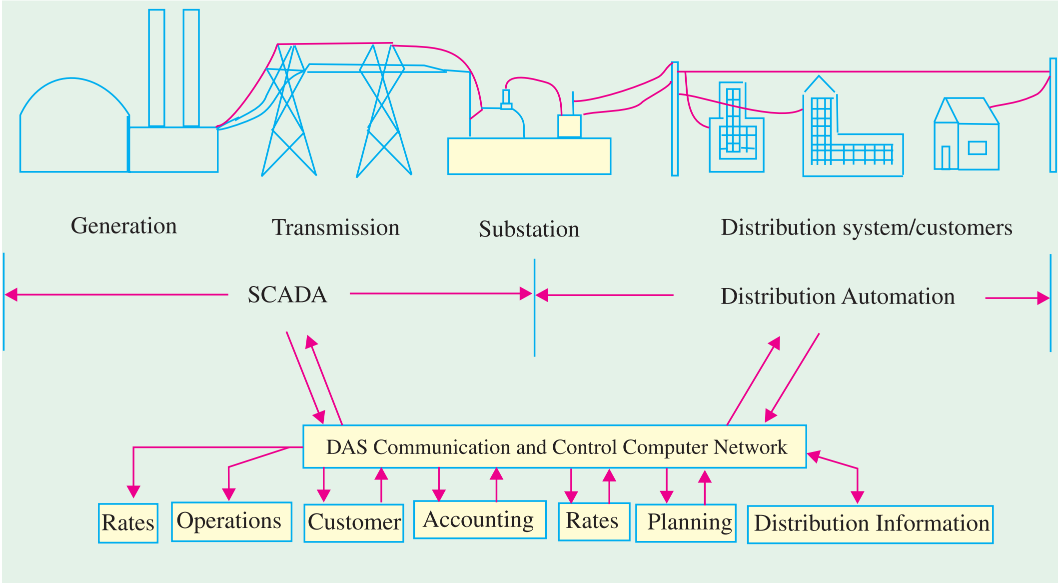


Fig. 42.1 Utility systems with SCADA and Distribution Automation

42.2.1. Advantages of NBEM

The distinct advantages of NBEM are:

(1) It ensures high reliability of supply to consumers meeting the specific demand effectively for periods of actual requirements.

(2) The system losses can be substantially reduced since line and equipment does not get overloaded at any point of time.

(3) The voltage profile at all levels is improved thus safeguarding the customer's equipment from losing their efficiency and performance at low voltage.

(4) The scheme facilitates the adoption of energy conservation schemes and energy audit policy.

(5) Power cuts are reduced and quality of power improves leading to better industrial and agricultural health and productivity.

42.3. Conventional Distribution Network

The power system network, which generally concerns (or which is in close proximity of) the common man, is the distribution network of 11 KV lines or feeders downstream of the 33 KV substation. Each 11 KV feeder, which emanates from the 33 KV substation branches further into several subsidiary 11 KV feeders to carry power close to the load points, where it is further step-down to either 230 V or 415 V.

The present structure of the distribution feeders doesn't support quick fault detection, isolation of faulty region and restoration of supply to the maximum outage area, which is healthy. In the absence of switches at different points in the distribution network, it is not possible to isolate certain loads for load shedding as and when required. The only option available in the present distribution network is the circuit breaker (one each for every main 11 KV feeder) at the 33 KV substation. However, these circuit breakers are actually provided as a means of protection to completely isolate the downstream network in the event of a fault. Using this as a tool for load management is not desirable, as it disconnects the power supply to a very large segment of consumers. Clearly, there is a need to put in place a system that can achieve a finer resolution in load management.

In the event of a fault on any feeder section downstream, the circuit breaker at the 33 KV substation trips (opens). As a result, there is a blackout over a large section of the distribution network. If the faulty segment could be precisely identified, it would be possible to substantially reduce the blackout area, by re-routing the power to the healthy feeder segments through the operation of sectionalizing switches, placed at strategic locations in various feeder segments.

Thus, lack of information at the base station (33 KV sub-station) of the loading and health status of the 11 KV/ 415 V distribution transformers and associated feeders is one primary cause of inefficient power distribution. Also, due to absence of monitoring, overloading occurs, which results in low voltage at the customer end and increases the risk of frequent breakdowns of transformers and feeders.

42.4. Automated System

The inefficient operation of the conventional distribution system can be mainly attributed to the frequent occurrence of faults and the uncertainty in detecting them. To enhance the electrical power distribution reliability, sectionalizing switches are provided along the way of primary feeders. Thus, by adding fault detecting relays to the sectionalizing switches along with circuit breaker and protective relays at the distribution substations, the system is capable to determine fault sections. To reduce the service disruption area in case of power failure, normally open (NO) sectionalizing switches called as route (tie) switches are used for supply restoration process. The operation of these switches is controlled from the control center through the Remote Terminal Units (RTU'S).

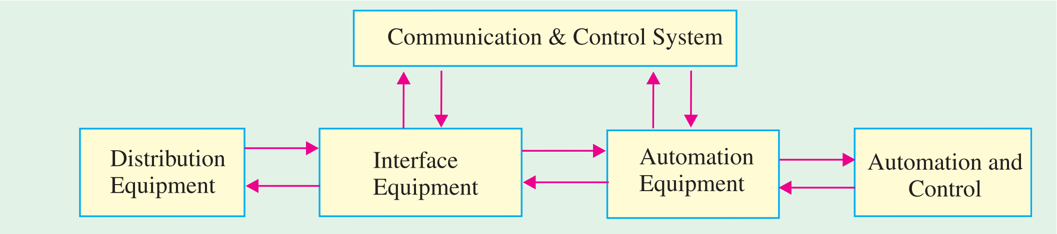


Fig. 42.2. Interconnection of distribution, control and communication system.

In distribution automation (DA) system, the various quantities (*e.g.*, voltage, current, switch status, temperature and oil level, etc.) are recorded in the field at the distribution transformers and feeders, using a data acquisition device called Remote Terminal Unit. These quantities are transmitted on-line to the base station through a communication media. The acquired data is processed at the base station for display at multiple computers through a Graphic user interface (GUI). In the event of a system quantity crossing a pre-defined threshold, an alarm is generated for operator intervention. Any control action, for opening or closing of the switch or circuit breaker, is initiated by the operator and transmitted from the 33 KV base station through the communication channel to the remote terminal unit associated with the corresponding switch or CB. The desired switching takes place and the action is acknowledged back to the operator.

Interconnection of distribution, control and communication system is shown in Figure 42.2.

All the above mentioned functions of data collection, data transmission, data monitoring, data processing, man-machine interface, etc. are realized using an integrated distribution SCADA (Supervisory Control And Data Acquisition) system.

The implementation of SCADA system in the electric utility involves the installation of following units :

1. Sectionalizing Switches
2. Remote Terminal Unit
3. Data Acquisition System
4. Communication Interface
5. Control Computer

42.5. Sectionalizing Switches

These sectionalizing switches are basically either air-brake switches or Load Break Switches (LBS) or Moulded Case Circuit Breaker (MCCB). These are remotely operable switches designed specifically for 11 KV and 415 V feeders. However, switches of appropriate rating corresponding to the rated feeder current can also be chosen.

Generally, 11 KV Vacuum break line sectionalizers are installed away from the substation and on the pole. A fault-indicating device is provided for location of fault at any section. To avoid opening of switches due to transient voltage drop or any other mal-operation, generally, 1.5 - 3 seconds delay is provided. Benefits obtained from installing these sectionalizing switches include :

1. Immediate isolation with indication of faulty section
2. Immediate restoration of supply to the healthy section
3. No down time for transient faults
4. Improvement of revenue due to lesser outage of section and low down time
5. Requires less man-power in the system and
6. Better Reliability of power supply

42.6. Remote Terminal Units (RTU's)

A typical SCADA system consists of remote terminal units, to record ad check, measured values and meter readings, before transmitting them to control station and in the opposite direction, to transmit commands, set point values and other signals to the switchgear and actuator.

The functions of RTU's can be given as following :

(*a*) Acquisition of information such as measured values, signals, meter readings, etc.

(*b*) Transmit commands or instructions (binary plus type or continuous), set points, control variables, etc. including their monitoring as a function of time.

(*c*) Recognition of changes in signal input states plus time data allocation and sequential recording of events by the master control station.

(*d*) Processing of information transmitted to and from the telecommunication equipment such as data compression, coding and protection.

(*e*) Communication with master control station.

A typical architecture RTU interfacing is shown in Fig 42.3.

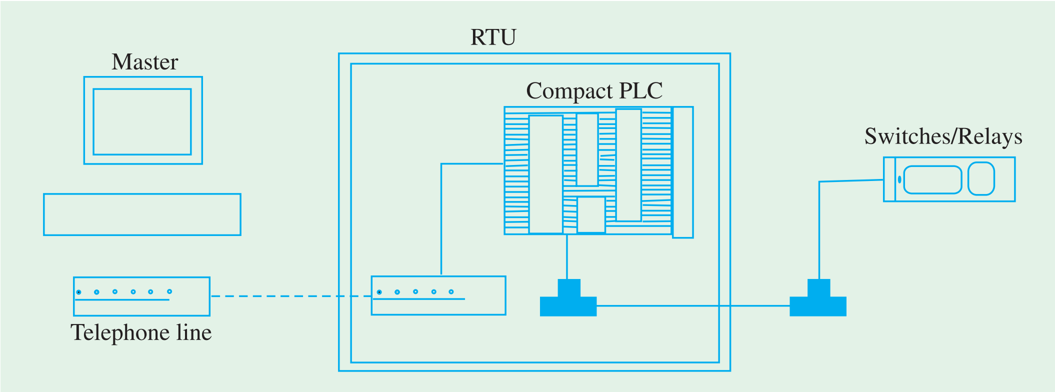


Fig. 42.3. Typical Architecture of RTU Interfacing

42.7. Data Acquisition System (DAS)

The data regarding the complete network consists of electrical and mechanical variables, on / off states, analog quantities, digital quantities, changes of state, sequence of events, time of occurrence and several other data, which the control room operators will like to know.

Data is acquired by means of current transformers (CTs), potential transformers (PTs), transducers and other forms of collecting information. Transducers convert the data into electrical form to enable easy measurement and transmission. Data may be collected at low level or high level. Then it is amplified in signal amplifier and conditioned in data signal conditioner. The data is transmitted from the process location to the control room and from the control room to the control center.

The large number of electrical, mechanical and other data are scanned at required interval, recorded and displayed as per the requirement. Some of the data is converted from analog to digital form through A/D (Analogue to Digital) converters.

The data loggers perform the following functions :

1. Input Scanning
2. Signal Amplification and A/D Conversion
3. Display, Recording and Processing

The input scanner is generally a multi-way device, which selects input signals at regular periodic intervals in a sequence decided by the rate of change of input data. Slow varying quantities are scanned with a lower period of time-intervals. Output of scanner is given to A/D converter. Digital signals are obtained through DSP by micro-controllers or the control computer. This acquired signal can be displayed, recorded and processed for appropriate actions to be performed later.

42.8. Communication Interface

A good data communication system to transmit the control commands and data between Distribution Control Centre (DCC) and a large number of device remotely located on the distribution network is a pre-requisite for the good performance of Distribution Automation System (DAS). The communication requirements of each DAS is unique, depending upon the Distribution Automation functions selected for the implementation. A wide range of communication technologies are available to perform the tasks of DAS. The choice of communication technology also has a big impact on the cost of DAS.

RTU's communicate with the control room through a communication interface, which could be any of the following :

(*a*) Power line carrier communication (PLCC) :

Each end of the transmission line is provided with identical carrier equipment in the frequency range of 30 to 500 kHz. The high frequency signals are transmitted through power lines. The carrier current equipment compromises the coupling capacitor and the tuning circuit.

(*b*) Fibre optics data communication :

Application of fibre optic communication is presently in infant stage and has a vast scope due to freedom from electromagnetic interference and enormous data handling capacity of a single pair of optical fibre. The information is exchanged in the form of digitised light signals transmitted through optical fibres.

Fibre optics, with its explicit downward cost trend in terms of product as well as installation and maintenance costs has become a widely accepted choice, as it offers both technical and commercial advantages over conventional systems that use metallic cables and radio links. The communication basically consists of a transmitter and a receiver for information signals coming from the user's device which is connected through copper wire to the switching center or exchange, where it is changed into a digital signal like 1s or 0 s for easy handling. The signal is then transferred to the transmitter. In the transmitter, the information signal which is electrical, drives an optical source : Laser or a light emitting diode (LED), which in turn, optically modulates the information signal, which gets coupled into the optical fibre. The receiver located at the other side of the link detects the original signal and demodulates or converts back the optical signal to the original information signal (Electrical). The signal is then connected through copper wires to the switching device or exchange for selection and connection to the proper user or user device. This has advantage of high data rate (9600 bits and much more) and immunity from noise.

(*c*) Radio communication :

Radio communication utilizes frequency bands between 85 MHz to 13 GHz, Point to point radio links, Multi terminal radio communication facility, Limited area radio scheme, Mobile radio sets, Emergency radio communication etc. are the types of radio communication facilities used in electrical power systems.

Nowadays even telephone connections are via wireless systems. The figure shows the transmission tower from where radio waves can be transmitted to other stations and satellites

(*d*) Public telephone communication :

Dial-up and dedicated leased telephone lines are often used for Distribution Automation. The dial-up lines are suitable for infrequent data trans-mission. The leased line are suitable for continuous communication but are expensive. The reliability of communication varies greatly and is dependent upon the telephone company. Telephone communication service through packet switching network, cellular radio are viable and may have the advantages of providing services in other-wise inaccessible places.

(*e*) Satellite communication :

A satellite communi-cation system using very small aperture terminal (VSAT) is suitable for DAS. VSAT is a point to multi-point star network like TDMA. It consists of one single Hub and number of remote Personal Earth Stations (PES). The communication between transponder and Hub is TDM access and between Hub and PES is TDMA access. The communi-cation system between Hub remote PES is through two separate radio links. The link from remote PES to Hub is called inroute and from Hub to the remote PES is called as outroute. No end user transmission either originates or terminates at the satellite. In India extended 'C' band with up link in the 6.15/ 6.815 and down link in the 4.09/ 4.59 KHz range is used. Fig. 42.4 shows schematic diagram of VSAT network.

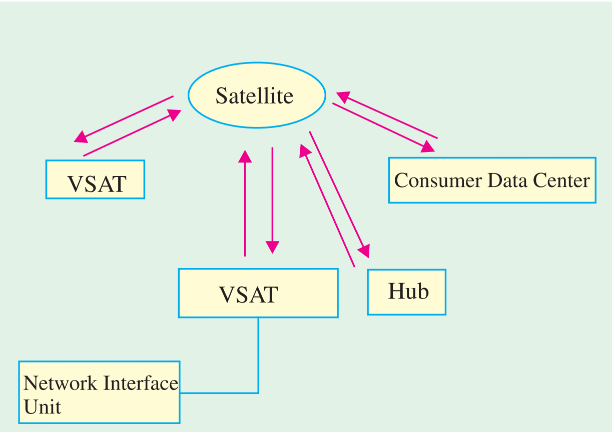
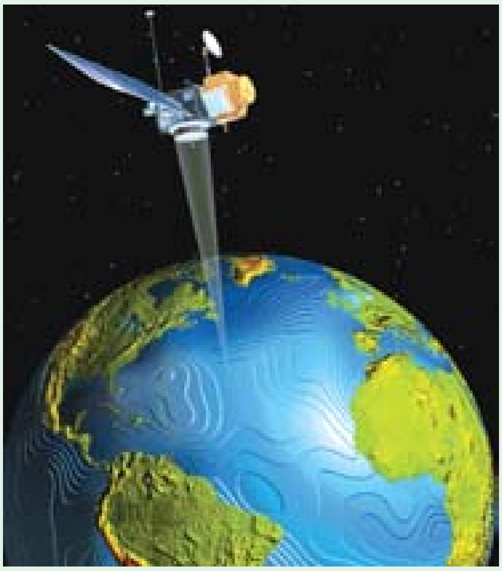


Fig. 42.4 Schematic diagram of VSAT network



A satellite helps to develop instant communication links with the stations spread across the globe

(*f*) Polling scheme :

SCADA systems intended for electric system operations almost universally use a polling scheme between the central master and individual RTUs.

The master station controls all activities and RTUs respond only to polling requests. Fig. 42.5 illustrates the most common communication arrangement. Multiple two or four wire telephone grade circuits radiate from the master. The media for these circuits may be leased telephone circuits from a common carrier, private microwave, fibre optic cable systems, two-way cable TV, power line carrier, or even satellite. Polling and command requests and RTU responses are time multiplexed on each circuit. Each circuit terminating at the master station is independently serviced on an asynchronous basis by the master station. The most commonly used information rates is 1200 bits/sec. using asynchronous byte-oriented message formats.

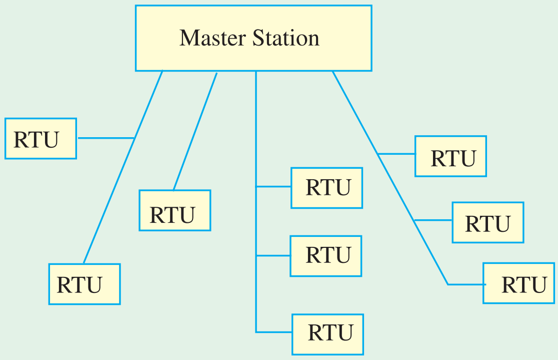


Fig. 42.5. Typical multidrop communication system

42.9. Distribution SCADA

Distribution SCADA involves collecting and analyzing information to take decisions, implementing the appropriate decisions and then verifying whether the desired results are achieved. Fig 42.6 gives flow diagram of the SCADA functions.

Data acquisition in an electric utility SCADA system concentrates on the power system performance quantities like bus line volts, transformer currents, real and reactive power flow, C.B. status (circuit breaker status), isolator status and secondary quantities such as transformer temperature, insulating gas pressure, tank oil levels, flow levels etc. Often transformer tap positions, usual positions or other multiple position quantities are also transmitted in analog format.

The usual reason for installing supervisory control is to provide the system with sufficient information and control to operate the power system or some part of it in a safe, secure and economic manner.

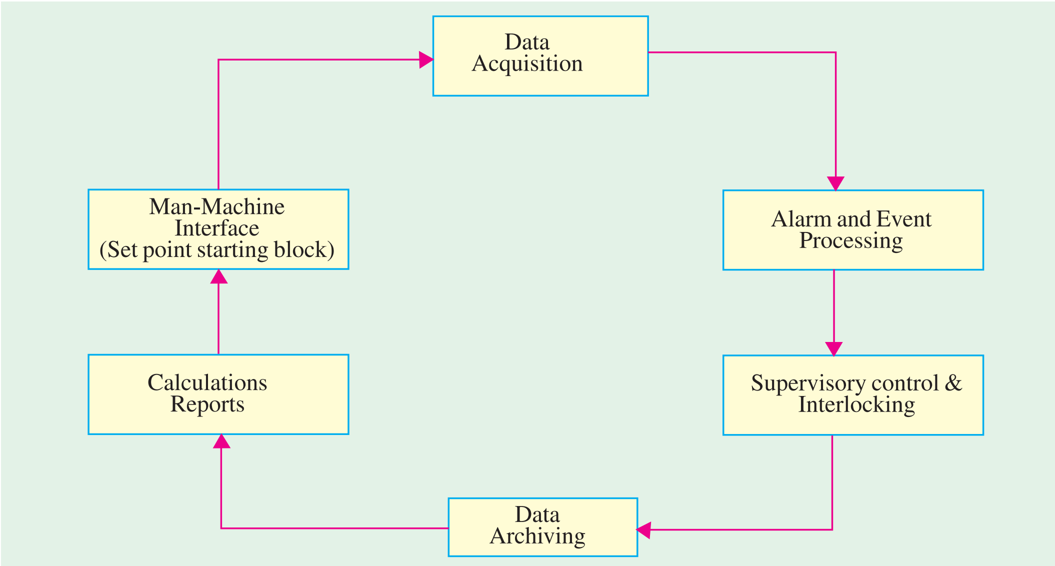


Fig. 42.6. SCADA functions

42.10. Man – Machine Interface

The implementation of SCADA system in an electric utility requires the installation of remote terminal units. RTU's are designed to acquire data and transfer the same to the master station through a communication link. They collect data from transducers, transmitters, connect input from equipment/ instruments, meter readings etc., performs analogue/ digital conversions, check data scaling and corrections (typically at I/O levels) performs preprocessing tasks and send/ receive messages from/ to master stations via interfaces. Fig 42.7 shows the flow diagram for man-machine interface.

Man-Machine Interface (MMI) is the interface between man and technology for control of the technical process. The computer system at master control centre or central control room integrates with RTU over the communication link with its transmission protocol, acquires the remote substation or distribution transformer/ feeder data and transfers the same to the computer system for man-machine interface.

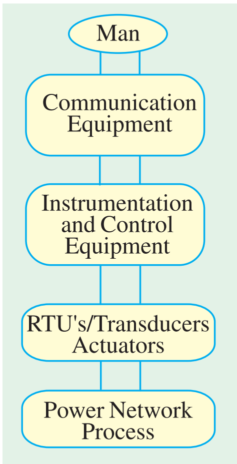


Fig. 42.7. Man - Machine Interface

42.11. A Typical SCADA System

A typical SCADA system may therefore comprises hardware and the software.

The hardware may include :

1. User friendly man-machine interface
2. Work station
3. Service having a particular function
4. Communication sub system
5. RTU's

All the above components communicate with each other through a local area network (LAN) with internationally standardised protocols. A flexible redundancy is provided, assigning hot standby server to any server fulfilling time critical functions.

A software must offer the following features :

1. Use of high level programming language
2. Modular structure with clear and standardised interfaces between software modules and the database
3. Inter module communication only via a 'soft bus' independent of their computer residency
4. Easy addition of further application programs
5. Comfortable on-line diagnostics, development tools and file editors.

Artificial Intelligence (AI) plays a pivotal role in the development of software, rule based experts systems, logic based systems like fuzzy logic, neural networks etc. fall under this category. Few applications of the recent times are stated on the next page :

1. Application of expert system to power system restoration using flow control rules, energisation rules, line switching rules, load shedding rules, voltage correction rules etc.
2. Knowledge-based expert systems for fault location and diagnosis on electric power distribution feeders.
3. Fuzzy based logic for reconfiguration of distribution networks.

The application of model based expert system results in an intelligent power distribution system with learning self organizing and diagnostic capabilities. In addition, it advises the control center operations in cases of emergency. Distribution automated systems can therefore be operated in two operational modes: Online and Off-line. The Online operation permits faulty equipment identification, restoration planning, network maintenance scheduling and emergency operations. The Off-line simulator mode allows the user to verify the validity of acquired knowledge by setting an imaginary fault on the system and provides a convenient way of training the inexperienced operator.

42.12. Distribution Automation

Distribution Automation functions provide a means to more effectively manage minute by minute continuous operation of a distribution system. Distribution Automation provides a tool to achieve a maximum utilization of the utility's physical plant and to provide the highest quality of service to its customers. Obviously, both the utility and its customers are beneficiaries of successful Distribution Automation.

Distribution Automation systems are modular, hence they may be implemented in stages, commencing from a modest degree of capability and complexity and growing as necessary to achieve tangible and intangible economic benefits. For example, a utility may start with a limited capability SCADA System for sub station monitoring and control, extend this to the feeders and finally implement a complete integration of automation functions. Systems implemented in this fashion must be designed to accommodate future expansion.

Distribution Automation System offers an integrated 'Distributed Management System' (DMS). The functions of DMS are shown in Fig 42.8. As in any other SCADA system, Distribution SCADA involves

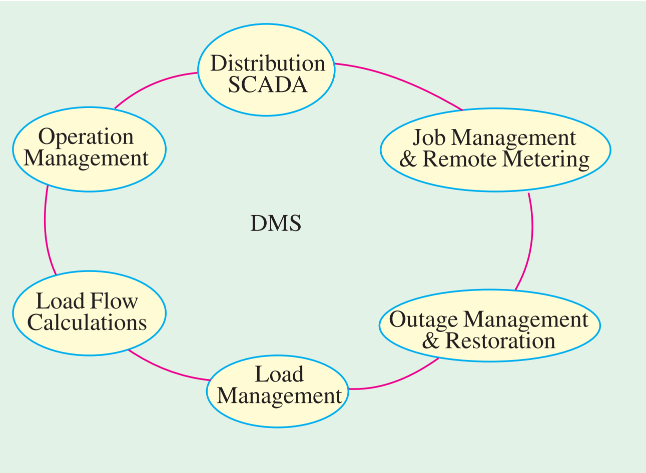


Fig. 42.8. Functions of distributed management system

collecting and analyzing information to take decisions, implementing the appropriate decisions and then verifying that the desired results are achieved. Operation management supports the analysis of distribution network. It models the load profile to present state of the network. Load flow calculations estimate voltage levels and power flows at each feeder. Job management makes switching order handling easier and work protection tagging ensures the safety of repair crew on duty. Outage management and service restoration facilitates to reduce outage time thereby increasing the reliability of the supply. Remote metering provides for the appropriate selection of energy registers where time-of-use rates are in effect, thus improving energy metering services to be more accurate and more frequent.

42.13. Load Management in DMS

This involves controlling system loads by remote control of individual customer loads. Control includes suppressing or biasing automatic control of cyclic loads, as well as load switching. Load Management can also be effected by inducing customers to suppress loads during utility selected daily periods by means of time-of-day rate incentives. Distribution Automation provides the control and monitoring ability required for both the load management scenarios -viz - direct control of customers loads and the monitoring necessary to verify that programmed levels are achieved. Execution of load management provides several possible benefits to the utility and its customers. Maximum utilization of the existing distribution system can lead to deferrals of capital expenditure. This is achieved by :

(*i*) Shaping the daily (or monthly, annual) load characteristic by suppressing loads at peak times and encouraging energy consumption at off-peak times.

(*ii*) Minimizing the requirement for more costly generation or power purchases by suppressing loads.

(*iii*) Relieving the consequences of significant loss of generation or similar emergency situations by suppressing load.

(*iv*) Reducing cold load pick-up during re-energization of circuits using devices with cold load pick-up features.

The effectiveness of direct control of customer loads is obviously enhanced by selecting the larger and more significant customer loads. These include electric space and water heating, air conditioning, washing machines, dryers and others of comparable magnitude.

More sophisticated customer's activated load management strategies are under study, taking advantage of the capabilities of Distribution Automation and of customers installed load control PC. With such an arrangement, the utility could vary rates throughout the day, reflecting actual generation costs and any system supply capability constraints and broadcast this information to all customer's PC. Each PC could then control it's loads to confirm to a customer selected cost bias. This is sometimes called as 'spot pricing'.

42.14. Automated Distribution System

In the conventional distribution system the abnormal conditions are detected manually which costs lots of time and money to both consumers and power industry. In order to maintain high service quality and reliability and minimize loss in revenues, automation is required. Automation may be applied to the power distribution system so that problems on the distribution network may be detected and operated upon so as to minimize the outage time.

The equipments (either fixed wired or /and programmable), which are used for distribution automation, include :

1. Data collection equipment
2. Data transmission (telemetering) equipment

3. Data monitoring equipment
4. Data processing equipment
5. Man-Machine interface.

All the above equipments are integrated through distribution SCADA system.

Distribution SCADA involves collecting and analyzing information to take decisions and then implementing them and finally verifying whether desired results are achieved .

The implementation of SCADA system in the electric utility involves the installation of following units :

(*a*) Data acquisition unit : The basic variables (data) required for control, monitoring and protection include current, voltage, frequency, time, power factor, reactive power and real power. The data may be tapped in analog or digital form as required. For data collection purpose CT's and PT's are used. Transducers may be needed to convert the data into electrical form to enable easy measurement and transmission .The data is amplified in signal amplifier and conditioned in data signal conditioner. The data is then transmitted from the process location to the transmission room. In the control room data processing and data logging are performed which includes input scanning at required intervals, recording, programming and display by microprocessor, PLC, PC etc.

(*b*) Remote terminal unit (RTU) : RTU is used to record and check signals, measured values and meter readings before transmitting them to control station and in the opposite direction to transmit commands, set point values and other signals to the switchgear and actuator.

(*c*) Communication unit : A good data communication system to transmit the control command and data between distribution control centre and large number of remotely located devices is a prerequisite for a good performance of DA system. Wide range of communication technologies are available to perform the task of DA system which include public telephone communication (leased or dedicated line), power line carrier communication (PLCC), UHF MARS (ultra high frequency multi address radio system) and VHF Radio, as discussed earlier.

Distribution Automation can be broadly classified as :

1. Substation Automation.
2. Feeder Automation.
3. Consumer side Automation

42.15. Substation Automation

Substation automation is the cutting edge technology in electrical engineering. It means having an intelligent, interactive power distribution network including :

1. Increased performance and reliability of electrical protection.
2. Advanced disturbance and event recording capabilities, aiding in detailed electrical fault analysis.
3. Display of real time substation information in a control center.
4. Remote switching and advanced supervisory control.
5. Increased integrity and safety of the electrical power network including advanced interlocking functions.
6. Advanced automation functions like intelligent load-shedding.

42.15.1 Requirements

The general requirements for selecting an automation system while designing a new substation are :

1. The system should be adaptable to any vendor's hardware.
2. It should incorporate distributed architecture to minimize wiring.
3. It should be flexible and easily set up by the user.

4. The substation unit should include a computer to store data and pre-process information.

42.15.2. Functioning

Bus voltages and frequencies, line loading, transformer loading, power factor, real and reactive power flow, temperature, etc. are the basic variables related with substation control and instrumentation. The various supervision, control and protection functions are performed in the substation control room. The relays, protection and control panels are installed in the controlled room. These panels along with PC aids in automatic operation of various circuit breakers, tap changers, autoreclosers, sectionalizing switches and other devices during faults and abnormal conditions. Thus, primary control in substation is of two categories.

1. Normal routine operation by operator's command with the aid of analog and digital control system.
2. Automatic operation by action of protective relays , control systems and PC.

The automated substation functioning can be treated as integration of two subsystems, as discussed below :

(*a*) Control System : The task of control system in a substation includes data collection, scanning, event reporting and recording; voltage control, power control, frequency control, other automatic and semiautomatic controls etc. The various switching actions like auto reclosing of line circuit breakers, operation of sectionalizing switches, on-load tap changers are performed by remote command from control room. The other sequential operations like load transfer from one bus to another, load shedding etc. are also taken care by control center.

(*b*) Protective System : The task of protective system includes sensing abnormal condition, annunciation of abnormal condition, alarm, automatic tripping, back-up protection, protective signaling.

The above two systems work in close co-operation with each other. Most of the above functions *i.e*. automatic switching sequences, sequential event recording, compiling of energy and other reports, etc. are integrated in software in the substation computer. This software is of modular design, which facilitates addition of new functions.

The communication between circuit breakers, autoreclosers and sectionalizing switches in the primary and secondary distribution circuits located in the field and the PC in distribution substation control room is through radio telecontrol or fibre optic channel or power line carrier channel as is feasible.

42.16. Feeder Automation

Automating the fault diagnosis and supply restoration process significantly reduces the duration of service interruptions. The key objective behind automating the service restoration process is to restore supply to maximum loads in out-of-service zones. This is achieved by reconfiguring the network such that the constraints of the system are not violated. Providing timely restoration of supply to outage areas of the feeder enhances the value of service to customers and retains the revenue for the power industry.

The system data consisting of the status signals and electrical analog quantities are obtained using a suitable Data Acquisition System and processed by the control computer for typical functions of fault detection, isolation and network reconfiguration for supply restoration. The equipments normally required in Feeder Automation are discussed below :

(*a*) Distribution equipment :

This includes transformers, breakers, load break switches and motor operators, power reclosures, voltage regulators, capacitor banks etc.

(*b*) Interface equipment :

Interface equipment is required for the purpose of data acquisition and control. Potential transformers, current transformer, watt, var meter and voltage transducers, relays are some examples.

(*c*) Automation equipment :

Automation Equipment includes a DAS, communication equipment, substation remote terminal units (RTU) and distribution feeder RTU, current-to-voltage converters, etc.

42.17. Consumer Side Automation

Consumer side automation is very important for a distribution company as almost 80% of all the losses are taking place on distribution side alone. It is needed to evaluate the performance of a specific area in the distribution system and judge the overall losses.

42.17.1. Energy Auditing

Energy audit has a very wide range of applications in the electrical systems. It means overall accounting of energy generated, transmitted and distributed. As far as distribution side is concerned energy audit would mean overall accounting of energy supplied to and utilized by the consumers. Energy audit can also be used for rethinking about billing strategy, usefulness of an individual subscriber, loading of a given feeder etc.

Remote metering is used in energy auditing in which the energy used by a consumer is billed from a remote (distant) location without actually going to the place. In remote metering, the concept of TOD (time of day) metering can be introduced wherein the electronic meters at consumer's service entrance point are programmed to read the following meter readings on monthly basis.

1. KW hrs. consumed during calendar month by the consumer during low tariff and high tariff hours.
2. KVA maximum demand by the consumer during the calendar month (based on maximum demand lasting for 30 minutes duration).
3. Low tariff for off peak hour consumption.

These readings are telemetered to the control room for the purpose of monthly billing and cash collection through the various modes of communication available (- viz - Telephonic and wireless communication) depending on the load condition of the consumer. Tampering of energy - meters (if done) is also telemetered for taking action/penalty and disconnection of service.

Thus energy audit, though a very cumbersome and tedious job, can make a non-profitable business of distribution into a highly profitable one.

42.18. Advantages of Distribution Automation

More and more electric utilities are looking to distribution automation as an answer to the three major economic challenges facing the industry : the rising cost of adding generating capacity, increased saturation of existing distribution networks and greater sensitivity to customer service. Therefore, utilities that employ distribution automation expect both cost and service benefits. These benefits accumulate in areas that are related to investments, interruptions and customer service, as well as in areas related to operational cost savings, as given below :

(*a*) Reduced line loss :

The distribution substation is the electrical hub for the distribution network. A close coordination between the substation equipment, distribution feeders and associated equipment is necessary to increase system reliability. Volt/VAR control is addressed through expert algorithms which monitors and controls substation voltage devices in coordination with down-line voltage devices to reduce line loss and increase line throughout.

(*b*) Power quality :

Mitigation equipment is essential to maintain power quality over distribution feeders. The substation RTU in conjunction with power monitoring equipment on the feeders monitors, detects, and corrects power-related problems before they occur, providing a greater level of customer satisfaction.

(*c*) Deferred capital expenses :

A preventive maintenance algorithm may be integrated into the system. The resulting ability to schedule maintenance, reduces labour costs, optimizes equipment use and extends equipment life.

(*d*) Energy cost reduction :

Real-time monitoring of power usage throughout the distribution feeder provides data allowing the end user to track his energy consumption patterns, allocate usage and assign accountability to first line supervisors and daily operating personnel to reduce overall costs.

(*e*) Optimal energy use :

Real-time control, as part of a fully-integrated, automated power management system, provides the ability to perform calculations to reduce demand charges. It also offers a load-shedding/ preservation algorithm to optimize utility and multiple power sources, integrating cost of power into the algorithm.

(*f*) Economic benefits :

Investment related benefits of distribution automation came from a more effective use of the system. Utilities are able to operate closer to the edge to the physical limits of their systems. Distribution automation makes this possible by providing increased availability of better data for planning, engineering and maintenance. Investment related benefits can be achieved by deferring addition of generation capacity, releasing transmission capacity and deferring the addition, replacement of distribution substation equipment. Features such as voltage/VAR control, data monitoring and logging and load management contribute to capital deferred benefits.

Distribution automation can provide a balance of both quantitative and qualitative benefits in the areas of interruption and customer service by automatically locating feeder faults, decreasing the time required to restore service to unfaulted feeder sections, and reducing costs associated with customer complaints.

(*g*) Improved reliability :

On the qualitative side, improved reliability adds perceived value for customer and reduce the number of complaints. Distribution automation features that provide interruption and customer service related benefits include load shedding and other automatic control functions.

Lower operating costs are another major benefits of distribution automation. Operating cost reduction are achieved through improved voltage profiles, controlled VAR flow, repairs and maintenance savings, generation fuel savings from reduced substation transformer load losses, reduced feeder primary and distribution transformer losses, load management and reduced spinning reserve requirements. In addition, data acquisition and processing and remote metering functions play a large role in reducing operating costs and should be considered an integral part of any distribution automation system.

Through real time operation, the control computer can locate the faults much faster and control the switches and reclosures to quickly reroute power and minimize the total time-out, thus increasing the system reliability.

(*h*) Compatibility :

Distribution automation spans many functional and product areas including computer systems, application software, RTUs, communication systems and metering products. No single vendor provides all the pieces. Therefore, in order to be able to supply a utility with a complete and integrated system, it is important for the supplier to have alliances and agreements with other vendors.

An effective distribution automation system combines complementary function and capabilities and require an architecture that is flexible or "open" so that it can accommodate products from different vendors. In addition, a distribution automation system often requires interfaces with existing system in order to allow migration and integration, still monitoring network security.

Tutorial Problem No. 42.1

1. Why we need Transmission System Interconnection? *(Nagpur University, Summer 2004)*
2. Describe different distribution automation methods and their advantages and disadvantages.

OBJECTIVE TESTS – 42

1. Need based energy management is better than demand based energy management
   (*a*) true (*b*) false
2. Round the clock power is required to
   (*a*) agricultural uses
   (*b*) shift based industries
   (*c*) commercial organisations
   (*d*) essential services
3. Need based energy management would include
   (*a*) plan power generation as per forecast
   (*b*) monitor matching of need with supply
   (*c*) identify the needs of various consumers
   (*d*) all of the above
4. Existing Distribution systems are
   (*a*) properly designed
   (*b*) chaotic
   (*c*) optimally operated
   (*d*) automated
5. Distribution systems are not radial
   (*a*) true (*b*) false
6. The major aim of distribution automation is to keep minimum area in dark for a minimum time
   (*a*) true (*b*) false
7. Distribution automation is used for
   (*a*) electrical quantities only
   (*b*) physical quantities only
   (*c*) both of the above
8. Sectionalizing switches are circuit breakers
   (*a*) true (*b*) false
9. RTU is a -
   (*a*) transmitter (*b*) receiver
   (*c*) trans-receiver
10. The main problem in implanting Distri-bution Automation in metros is -
    (*a*) RTUs (*b*) communication
    (*c*) D.A.S. (*d*) control computers
11. Distribution SCADA means –
    (*a*) data acquisition
    (*b*) man – machine interface
    (*c*) data archiving
    (*d*) all of the above
12. Now - a - days the trend is towards –
    (*a*) manned substation
    (*b*) unmanned substation
13. Artificial Intelligence plays a pivotal data in distribution automation
    (*a*) true (*b*) false
14. Distribution Automation means –
    (*a*) substation Automation
    (*b*) feeder Automation
    (*c*) consumer side Automation
    (*d*) all of the above
15. Distribution Automotive system are modular
    (*a*) true (*b*) false
16. Existing distribution systems are controlled
    (*a*) automatically
    (*b*) manually
17. Distribution Automotive systems operate in real time
    (*a*) true (*b*) false
18. Until recent years, much of the investment and technical exploitation was done in –
    (*a*) generation
    (*b*) transmission
    (*c*) generation and transmission
    (*d*) distribution
19. Demand supply gap in power sector is expected to grow by - - - - percent every year.
    (*a*) 5 (*b*) 10
    (*c*) 15 (*d*) 20
20. Major revenue losses in distribution system occur in –
    (*a*) domestic sector
    (*b*) industrial sector
    (*c*) agricultural sector
    (*d*) commercial sector

ANSWERS

1. (*a*)	2. (*d*)	3. (*d*)	4. (*b*)	5. (*b*)	6. (*a*)	7. (*c*)	8. (*b*)	9. (*c*)	10. (*b*)
11. (*d*)	12. (*b*)	13. (*a*)	14. (*d*)	15. (*a*)	16. (*b*)	17. (*a*)	18. (*c*)	19. (*d*)	20. (*c*)

GO To FIRST

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CHAPTER 43

ELECTRIC TRACTION

Learning Objectives

- General
- Traction System
- Direct Steam Engine Drive
- Advantages of Electric Traction
- Saving in High Grade Coal
- Disadvantages of Electric Traction
- System of Railway Electrification
- Three Phase Low-Frequency A.C. System
- Block Diagram of an AC Locomotive
- The Tramways
- Collector Gear for OHE
- Confusion Regarding Weight and Mass of Train
- Tractive Efforts for Propulsion of a Train
- Power Output from Driving Axles
- Energy Output from Driving Axles
- Specific Energy Output
- Evaluation of Specific Energy Output
- Energy Consumption
- Specific Energy Consumption
- Adhesive Weight
- Coefficient of Adhesion

In electric traction, driving force (or tractive force) is generated by electricity, using electric motors. Electric trains, trams, trolley, buses and battery run cars are some examples where electric traction is employed.

CONTENTS

43.1. General

By electric traction is meant locomotion in which the driving (or tractive) force is obtained from electric motors. It is used in electric trains, tramcars, trolley buses and diesel-electric vehicles etc. Electric traction has many advantages as compared to other non-electrical systems of traction including steam traction.

43.2. Traction Systems

Broadly speaking, all traction systems may be classified into two categories :

(*a*) non-electric traction systems

They do not involve the use of electrical energy at ***any stage***. Examples are : steam engine drive used in railways and internal-combustion-engine drive used for road transport.

The above picture shows a diesel train engine. These engines are now being rapidly replaced by electric engines

(*b*) electric traction systems

They involve the use of electric energy at some stage or the other. They may be further subdivided into two groups :

1. First group consists of self-contained vehicles or locomotives. Examples are : battery-electric drive and diesel-electric drive etc.
2. Second group consists of vehicles which receive electric power from a distribution network fed at suitable points from either central power stations or suitably-spaced sub-stations. Examples are : railway electric locomotive fed from overhead ac supply and tramways and trolly buses supplied with dc supply.

43.3. Direct Steam Engine Drive

Though losing ground gradually due to various reasons, steam locomotive is still the most widely-adopted means of propulsion for railway work. Invariably, the reciprocating engine is employed because

1. it is inherently simple.
2. connection between its cylinders and the driving wheels is simple.
3. its speed can be controlled very easily.

However, the steam locomotive suffers from the following disadvantages :

1. since it is difficult to install a condenser on a locomotive, the steam engine runs non-condensing and, therefore, has a very low thermal efficiency of about 6-8 percent.
2. it has strictly limited overload capacity.
3. it is available for hauling work for about 60% of its working days, the remaining 40% being spent in preparing for service, in maintenance and overhaul.

43.4. Diesel-electric Drive

It is a self-contained motive power unit which employs a diesel engine for direct drive of a dc generator. This generator supplies current to traction motors which are geared to the driving axles.

In India, diesel locomotives were introduced in 1945 for shunting service on broad-guage (BG) sections and in 1956 for high-speed main-line operations on metre-guage (MG) sections. It was only in 1958 that Indian Railways went in for extensive main-line dieselisation.*

Diesel-electric traction has the following advantages :

1. no modification of existing tracks is required while converting from steam to diesel-electric traction.
2. it provides greater tractive effort as compared to steam engine which results in higher starting acceleration.
3. it is available for hauling for about 90% of its working days.
4. diesel-electric locomotive is more efficient than a steam locomotive (though less efficient than an electric locomotive).

Disadvantages

1. for same power, diesel-electric locomotive is costlier than either the steam or electric locomotive.
2. overload capacity is limited because diesel engine is a constant-kW output prime mover.
3. life of a diesel engine is comparatively shorter.
4. diesel-electric locomotive is heavier than plain electric locomotive because it carries the main engine, generator and traction motors etc.
5. regenerative braking cannot be employed though rheostatic braking can be.

43.5. Battery-electric Drive

In this case, the vehicle carries secondary batteries which supply current to dc motors used for driving the vehicle. Such a drive is well-suited for shunting in railway yards, for traction in mines, for local delivery of goods in large towns and large industrial plants. They have low maintenance cost and are free from smoke. However, the scope of such vehicles is limited because of the small capacity of the batteries and the necessity of charging them frequently.

The above picture shows a battery run car. Battery run vehicles are seen as alternatives for future transport due to their pollution-free locomotion

43.6. Advantages of Electric Traction

As compared to steam traction, electric trac-

* The Diesel Locomotive Works at Varanasi turns out 140 locomotives of 2700 hp (2015 kW) annually. Soon it will be producing new generation diesel engines of 4000 hp (2985 kW).

tion has the following advantages :

1. Cleanliness. Since it does not produce any smoke or corrosive fumes, electric traction is most suited for underground and tube railways. Also, it causes no damage to the buildings and other apparatus due to the absence of smoke and flue gases.

2. Maintenance Cost. The maintenance cost of an electric locomotive is nearly 50% of that for a steam locomotive. Moreover, the maintenance time is also much less.

3. Starting Time. An electric locomotive can be started at a moment's notice whereas a steam locomotive requires about two hours to heat up.

4. High Starting Torque. The motors used in electric traction have a very high starting torque. Hence, it is possible to achieve higher accelerations of 1.5 to 2.5 km/h/s as against 0.6 to 0.8 km/h/s in steam traction. As a result, we are able to get the following additional advantages:

(*i*) high schedule speed

(*ii*) increased traffic handling capacity

(*iii*) because of (*i*) and (*ii*) above, less terminal space is required—a factor of great importance in urban areas.

5. Braking. It is possible to use regenerative braking in electric traction system. It leads to the following advantages :

(*i*) about 80% of the energy taken from the supply during ascent is returned to it during descent.

(*ii*) goods traffic on gradients becomes safer and speedier.

(*iii*) since mechanical brakes are used to a very small extent, maintenance of brake shoes, wheels, tyres and track rails is considerably reduced because of less wear and tear.

6. Saving in High Grade Coal. Steam locomotives use costly high-grade coal which is not so abundant. But electric locomotives can be fed either from hydroelectric stations or pit-head thermal power stations which use cheap low-grade coal. In this way, high-grade coal can be saved for metallurgical purposes.

7. Lower Centre of Gravity. Since height of an electric locomotive is much less than that of a steam locomotive, its centre of gravity is comparatively low. This fact enables an electric locomotive to negotiate curves at higher speeds quite safely.

8. Absence of Unbalanced Forces. Electric traction has higher coefficient of adhesion since there are no unbalanced forces produced by reciprocating masses as is the case in steam traction. It not only reduces the weight/kW ratio of an electric locomotive but also improves its riding quality in addition to reducing the wear and tear of the track rails.

43.7. Disadvantages of Electric Traction

1. The most vital factor against electric traction is the initial high cost of laying out overhead electric supply system. Unless the traffic to be handled is heavy, electric traction becomes uneconomical.

2. Power failure for few minutes can cause traffic dislocation for hours.

3. Communication lines which usually run parallel to the power supply lines suffer from electrical interference. Hence, these communication lines have either to be removed away from the rail track or else underground cables have to be used for the purpose which makes the entire system still more expensive.

4. Electric traction can be used only on those routes which have been electrified. Obviously, this restriction does not apply to steam traction.

5. Provision of a negative booster is essential in the case of electric traction. By avoiding the flow of return currents through earth, it curtails corrosion of underground pipe work and interference with telegraph and telephone circuits.

43.8. Systems of Railway Electrification

Presently, following four types of track electrification systems are available :

1. Direct current system—600 V, 750 V, 1500 V, 3000 V
2. Single-phase ac system—15-25 kV, $16\frac{2}{3}$, 25 and 50 Hz
3. Three-phase ac system—3000-3500 V at $16\frac{2}{3}$ Hz
4. Composite system—involving conversion of single-phase ac into 3-phase ac or dc.

43.9. Direct Current System

Direct current at 600-750 V is universally employed for tramways in urban areas and for many suburban railways while 1500-3000 V dc is used for main line railways. The current collection is from third rail (or conductor rail) up to 750 V, where large currents are involved and from overhead wire for 1500 V and 3000 V, where small currents are involved. Since in majority of cases, track (or running) rails are used as the return conductor, only one conductor rail is required. Both of these contact systems are fed from substations which are spaced 3 to 5 km for heavy suburban traffic and 40-50 km for main lines operating at higher voltages of 1500 V to 3000 V. These sub-stations themselves receive power from 110/132 kV, 3-phase network (or grid). At these substations, this high-voltage 3-phase supply is converted into low-voltage 1-phase supply with the help of Scott-connected or V-connected 3-phase transformers (Art. 31.9). Next, this low ac voltage is converted into the required dc voltage by using suitable rectifiers or converters (like rotary converter, mercury-arc, metal or semiconductor rectifiers). These substations are usually automatic and are remote-controlled.

The dc supply so obtained is fed via suitable contact system to the traction motors which are either dc series motors for electric locomotive or compound motors for tramway and trolley buses where regenerative braking is desired.

It may be noted that for ***heavy suburban service***, low voltage dc system is undoubtedly superior to 1-phase ac system due to the following reasons :

1. dc motors are better suited for frequent and rapid acceleration of heavy trains than ac motors.
2. dc train equipment is lighter, less costly and more efficient than similar ac equipment.
3. when operating under similar service conditions, dc train consumes less energy than a 1-phase ac train.
4. the conductor rail for dc distribution system is less costly, both initially and in maintenance than the high-voltage overhead ac distribution system.
5. dc system causes no electrical interference with overhead communication lines.

The only disadvantage of dc system is the necessity of locating ac/dc conversion sub-stations at relatively short distances apart.

43.10. Single-Phase Low-frequency AC System

In this system, ac voltages from 11 to 15 kV at $16\frac{2}{3}$ or 25 Hz are used. If supply is from a generating station exclusively meant for the traction system, there is no difficulty in getting the electric supply of $16\frac{2}{3}$ or 25 Hz. If, however, electric supply is taken from the high voltage transmission lines at 50 Hz, then in addition to step-down transformer, the substation is provided with a frequency converter. The frequency converter equipment consists of a 3-phase synchronous motor which drives a I-phase alternator having or 25 Hz frequency.

The 15 kV $16\frac{2}{3}$ or 25 Hz supply is fed to the electric locomotor via a single over-head wire (running rail providing the return path).

A step-down transformer carried by the locomotive reduces the 15-kV voltage to 300-400 V for feeding the ac series motors. Speed regulation of ac series motors is achieved by applying variable voltage from the tapped secondary of the above transformer.

Low-frequency ac supply is used because apart from improving the commutation properties of ac motors, it increases their efficiency and power factor. Moreover, at low frequency, line reactance is less so that line impedance drop and hence line voltage drop is reduced. Because of this reduced line drop, it is feasible to space the substations 50 to 80 km apart. Another advantage of employing low frequency is that it reduces telephonic interference.

41.11. Three-phase Low-frequency AC System

It uses 3-phase induction motors which work on a 3.3 kV, $16\frac{2}{3}$ Hz supply. Sub-stations receive power at a very high voltage from 3-phase transmission lines at the usual industrial frequency of 50 Hz. This high voltage is stepped down to 3.3 kV by transformers whereas frequency is reduced from 50 Hz to $16\frac{2}{3}$ Hz by frequency converters installed at the sub-stations. Obviously, this system employs *two* overhead contact wires, the track rail forming the third phase (of course, this leads to insulation difficulties at the junctions).

Induction motors used in the system are quite simple and robust and give trouble-free operation. They possess the merits of high efficiency and of operating as a generator when driven at speeds above the synchronous speed. Hence, they have the property of automatic regenerative braking during the descent on gradients. However, it may be noted that despite all its advantages, this system has not found much favour and has, in fact, become obsolete because of its certain inherent limitations given below :

1. the overhead contact wire system becomes complicated at crossings and junctions.
2. constant-speed characteristics of induction motors are not suitable for traction work.
3. induction motors have speed/torque characteristics similar to dc shunt motors. Hence, they are not suitable for parallel operation because, even with little difference in rotational speeds caused by unequal diameters of the wheels, motors will becomes loaded very unevenly.

43.12. Composite System

Such a system incorporates good points of two systems while ignoring their bad points. Two such composite systems presently in use are :

1. 1-phase to 3-phase system also called Kando system
2. 1-phase to dc system.

43.13. Kando System

In this system, single-phase 16-kV, 50 Hz supply from the sub-station is picked up by the locomotive through the single overhead contact wire. It is then converted into 3-phase ac supply at the same frequency by means of phase converter equipment carried on the locomotives. This 3-phase supply is then fed to the 3-phase induction motors.

As seen, the complicated overhead two contact wire arrangement of ordinary 3-phase system is replaced by a single wire system. By using silicon controlled rectifier as inverter, it is possible to get variable-frequency 3-phase supply at 1/2 to 9 Hz frequency. At this low frequency, 3-phase motors develop high starting torque without taking excessive current. In view of the above, Kando system is likely to be developed further.

43.14. Single-phase AC to DC System

This system combines the advantages of high-voltage ac distribution at industrial frequency with the dc series motors traction. It employs overhead 25-kV, 50-Hz supply which is stepped down by the transformer installed in the locomotive itself. The low-voltage ac supply is then converted into dc supply by the rectifier which is also carried on the locomotive. This dc supply is finally fed to dc series traction motor fitted between the wheels. The system of traction employing 25-kV, 50-Hz, 1-phase ac supply has been adopted for all future track electrification in India.

43.15. Advantages of 25-kV, 50-Hz AC System

Advantages of this system of track electrification over other systems particularly the dc system are as under :

1. Light Overhead Catenary

Since voltage is high (25 kV), line current for a given traction demand is less. Hence, cross-section of the overhead conductors is reduced. Since these small-sized conductors are light, supporting structures and foundations are also light and simple. Of course, high voltage needs higher insulation which increases the cost of overhead equipment (OHE) but the reduction in the size of conductors has an overriding effect.

2. Less Number of Substations

Since in the 25-kV system, line current is less, line voltage drop which is mainly due to the resistance of the line is correspondingly less. It improves the voltage regulation of the line which fact makes larger spacing of 50-80 km between sub-stations possible as against 5-15 km with 1500 V dc system and 15-30 km with 3000 V dc sysem. Since the required number of substations along the track is considerably reduced, it leads to substantial saving in the capital expenditure on track electrification.

3. Flexibility in the Location of Substations

Larger spacing of substations leads to greater flexibility in the selection of site for their proper location. These substations can be located near the national high-voltage grid which, in our country, fortunately runs close to the main railway routes. The substations are fed from this grid thereby saving the railway administration lot of expenditure for erecting special transmission lines for their substations. On the other hand, in view of closer spacing of dc substations and their far away location, railway administration has to erect its own transmission lines for taking feed from the national grid to the substations which consequently increases the initial cost of electrification.

4. Simplicity of Substation Design

In ac systems, the substations are simple in design and layout because they do not have to install and maintain rotary converters or rectifiers as in dc systems. They only consist of static transformers alongwith their associated switchgear and take their power directly from the high-voltage national grid running over the length and breadth of our country. Since such sub-stations are remotely controlled, they have few attending personnel or even may be unattended.

5. Lower Cost of Fixed Installations

The cost of fixed installations is much less for 25 kV ac system as compared to dc system. In fact, cost is in ascending order for 25 kV ac, 3000 V dc and 1500 V dc systems. Consequently, traffic densities for which these systems are economical are also in the ascending order.

6. Higher Coefficient of Adhesion

The straight dc locomotive has a coefficient of adhesion of about 27% whereas its value for ac rectifier locomotive is nearly 45%. For this reason, a lighter ac locomotive can haul the same load as a heavier straight dc locomotive. Consequently, ac locomotives are capable of achieving higher speeds in coping with heavier traffic.

7. Higher Starting Efficiency

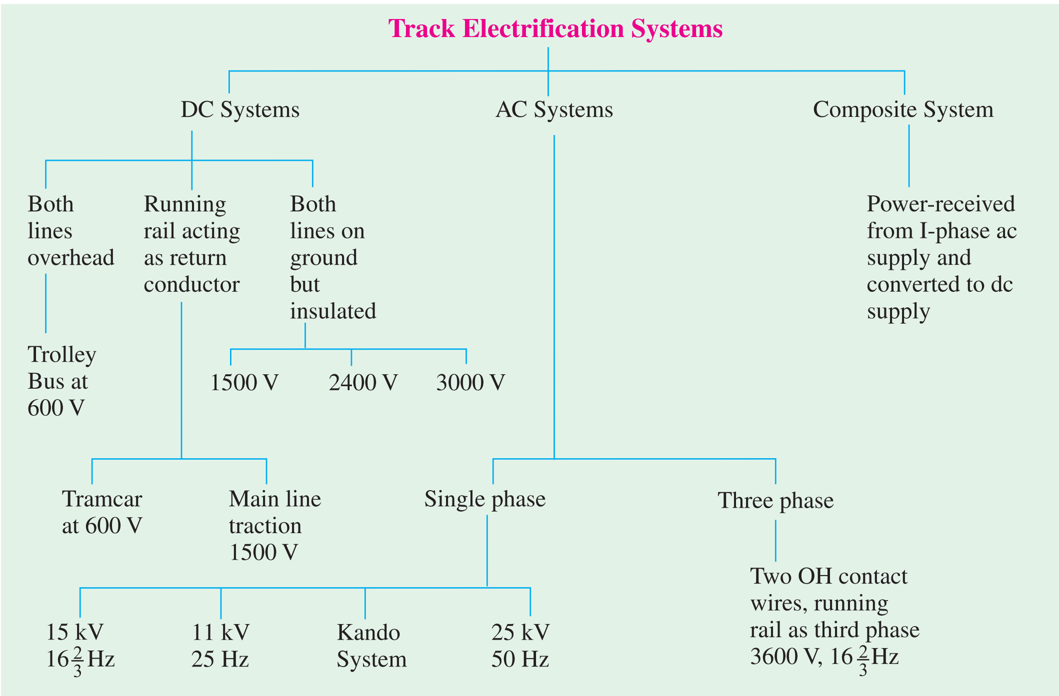
An ac locomotive has higher starting efficiency than a straight dc locomotive. In dc locomotive supply voltage at starting is reduced by means of ohmic resistors but by on-load primary or secondary tap-changer in ac locomotives.

43.16. Disadvantages of 25-kV AC System

1. Single-phase ac system produces both current and voltage unbalancing effect on the supply.

2. It produces interference in telecommunication circuits. Fortunately, it is possible at least to minimize both these undesirable effects.

Different track electrification systems are summarised below :



43.17. Block Diagram of an AC Locomotive

The various components of an ac locomotive running on single-phase 25-kV, 50-Hz ac supply are numbered in Fig. 43.1.

1. OH contact wire

2. pantograph
3. circuit breakers
4. on-load tap-changers
5. transformer
6. rectifier
7. smoothing choke
8. dc traction motors.

As seen, power at 25 kV is taken via a pantograph from the overhead contact wire and fed to the step-down transformer in the locomotive. The low ac voltage so obtained is converted into pulsating dc voltage by means of the rectifier. The pulsations in the dc voltage are then removed by the smoothing choke before it is fed to dc series traction motors which are mounted between the wheels.

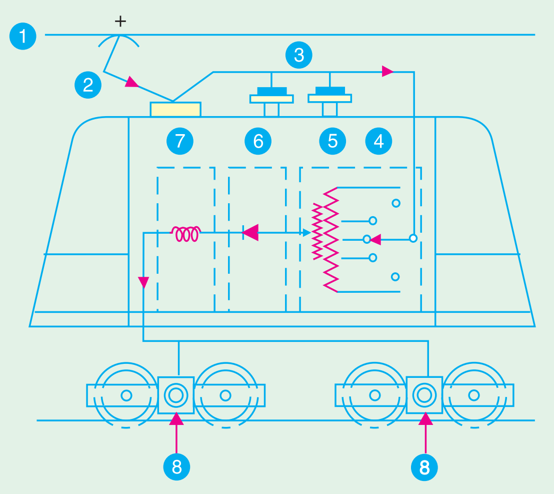


Fig. 43.1

The function of circuit breakers is to immediately disconnect the locomotive from the overhead supply in case of any fault in its electrical system. The on-load tap-changer is used to change the voltage across the motors and hence regulate their speed.

43.18. The Tramways

It is the most economical means of transport for very dense traffic in the congested streets of large cities. It receives power through a bow collector or a grooved wheel from an overhead conductor at about 600 V dc, the running rail forming the return conductor. It is provided with at least two driving axles in order to ***(i)*** secure necessary adhesion ***(ii)*** start it from either end and ***(iii)*** use two motors with series-parallel control. Two drum-type controllers, one at each end, are used for controlling the tramcar. Though these controllers are connected in parallel, they have suitable interlocking arrangement meant to prevent their being used simultaneously.

Tramcars are being replaced by trolley-buses and internal-combustion-engined omnibuses because of the following reasons :

1. tramcars lack flexibility of operation in congested areas.
2. the track constitutes a source of danger to other road users.


Trolley Bus

43.19. The Trolleybus

It is an electrically-operated pneumatic-tyred vehicle which needs ***no track in the roadway***. It receives its power at 600 V dc from two overhead contact wires. Since adhesion between a rubber-tyred wheel and ground is sufficiently high, only a single driving axle and, hence, a single motor is used. The trolleybus can manoeuvre through traffic a metre or two on each side of the centre line of the trolley wires.

43.20. Overhead Equipment (OHE)

Broadly speaking, there are two systems of current collection by a traction unit :

(i) third rail system and *(ii)* overhead wire system.

It has been found that current collection from overhead wire is far superior to that from the third rail. Moreover, insulation of third rail at high voltage becomes an impracticable proposition and endangers the safety of the working personnel.

The simplest type of OHE consists of a single contact wire of hard drawn copper or silico-bronze supported either by bracket or an overhead span. To facilitate connection to the supports, the wire is grooved as shown in Fig. 43.2. Because there is appreciable sag of the wire between supports, it limits the speed of the traction unit to about 30 km/h. Hence, single contact wire system is suitable for tramways and in complicated yards and terminal stations where speeds are low and simplicity of layout is desirable.

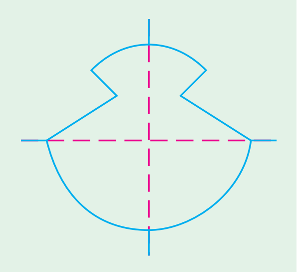


Fig. 43.2

For collection of current by high-speed trains, the contact (or trolley) wire has to be kept level without any abrupt changes in its height between the supporting structures. It can be done by using the single catenary system which consists of one catenary or messenger wire of steel with high sag and the trolley (or contact) wire supported from messenger wire by means of droppers clipped to both wires as shown in Fig. 43.3.

43.21. Collector Gear for OHE

The most essential requirement of a collector is that it should keep continuous contact with trolley wire at all speeds. Three types of gear are in common use :

1. trolley collector **2.** bow collector and **3.** pantograph collector.

To ensure even pressure on OHE, the gear equipment must, be flexible in order to follow variations in the sag of the contact wire. Also, reasonable precautions must be taken to prevent the collector from leaving the overhead wire at points and crossings.

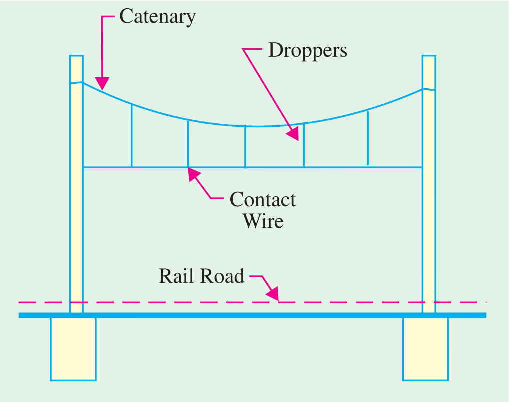


Fig. 43.3

43.22. The Trolley Collector

This collector is employed on tramways and trolley buses and is mounted on the roof of the vehicle. Contact with the OH wire is made by means of either a grooved wheel or a sliding shoe carried at the end of a light trolley pole attached to the top of the vehicle and held in contact with OH wire by means of a spring. The pole is hinged to a swivelling base so that it may be reversed for reverse running thereby making it unnecessary for the trolley wire to be accurately maintained above the centre of the track. Trolley collectors always operate in the trailing position.

The trolley collector is suitable for low speeds upto 32 km/h beyond which there is a risk of its jumping off the OH contact wire particularly at points and crossing.

43.23. The Bow Collector

It can be used for higher speeds. As shown in Fig. 43.4, it consists of two roof-mounted trolley poles at the ends of which is placed a light metal strip (or bow) about one metre long for current collection. The collection strip is purposely made of soft material (copper, aluminium or carbon) in order that most of the wear may occur on it rather than on the trolley wire. The bow collector also operates in the trailing position. Hence, it requires provision of either duplicate bows or an arrangement for reversing the bow for running in the reverse direction. Bow collector is not suitable for railway work where speeds up to 120 km/h and currents up to 3000 A are encountered. It is so because the inertia of the bow collector is too high to ensure satisfactory current collection.

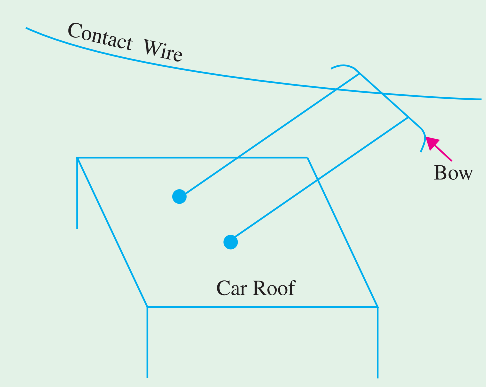


Fig. 43.4

43.24. The Pantograph Collector

Its function is to maintain link between overhead contact wire and power circuit of the electric locomotive at different speeds under all wind conditions and stiffness of OHE. It means that positive pressure has to be maintained at all times to avoid loss of contact and sparking but the pressure must be as low as possible in order to minimize wear of OH contact wire.

A 'diamond' type single-pan pantograph is shown in Fig. 43.5. It consists of a pentagonal framework of high-tensile alloy-steel tubing. The contact portion consists of a pressed steel pan fitted with renewable copper wearing strips which are forced against the OH contact wire by the upward action of pantograph springs. The pantograph can be raised or lowered from cabin by air cylinders.

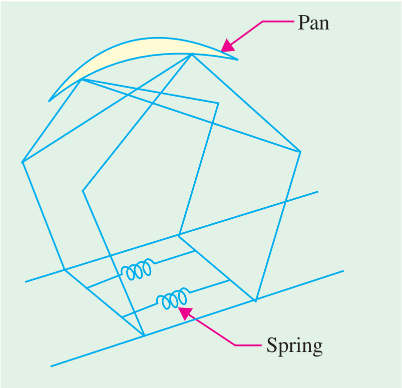


Fig. 43.5

43.25. Conductor Rail Equipment

The conductor rails may be divided into three classes depending on the position of the contact surface which may be located at the top, bottom or side of the rail. The top contact rail is adopted universally for 600 V dc electrification. The side contact rail is used for 1200 V dc supply. The under contact rail has the advantage of being protected from snow, sleet and ice.

Fig. 43.6 shows the case when electric supply is collected from the top of an insulated conductor rail *C* (of special high-conductivity steel) running parallel to the track at a distance of 0.3 to 0.4 m from the running rail (*R*) which forms the return path. *L* is the insulator and *W* is the wooden protection used at stations and crossings.

The current is collected from top surface of the rail by flat steel shoes (200 mm × 75 mm), the necessary contact pressure being obtained by gravity. Since it is not always possible to provide conductor rail on the same side of the track, shoes are provided on both sides of the locomotive or train. Moreover two shoes are provided on each side in order to avoid current interruption at points and crossings where there are gaps in the running rail.

The pantograph mechanism helps to maintain a link between the overhead contact wire and power circuit of the electric locomotive

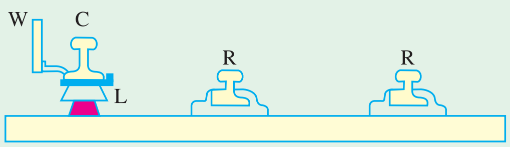


Fig. 43.6

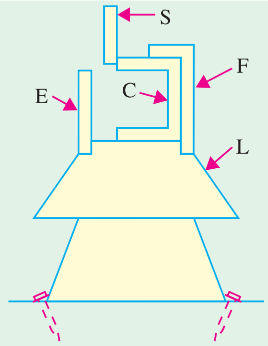


Fig. 43.7

Fig. 43.7 shows the side contact rail and the method of the mounting. The conductor rail (*C*) rests upon a wooden block recessed into the top of the procelain insulator *L*. Current is collected by steel shoes (*S*) which are kept pressed on the contact rail by springs. *E* and *F* are the guards which rest upon ledges on the insulator.

43.26. Types of Railway Services

There are three types of passenger services offered by the railways :

1. City or Urban Service. In this case, there are frequent stops, the distance between stops being nearly 1 km or less. Hence, high acceleration and retardation are essential to achieve moderately high schedule speed between the stations.

2. Suburban Service. In this case, the distance between stops averages from 3 to 5 km over a distance of 25 to 30 km from the city terminus. Here, also, high rates of acceleration and retardation are necessary.

3. Main Line Service. It involves operation over long routes where stops are infrequent. Here, operating speed is high and accelerating and braking periods are relatively unimportant.

On goods traffic side also, there are three types of services ***(i)*** main-line freight service ***(ii)*** local or pick-up freight service and ***(iii)*** shunting service.

43.27. Train Movement

The movement of trains and their energy consumption can be conveniently studied by means of speed/time and speed/distance curves. As their names indicate, former gives speed of the train at various ***times*** after the start of the run and the later gives speed at various ***distances*** from the starting point. Out of the two, speed/time curve is more important because

1. its slope gives acceleration or retardation as the case may be.
2. area between it and the horizontal (*i.e.* time) axis represents the distance travelled.
3. energy required for propulsion can be calculated if resistance to the motion of train is known.

43.28. Typical Speed/Time Curve

Typical speed/time curve for electric trains operating on passenger services is shown in Fig. 43.8. It may be divided into the following **five** parts :

1. Constant Acceleration Period (0 to t_1)

It is also called notching-up or starting period because during this period, starting resistance of the motors is gradually cut out so that the motor current (and hence, tractive effort) is maintained nearly constant which produces constant acceleration alternatively called 'rheostatic acceleration' or 'acceleration while notching'.

2. Acceleration on Speed Curve (t_1 to t_2)

This acceleration commences after the starting resistance has been all cut-out at point t_1 and full supply voltage has been applied to the motors. During this period, the motor current and torque decrease as train speed increases. Hence, acceleration gradually ***decreases*** till torque developed by motors exactly balances that due to resistance to the train motion. The shape of the portion *AB* of the speed/time curve depends primarily on the torque/speed characteristics of the traction motors.

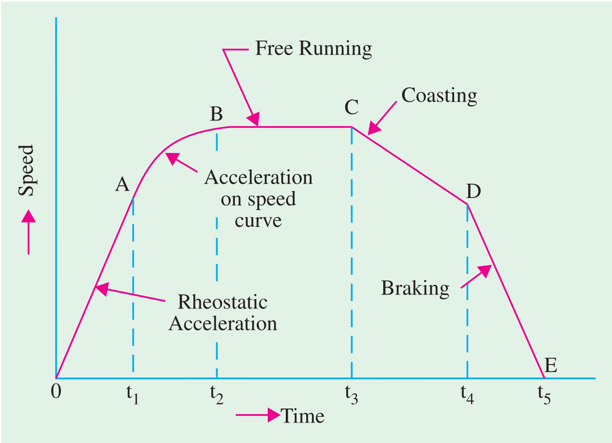


Fig. 43.8

3. Free-running Period (t_2 to t_3)

The train continues to run at the speed reached at point t_2. It is represented by portion *BC* in Fig. 43.8 and is a constant-speed period which occurs on level tracks.

4. Coasting (t_3 to t_4)

Power to the motors is cut off at point t_3 so that the train runs under its momentum, the speed gradually falling due to friction, windage etc. (portion *CD*). During this period, retardation remains practically constant. Coasting is desirable because it utilizes some of the kinetic energy of the train which would, otherwise, be wasted during braking. Hence, it helps to reduce the energy consumption of the train.

5. Braking (t_4 to t_5)

At point t_4, brakes are applied and the train is brought to rest at point t_5.

It may be noted that coasting and braking are governed by train resistance and allowable retardation respectively.

43.29. Speed/Time Curves for Different Services

Fig. 43.9 (*a*) is representative of city service where relative values of acceleration and retardation are high in order to achieve moderately high average speed between stops. Due to short

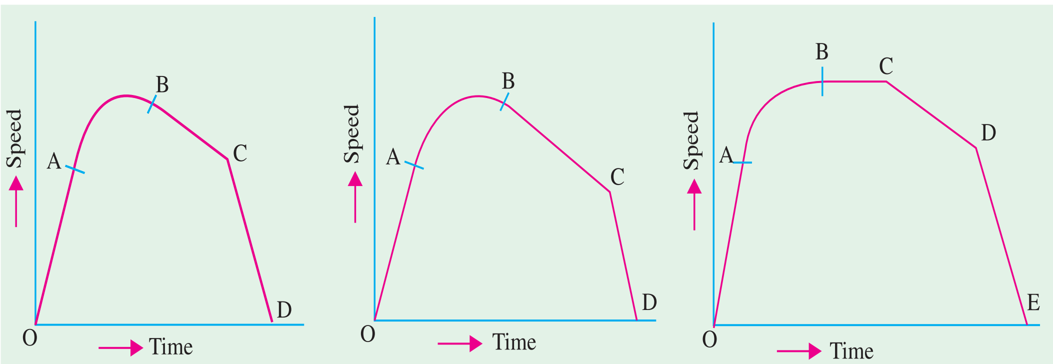


Fig. 43.9

distances between stops, there is no possibility of free-running period though a short coasting period is included to save on energy consumption.

In suburban services [Fig. 43.9 (*b*)], again there is no free-running period but there is comparatively ***longer*** coasting period because of longer distances between stops. In this case also, relatively high values of acceleration and retardation are required in order to make the service as attractive as possible.

For main-line service [Fig. 43.9 (c)], there are long periods of free-running at high speeds. The accelerating and retardation periods are relatively unimportant.

43.30. Simplified Speed/Time Curve

For the purpose of comparative performance for a given service, the actual speed/time curve of Fig. 43.8 is replaced by a simplified speed/time curve which does not involve the knowledge of motor characteristics. Such a curve has simple geometric shape so that simple mathematics can be used to find the relation between acceleration, retradation, average speed and distance etc. The simple curve would be fairly accurate provided it ***(i) retains the same acceleration and retardation and (ii) has the same area as the actual speed/time curve***. The simplified speed/time curve can have either of the two shapes :

(i) trapezoidal shape OA_1B_1C of Fig. 43.10 where speed-curve running and coasting periods of the actual speed/time curve have been replaced by a constant-speed period.

(ii) quadrilateral shape OA_2B_2C where the same two periods are replaced by the extensions of initial constnat acceleration and coasting periods.

It is found that trapezodial diagram OA_1B_1C gives simpler relationships between the principal quantities involved in train movement and also gives closer approximation of actual energy consumed during ***main-line service on level track***. On the other hand, quadrilateral diagram approximates more closely to the actual conditions in ***city and suburban services***.

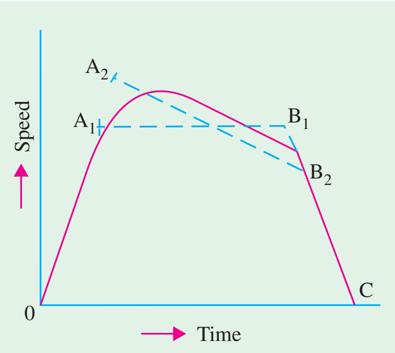


Fig. 43.10

43.31. Average and Schedule Speed

While considering train movement, the following three speeds are of importance :

1. Crest Speed. It is the maximum speed (V_m) attained by a train during the run.

2. Average Speed $= \dfrac{\text{distance between stops}}{\text{actual time of run}}$

In this case, only running time is considered but *not the stop time*.

3. Schedule Speed $= \dfrac{\text{distance between stops}}{\text{actual time of run + stop time}}$

Obviously, schedule speed can be obtained from average speed by including the duration of stops. For a given distance between stations, higher values of acceleration and retardation will mean lesser running time and, consequently, higher schedule speed. Similarly, for a given distance between stations and for fixed values of acceleration and retardation, higher crest speed will result in higher schedule speed. For the same value of average speed, increase in duration of stops decreases the schedule speed.

43.32. SI Units in Traction Mechanics

In describing various quantities involved in the mechanics of train movement, only the latest SI system will be used. Since SI system is an 'absolute system', only absolute units will be used while gravitational units (used hitherto) will be discarded.

1. Force. It is measured in newton (N)

2. Mass. Its unit is kilogram (kg). Commonly used bigger units is tonne (t), 1 tonne = 1000 kg

3. Energy. Its basic unit is joule (J). Other units often employed are watt-hour (Wh) and kilowatt-hour (kWh).

$$1\text{ Wh} = 1\frac{\text{J}}{\text{s}}\times 3600\text{ s} = 3600\text{ J} = 3.6\text{ kJ}$$

$$1\text{kWh} = 1000\times 1\frac{\text{J}}{\text{s}}\times 3600\text{ s} = 36\times 10^5\text{ J} = 3.6\text{ MJ}$$

4. Work. Its unit is the same as that of energy.

5. Power. Its unit is watt (W) which equals 1 J/s. Other units are kilowatt (kW) and megawatt (MW).

6. Distance. Its unit is metre. Other unit often used is kilometre (km).

7. Velocity. Its absolute unit is metre per second (m/s). If velocity is given in km/h (or km.ph), it can be easily converted into the SI unit of m/s by multiplying it with a factor of (1000/3600) = 5/18 = 0.2778. For example, 72 km.ph = 72 × 5/18 = 72 × 0.2778 = 20 m/s.

8. Acceleration. Its unit is metre/second^2 (m/s^2). If acceleration is given in km/h/s (or km-ph.ps), then it can be converted into m/s^2 by simply multiplying it by the factor (1000/3600) = 5/18 = 0.2778 *i.e.* the same factor as for velocity.

For example, 1.8 km.ph.ps = 1.8 × 5/18 = 1.8 × 0.2778 = 0.5 m/s^2

43.33. Confusion Regarding Weight and Mass of a Train

Many students often get confused regarding the correct meaning of the terms 'weight' and 'mass' and their units while solving numericals on train movement particularly when they are not expressed clearly and consistently in their absolute units. It is primarily due to the mixing up of absolute units with gravitational units. There would be no confusion at all if ***we are consistent in using only absolute units*** as required by the SI system of units which disallows the use of gravitational units.

Though this topic was briefly discussed earlier, it is worth repeating here.

1. Mass (M). It is the quantity of matter contained in a body.

Its absolute unit is kilogram (kg). Other multiple in common use is tonne.

2. Weight (W). It is the **force** with which earth pulls a body downwards.

The weight of a body can be expressed in (*i*) the ***absolute*** unit of newton (N) or (*ii*) the ***gravitational*** unit of kilogram-weight (kg. wt) which is often writing as 'kgf' in engineering literature.

Another still bigger ***gravitational*** unit commonly used in traction work is tonne-weight (t-wt)

$$1\text{ t-wt} = 1000\text{ kg-wt} = 1000\times 9.8\text{ N} = 9800\text{ N}$$

(*i*) Absolute Unit of Weight

It is called newton (N) whose definition may be obtained from Newton's Second Law of Motion.

Commonly used multiple is kilo-newton (kN). Obviously, 1 kN = 1000 N = 10^3 N.

For example, if a mass of 200 kg has to be given an acceleration of 2.5 m/s^2, force required is $F = 200\times 2.5 = 500$ N.

If a train of mass 500 tonne has to be given an acceleration of 0.6 m/s^2, force required is

$$F = ma = (500\times 1000)\times 0.6 = 300{,}000\text{ N} = 300\text{ kN}$$

(*ii*) Gravitational Unit of Weight

It is '*g*' times bigger than newton. It is called kilogram-weight (kg.wt.)

$$1\text{ kg.wt} = g\text{ newton} = 9.81\text{ N} \cong 9.8\text{ N}$$

Unfortunately, the word 'wt' is usually omitted from kg-wt when expressing the weight of the body on the assumption that it can be understood or inferred from the language used.

Take the statement "a body has a *weight* of 100 kg". It looks as if the weight of the body has been expressed in terms of the mass unit 'kg'. To avoid this confusion, statement should be 'a body has a weight of 100 kg. wt.' But the first statement is justified by the writers on the ground that since the word 'weight' has already been used in the statement, it should be automatically understood by the readers that 'kg' is not the 'kg' of mass but is kg-wt. It would be mass kg if the statement is 'a body has a mass of 100 kg'. Often kg-wt is written as 'kgf' where 'f' is the first letter of the word force and is added to distinguish it from kg of mass.

Now, consider the statement "a body *weighing* 500 kg travels with a speed of 36 km/h.........."

Now, weight of the body W = 500 kg.wt. = 500×9.8 N

Since we know the weight of the body, we can find its mass from the relation $W = mg$. But while using this equation, it is essential that we must ***consistently* use** the ***absolute*** units only. In this equation, W must be in newton (not in kg. wt), m in kg and g in m/s^2.

$\therefore \quad 500 \times 9.8 = m \times 9.8 \; ; \qquad \therefore \quad m = 500$ kg

It means that a body which *weighs* 500 kg (wt) has a *mass* of 500 kg.

As a practical rule, weight of a body in **gravitational** units is numerically equal to its mass in **absolute** units. This simple fact must be clearly understood to avoid any confusion between weight and mass of a body.

A train which weighs 500 tonne has a mass of 500 tonne as proved below :

train weight, $\quad W = 500$ tonne-wt $= 500 \times 1000$ kg-wt $= 500 \times 1000 \times 9.8$ N

Now, $\quad W = mg \; ; \qquad \therefore \quad 500 \times 1000 \times 9.8 = m \times 9.8$

$\therefore \quad m = 500 \times 1000$ kg $= 500 \times 1000/1000 = 500$ tonne

To avoid this unfortunate confusion, it would be helpful to change our terminology. For example, instead of saying "a train weighing 500 tonne is........" it is better to say "a 500-t train is ...." or "a train having a mass of 500 t is ...."

In order to remove this confusion, SI system of units has disallowed the use of gravitational units. There will be no confusion if ***we consistently use only absolute units***.

43.34. Quantities Involved in Traction Mechanics

Following principal quantities are involved in train movement :

D = distance between stops

M_e = effective mass of the train

W_e = effective weight of the train

β_c = retardation during coasting

V_a = average speed

t = total time for the run

t_2 = time of free running = $t - (t_1 + t_3)$

F_t = tractive effort

M = dead mass of the train

W = dead weight of the train

α = accelaration during starting period

β = retardation during braking

V_m = maximum (or crest) speed.

t_1 = time of acceleration

t_3 = time of braking

T = torque

43.35. Relationship Between Principal Quantities in Trapezoidal Diagram

As seen from Fig. 43.11.

$\alpha = V_m/t_1 \quad$ or $\quad t_1 = V_m/\alpha$

$\beta = V_m/t_3 \quad$ or $\quad t_3 = V_m/\beta$

As we know, total distance D between the two stops is given by the area of trapezium $OABC$.

$$\therefore \quad D = \text{area } OABC$$
$$= \text{area } OAD + \text{area } ABED + \text{area } BCE$$
$$= \frac{1}{2} V_m t_1 + V_m t_2 + \frac{1}{2} V_m t_3$$

$$= \frac{1}{2}\, V_m t_1 + V_m[t - (t_1 + t_3)] + \frac{1}{2}\, V_m t_3$$

$$= V_m\left[\frac{t_1}{2} + t - t_1 - t_3 + \frac{t_3}{2}\right]$$

$$= V_m\left[t - \frac{1}{2}(t_1 + t_3)\right]$$

$$= V_m\left[t - \frac{V_m}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right]$$

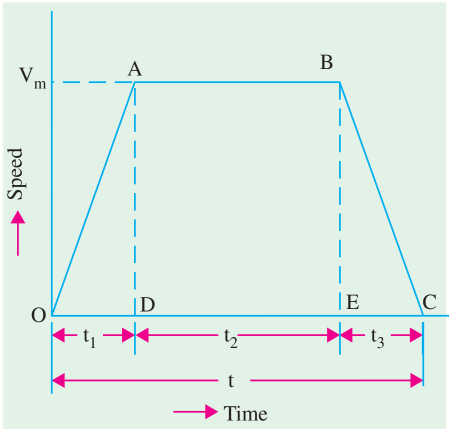


Fig. 43.11

Let, $K = \frac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$. Substituting this value of K in the above equation, we get

$$D = V_m (t - KV_m)$$

or $KV_m^2 - V_m t + D = 0$ ...(*i*)

$$\therefore \quad V_m = \frac{t \pm \sqrt{t^2 - 4KD}}{2K}$$

Rejecting the positive sign which gives impracticable value, we get

$$V_m = \frac{t \pm \sqrt{t^2 - 4KD}}{2K}$$

From Eq. (*i*) above, we get

$$KV_m^2 = V_m t - D \qquad \text{or} \qquad K = \frac{t}{V_m} - \frac{D}{V_m^2} = \frac{D}{V_m^2}\left(V_m \cdot \frac{t}{D} - 1\right)$$

Now, $\boxed{V_a = \frac{D}{t}}$ $\quad\therefore\quad$ $\boxed{K = \frac{D}{V_m^2}\left(\frac{V_m}{V_a} - 1\right)}$

Obviously, if V_m, V_a and D are given, then value of K and hence of α and β can be found (Ex. 43.2).

43.36. Relationship Between Principal Quantities in Quadrilateral Diagram

The diagram is shown in Fig. 43.12. Let β_c represent the retardation during coasting period. As before,

$$t_1 = V_1/\alpha,\ t_2 = (V_2 - V_1)/\beta_c \text{ and } t_3 = V_2/\beta$$

$$D = \text{area } OABC$$

$$= \text{area } OAD + \text{area } ABED + \text{area } BCE$$

$$= \frac{1}{2} V_1 t_1 + t_2\left(\frac{V_1 + V_2}{2}\right) + \frac{1}{2} V_2 t_3$$

$$= \frac{1}{2} V_1 (t_1 + t_2) + \frac{1}{2} V_2 (t_2 + t_3)$$

$$= \frac{1}{2}\, V_1 (t - t_3) + \frac{1}{2} V_2 (t - t_1)$$

$$= \frac{1}{2}\, t\,(V_1 + V_2) - \frac{V_1 t_1}{2} - \frac{V_1 t_3}{2}$$

$$= \frac{1}{2}\, t\,(V_1 + V_2) - \frac{1}{2}\, V_1 V_2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= \frac{1}{2}\, t\,(V_1 + V_2) - KV_1 V_2$$

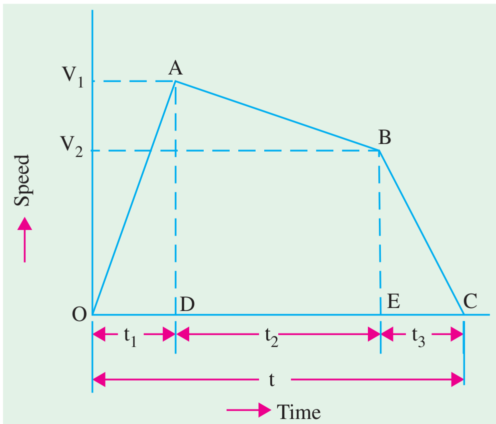


Fig. 43.12

where $K = \frac{1}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) = \frac{\alpha+\beta}{2\alpha\beta}$ Also, $\beta_c = \frac{(V_1 - V_2)}{t_2}$

$$\therefore \quad V_2 = V_1 - \beta_c\, t_2 = V_1 - \beta_c\,(t - t_1 - t_3)$$

$$= V_1 - \beta_c\left(t - \frac{V_1}{\alpha} - \frac{V_2}{\beta}\right) = V_1\,\beta_c\left(t - \frac{V_1}{\alpha}\right) + \beta_c\,\frac{V_2}{\beta}$$

or $$V_2\left(1 - \frac{\beta_c}{\beta}\right) = V_1 - \beta_c\left(t - \frac{V_1}{\alpha}\right) \qquad \therefore \quad V_2 = \frac{V_1 - \beta_c\,(t - V_1/\alpha)}{(1 - \beta_c/\beta)}$$

Example 43.1. *A suburban train runs with an average speed of 36 km/h between two stations 2 km apart. Values of acceleration and retardation are 1.8 km/h/s and 3.6 km/h/s.*

Compute the maximum speed of the train assuming trapezoidal speed/time curve.

(Electric Traction, Punjab Univ. 1994)

Solution. Now, V_a = 36 km/h = 36 × 5/18 = 10 m/s

$$\alpha = 1.8 \text{ km/h/s} = 1.8 \times 5/18 = 0.5 \text{ m/s}^2,\ \beta = 3.6 \text{ km/h/s} = 3.6 \times 5/18 = 1.0 \text{ m/s}^2$$

$$t = D/V_a = 2000/10 = 200 \text{ s} ;\ K = (\alpha + \beta)/2\alpha\beta = (0.5 + 1.0)/2 \times 0.5 \times 1 = 1.5$$

$$V_m = \frac{t - \sqrt{t^2 - 4KD}}{2K} = \frac{200 - \sqrt{200^2 - 4 \times 1.5 \times 2000}}{2 \times 1.5}$$

$$= 11 \text{ m/s} = 11 \times 18/5$$

$$= \mathbf{39.6 \text{ km/h}}$$

Electric traction provides high starting torque and low maintenance costs making it the best choice for trains

Example 43.2. *A train is required to run between two stations 1.5 km apart at a schedule speed of 36 km/h, the duration of stops being 25 seconds. The braking retardation is 3 km/h/s. Assuming a trapezoidal speed/time curve, calculate the acceleration if the ratio of maximum speed to average speed is to be 1.25*

(Elect. Power, Bombay Univ. 1980)

Solution. Here, D = 1500 m ; schedule speed = 36 km/h = 36 × 5/18 = 10 m/s

$$\beta = 3 \text{ km/h/s} = 3 \times 5/18 = 5/6 \text{ m/s}^2$$

Schedule time of run = 1500/10 = 150 s ; Actual time of run = 150 – 25 = 125 s

$$\therefore \quad V_a = 1500/125 = 12 \text{ m/s} ;\ V_m = 1.25 \times 12 = 15 \text{ m/s}$$

Now, $$K = \frac{D}{V_m^2}\left(\frac{V_m}{V_a} - 1\right) = \frac{1500}{15^2}\,(1.25 - 1) = \frac{5}{3}$$

Also, $$K = \frac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \quad \text{or} \quad \frac{5}{3} = \frac{1}{2}\left(\frac{1}{\alpha} + \frac{6}{5}\right)$$

$$\therefore \quad \alpha = 0.47 \text{ m/s}^2 = 0.47 \times 18/5 = \mathbf{1.7 \text{ km/h/s}}$$

Example 43.3. *Find the schedule speed of an electric train for a run of 1.5 km if the ratio of its maximum to average speed is 1.25. It has a braking retardation of 3.6 km/h/s, acceleration of 1.8 km/h/s and stop time of 21 second. Assume trapezoidal speed/time curve.*

Solution. $\alpha = 1.8 \times 5/18 = 0.5 \text{ m/s}^2$, $\beta = 3.6 \times 5/18 = 1.0 \text{ m/s}^2$

$D = 1.5 \text{ km} = 1500 \text{ m}$

$$K = \frac{1}{2}\left(\frac{1}{0.5}+\frac{1}{1}\right) = \frac{3}{2} \qquad \text{Now,} \quad K = \frac{D}{V_{m^2}}\left(\frac{V_m}{V_a}-1\right)$$

or $$V_m^2 = \frac{D}{K}\left(\frac{V_m}{V_a}-1\right) \qquad \therefore \quad V_m^2 = \frac{1500}{3/2}(1.25-1) = 250 \text{ ; } V_m = 15.8 \text{ m/s}$$

$V_a = V_m/1.25 = 15.8/1.25 = 12.6 \text{ m/s}$

Actual time of run = 1500/12.6 = 119 seconds

Schedule time = 119 + 21 = 140 second

$\therefore$ Schedule speed = 1500/140 = 10.7 m/s = **38.5 km/h**

Example 43.4. *A train runs between two stations 1.6 km apart at an average speed of 36 km/h. If the maximum speed is to be limited to 72 km/h, acceleration to 2.7 km/h/s, coasting retardation to 0.18 km/h/s and braking retardation to 3.2 km/h/s, compute the duration of acceleration, coasting and braking periods.*

Assume a simplified speed/time curve.

Solution. Given : $D = 1.6 \text{ km} = 1600 \text{ m}$, $V_a = 36 \text{ km/h} = 10 \text{ m/s}$

$V_1 = 72 \text{ km/h} = 20 \text{ m/s}$; $\alpha = 2.7 \text{ km/h/s} = 0.75 \text{ m/s}^2$

$\beta_c = 0.18 \text{ km/h/s} = 0.05 \text{ m/s}^2$; $\beta = 3.6 \text{ km/h/s} = 1.0 \text{ m/s}^2$

With reference to Fig. 43.12, we have

Duration of acceleration, $t_1 = V_1/\alpha = 20/0.75 = 27 \text{ s}$

Actual time of run, t = 1600/10 = **160 s**

Duration of braking, $t_3 = V_2/1.0 = V_2$ second

Duration of coasting, $t_2 = (V_1 - V_2)/\beta_c = (20 - V_2)/0.05 = (400 - 20\,V_2)$ second

Now, $t = t_1 + t_2 + t_3$ or $160 = 27 + (400 - 20\,V_2) + V_2$ $\quad \therefore \quad V_2 = 14 \text{ m/s}$

$\therefore$ $t_2 = (20 - 14)/0.05 = 120 \text{ s}$; $t_3 = 14/1.0 =$ **14 s**

43.37. Tractive Effort for Propulsion of a Train

The tractive effort (F_t) is the force developed by the traction unit at the rim of the driving wheels for moving the unit itself and its train (trailing load). The tractive effort required for train propulsion on a *level track* is

$$F_t = F_a + F_r$$

If gradients are involved, the above expression becomes

$$F_t = F_a + F_g + F_r \qquad \text{— for ascending gradient}$$
$$= F_a - F_g + F_r \qquad \text{— for descending gradient}$$

where F_a = force required for giving linear acceleration to the train
F_g = force required to overcome the effect of gravity
F_r = force required to overcome resistance to train motion.

(*a*) Value of F_a

If M is the dead (or stationary) mass of the train and a its linear acceleration, then

$$F_a = Ma$$

Since a train has rotating parts like wheels, axles, motor armatures and gearing etc., its *effective* (or accelerating) mass M_e is more (about 8 – 15%) than its stationary mass. These parts have to be given angular acceleration at the same time as the whole train is accelerated in the linear direction. Hence, $F_e = M_e a$

(*i*) If M_e is in kg and α in m/s^2, then $F_a = M_e\, a$ newton

(*ii*) If M_e is in tonne and α in km/h/s, then converting them into absolute units, we have

$$F_a = (1000\, M_e) \times (1000/3600)\, a = 277.8\, M_e\, a \text{ newton}$$

(*b*) Value of F_g

As seen from Fig. 43.13, $F_g = W \sin\theta = Mg \sin\theta$

In railway practice, gradient is expressed as the rise (in metres) a track distance of 100 m and is called percentage gradient.

$$\therefore \qquad \%\, G = \frac{BC}{AC/100} = 100\,\frac{BC}{AC} = 100 \sin\theta$$

Substituting the value of sin θ in the above equation, we get

$$F_g = Mg\, G/100 = 9.8 \times 10^{-2}\, MG$$

(*i*) When M is in kg, $F_g = 9.8 \times 10^{-2}\, MG$ newton

(*ii*) When M is given in tonne, then

$$F_g = 9.8 \times 10^{-2}\,(1000\, M)\, G = 98\, MG \text{ newton}$$

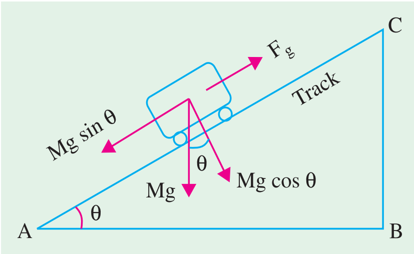


Fig. 43.13

(*c*) Value of F_r

Train resistance comprises all those forces which oppose its motion. It consists of mechanical resistance and wind resistance. Mechanical resistance itself is made up of internal and external resistances. The internal resistance comprises friction at journals, axles, guides and buffers etc. The external resistance consists of friction between wheels and rails and flange friction etc. Mechanical resistance is almost independent of train speed but depends on its weight. The wind friction varies directly as the square of the train speed.

If r is specific resistance of the train *i.e.* resistance offered per unit mass of the train, then $F_r = M.r$.

(*i*) If r is in newton per kg of train mass and M is the train mass in kg, then

$$F_r = M.r \text{ newton}$$

(*ii*) If r is in newton per tonne train mass (N/t) and M is in tonne (t), then

$$F_r = M \text{ tonne} \times r = M_r \text{ newton}^*$$

Hence, expression for total tractive effort becomes

$$F_t = F_a \pm F_g + F_r = (277.8\, \alpha\, M_e \pm 98\, MG + Mr) \text{ newton}$$

Please remember that here M is in tonne, α in km/h/s, G is in metres per 100 m of track length (*i.e.* % G) and r is in newton/tonne (N/t) of train mass.

The positive sign for F_g is taken when motion is along an ascending gradient and negative sign when motion is along a descending gradient.

43.38. Power Output from Driving Axles

If F_t is the tractive effort and ν is the train velocity, then

$$\text{output power} = F_t \times \nu$$

(*i*) If F_t is in newton and ν in m/s, then

$$\text{output power} = F_t \times \nu \text{ watt}$$

(*ii*) If F_t is in newton and ν is in km/h , then converting ν into m/s, we have

* If r is in kg (wt) per tonne train mass and M is in tonne, then $F_r = M$ tonne × (r × 9.8) newton/tonne = 9.8 Mr newton.

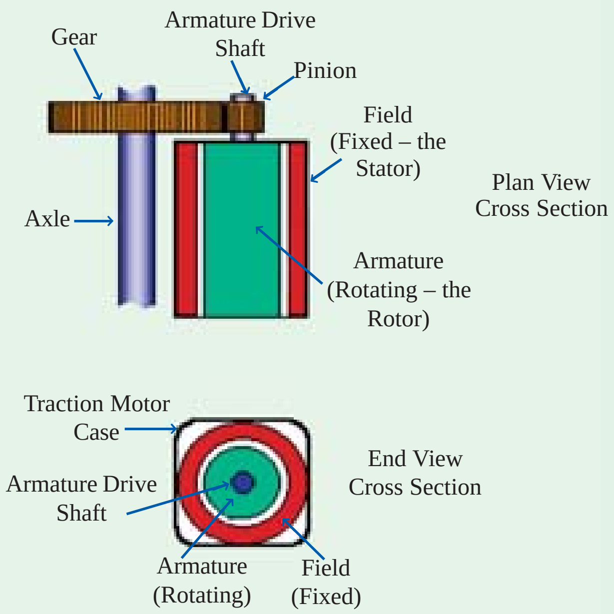


Diagram Shows how a DC motor drives the axle through a pinion and gearwheel

Traction motor schematic diagram

$$\text{output power} \;=\; F_t \times \left(\frac{1000}{3600}\right) \nu \text{ watt} = \frac{F_t\,\nu}{3600} \text{ kW}$$

If η is the efficiency of transmission gear, then power output of motors is

$$= F_t \,.\, \nu/\eta \text{ watt} \qquad \text{— } \nu \text{ in m/s}$$

$$= \frac{F_t\,\nu}{3600\,\eta} \text{ kW} \qquad \text{— } \nu \text{ in km/h}$$

43.39. Energy Output from Driving Axles

Energy (like work) is given by the product of power and time.

$$E \;=\; (F_t \times \nu\,) \times t = F_t \times (\nu \times t) = F_t \times D$$

where D is the distance travelled in the direction of tractive effort.

Total energy output from driving axles for the run is

$$E \;=\; \text{energy during acceleration} + \text{energy during free run}$$

As seen from Fig. 43.11

$$E \;=\; F_t \times \text{area } OAD + F_t' \times \text{area } ABED = F_t \times \frac{1}{2}\, V_m\, t_1 + F_t' \times \frac{1}{2}\, V_m\, t_2$$

where F_t is the tractive effort during accelerating period and F_t' that during free-running period. Incidentally, F_t will consist of all the three components given in Art. 43.37 whereas F_t' will consist of (98 MG + Mr) provided there is an ascending gradient.

43.40. Specific Energy Output

It is the energy output of the driving wheel expressed in watt-hour (Wh) per tonne-km (t-km) of the train. It can be found by first converting the energy output into Wh and then dividing it by the mass of the train in tonne and route distance in km.

Hence, unit of specific energy output generally used in railway work is : Wh/tonne-km (Wh/t-km).

43.41. Evaluation of Specific Energy Output

We will first calculate the total energy output of the driving axles and then divide it by train mass in tonne and route length in km to find the specific energy output. It will be presumed that :

(*i*) there is a gradient of G throughout the run and

(*ii*) power remains ON upto the end of free run in the case of trapezoidal curve (Fig. 43.11) and upto the accelerating period in the case of quadrilateral curve (Fig. 43.12).

Now, output of the driving axles is used for the following purposes :

1. for accelerating the train
2. for overcoming the gradient
3. for overcoming train resistance.

(*a*) Energy required for train acceleration (E_a)

As seen from trapezoidal diagram of Fig. 43.11,

$$E_a = F_a \times \text{distance } OAD = 277.8\,\alpha\, M_e \times \frac{1}{2}\, V_m.t_1 \text{ joules}$$

$$= 277.8\,\alpha\, M_e \times \frac{1}{2}\, V_m \times \frac{V_m}{\alpha} \text{ joules} \qquad \left(\text{फ} t_1 = \frac{V_m}{\alpha}\right)$$

$$= 277.8\,\alpha\, M_e \times \left[\frac{1}{2} \cdot \frac{V_m \times 1000}{3600} \times \frac{V_m}{\alpha}\right] \text{ joules}$$

It will be seen that since V_m is in km/h, it has been converted into m/s by multiplying it with the conversion factor of (1000/3600). In the case of (V_m /*t*), conversion factors for V_m and a being the same, they cancel out. Since 1 Wh = 3600 J.

$$\therefore \qquad E_a = 277.8\,\alpha\, M_e \left[\frac{1}{2} \cdot \frac{V_m \times 1000}{3600} \times \frac{V_m}{\alpha}\right] \text{Wh} = 0.01072\, \frac{V_m^2}{M_e} \text{ Wh}$$

(*b*) Energy required for over coming gradient (E_g)

$$\boldsymbol{E_g = F_g \times D'}$$

where '*D*′' is the ***total distance over which power remains*** ON. Its maximum value equals the distance represented by the area *OABE* in Fig. 43.11 *i.e.* from the start to the end of free-running period in the case of trapezoidal curve [as per assumption (*i*) above].

Substituting the value of F_g from Art. 43.37, we get

$$E_g = 98\, MG.\,(1000\, D') \text{ joules} = 98{,}000\, MGD' \text{ joules}$$

It has been assumed that *D*′ is in km.

When expressed in Wh, it becomes

$$E_g = 98{,}000\, MGD' \frac{1}{3600} \text{ Wh} = 27.25\, MGD' \text{ Wh}$$

(*c*) Energy required for overcoming resistance (E_r)

$$E_r = F_r \times D' = M\,.\,r \times (1000\, D') \text{ joules} \qquad \textbf{— } \boldsymbol{D'} \textbf{ in km}$$

$$= \frac{1000\, Mr\, D'}{3600} \text{ Wh} = 0.2778\, Mr\, D' \text{ Wh} \qquad \textbf{— } \boldsymbol{D'} \textbf{ in km}$$

∴ total energy output of the driving axles is

$$E = E_a + E_g + E_r$$
$$= (0.01072\, V_m^2/M_e + 27.25\, MGD' + 0.2778\, Mr\, D' \text{ Wh}$$

Specific energy output

$$E_{spo} = \frac{E}{M \times D} \qquad \text{— } D \text{ is the } total \text{ run length}$$
$$= \left(0.01072 \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 27.25\, G\, \frac{D'}{D} + 0.2778\, r\, \frac{D'}{D}\right) \text{Wh/t-km}$$

It may be noted that *if there is no gradient*, then

$$E_{spo} = \left(0.01072 \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 0.2778\, r\, \frac{D'}{D}\right) \text{Wh/t-km}$$

Alternative Method

As before, we will consider the trapezoidal speed/time curve. Now, we will calculate energy output not **force-wise** but **period-wise**.

(*i*) Energy output during accelerating period

$$E_a = F_t \times \text{distance travelled during accelerating period}$$
$$= F_t \times \text{area } OAD \qquad \text{—Fig. 43.11}$$
$$= F_t \times \frac{1}{2} V_m\, t_1 = \frac{1}{2} F_t \cdot V_m \cdot \frac{V_m}{\alpha}$$
$$= \frac{1}{2} \cdot F_t \left(\frac{1000}{3600} \cdot V_m\right) \cdot \frac{V_m}{\alpha} \text{ joules}$$
$$= \frac{1}{2} \cdot F_t \left(\frac{1000}{3600} \cdot V_m\right) \cdot \frac{V_m}{\alpha} \cdot \frac{1}{3600} \text{ Wh}$$

Substituting the value of F_t, we get

$$E_a = \frac{1000}{(3600)^2} \cdot \frac{V_m^2}{2\alpha} (277.8\, \alpha\, M_e + 98\, MG + Mr) \text{ Wh}$$

It must be remembered that during this period, ***all the three forces are at work*** (Art. 43.37)

(*ii*) Energy output during free-running period

Here, work is required only against two forces *i.e.* gravity and resistance (as mentioned earlier).

Energy $\quad E_{fr} = F_t' \times \text{area } ABED \qquad$ —Fig. 43.11

$$= F_t' \times (V_m \times t_2) = F_t' \times \left(\frac{1000}{3600} V_m\right) \cdot t_2 \text{ joules}$$
$$= F_t' \times \left(\frac{1000}{3600} V_m\right) \times t_2 \times \frac{1}{3600} \text{ Wh} = \left(\frac{1000}{3600}\right) F_t' \times V_m\, t_2 \cdot \frac{1}{3600} \text{ Wh}$$
$$= \left(\frac{1000}{3600}\right) \cdot F_t' \times D_{fr} \text{ Wh} = \left(\frac{1000}{3600}\right) (98\, MG + Mr)\, D_{fr} \text{ Wh}$$

where D_{fr} is the distance in km travelled during the free-running period*

Total energy required is the sum of the above two energies.

$$\therefore \quad E = E_a + E_{fr}$$
$$= \frac{1000}{(3600)^2} \frac{V_m^2}{2\,\alpha} (277.8\, \alpha \,.\, M_e + 98\, MG + Mr) + \frac{1000}{3600} (98\, MG + Mr)\, D_{fr} \text{ Wh}$$

---

* D_{fr} = velocity in km/h × time in hours
$= V_m \times (t_2/3600)$ because times are always taken in seconds.

$$= \frac{1000}{(3600)^2}\frac{V_m^2}{2\,\alpha}\,277.8\,\alpha\,M_e + \frac{1000}{(3600)^2}\cdot\frac{V_m^2}{2\,\alpha}\,(98\,MG + Mr) + \frac{1000}{3600}\,(98\,MG + Mr)\,.\,D_{fr}\text{ Wh}$$

$$= 0.01072\,V_m^2\,.\,M_e + \frac{1000}{3600}\,(98\,MG + Mr)\left(\frac{V_m^2}{2\alpha\times 3600} + D_{fr}\right)\text{Wh}$$

Now, $$\frac{V_m^2}{2\alpha\times 3600} = \frac{1}{2}\left(\frac{V_m}{3600}\right)\cdot\frac{V_m}{\alpha} = \frac{1}{2}\left(\frac{V_m}{3600}\right)\cdot t_1$$

$= $ distance travelled during accelerating period *i.e.* D_a

$$\therefore \qquad E = 0.01072\,V_m^2\,.\,M_e + \frac{1000}{3600}\,(98\,MG + Mr)\,(D_a + D_{fr})\text{ Wh}$$

$$= 0.01072\,V_m^2\,.\,M_e + (27.25\,MG + 0.2778\,Mr)\,D'\text{ Wh}$$

It is the same expression as found above.

43.42. Energy Consumption

It equals the total energy input to the traction motors from the supply. It is usually expressed in Wh which equals 3600 J. It can be found by dividing the energy output of the driving wheels with the combined efficiency of transmission gear and motor.

$$\therefore \qquad \text{energy consumption} = \frac{\text{output of driving axles}}{\eta_{motor}\times\eta_{gear}}$$

43.43. Specific Energy Consumption

It is the energy consumed (in Wh) per tonne mass of the train per km length of the run.

Specific energy consumption,

$$E_{spc} = \frac{\text{total energy consumed in Wh}}{\text{train mass in tonne}\times\text{run length in km}} = \frac{\text{specific energy output}}{\eta}$$

where η = overall efficiency of transmission gear and motor = $\eta_{gear}\times\eta_{motor}$

As seen from Art. 43.41, specific energy consumption is

$$E_{spc} = \left(0.01072\,.\,\frac{V_{m^2}}{\eta D}\,.\,\frac{M_e}{M} + 27.25\,\frac{G}{\eta}\,.\,\frac{D'}{D} + 0.2778\,\frac{r}{\eta}\,.\,\frac{D'}{D}\right)\text{Wh/t-km}$$

If no gradient is involved, then specific enrgy consumption is

$$E_{spc} = \left(0.01072\,.\,\frac{V_{m^2}}{\eta D}\,.\,\frac{M_e}{M} + 0.2778\,\frac{r}{\eta}\,.\,\frac{D'}{D}\right)\text{Wh/t-km}$$

The specific energy consumption of a train running at a given schedule speed is influenced by **1.** Distance between stops **2.** Acceleration **3.** Retardation **4.** Maximum speed **5.** Type of train and equipment **6.** Track configuration.

43.44. Adhesive Weight

It is given by the total weight carried on the driving wheels. Its value is $W_a = x\,W$, where W is dead weight and x is a fraction varying from 0.6 to 0.8.

43.45. Coefficient of Adhesion

Adhesion between two bodies is due to interlocking of the irregularities of their surfaces in contact. The adhesive weight of a train is ***equal to the total weight to be carried on the driving***

wheels. It is less than the dead weight by about 20 to 40%.

If $x = \dfrac{\text{adhesive weight, } W_a}{\text{dead weight } W}$, then, $W_a = x\,W$

Let, F_t = tractive effort to slip the wheels

or

= maximum tractive effort possible without wheel slip

Coefficient of adhesion, $\mu_a = F_t/W_a$

$\therefore \quad F_t = \mu_a W_a = \mu_a xW = \mu_a x Mg$

If M is in tonne, then

$$F_t = 1000 \times 9.8\, x\, \mu_a M = 9800\, \mu_a x M \text{ newton}$$

It has been found that tractive effort can be increased by increasing the motor torque but only upto a certain point. Beyond this point, any increase in motor torque does not increase the tractive effort but merely causes the driving wheels to slip. It is seen from the above relation that for increasing F_t, it is not enough to increase the kW rating of the traction motors alone but the weight on the driving wheels has also to be increased.

Adhesion also plays an important role in braking. If braking effort exceeds the adhesive weight of the vehicle, skidding takes place.

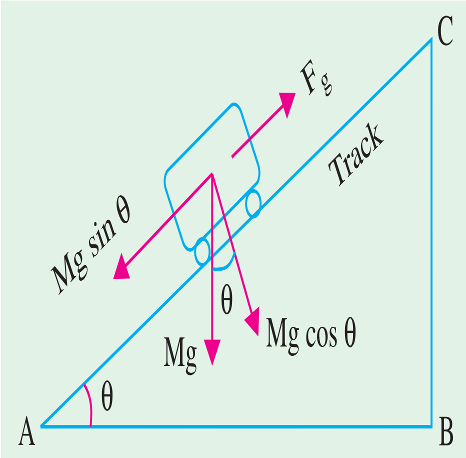


Fig. 43.14

43.46. Mechanism of Train Movement

The essentials of driving mechanism in an electric vehicle are illustrated in Fig. 43.14. The armature of the driving motor has a pinion which meshes with the gear wheel keyed to the axle of the driving wheel. In this way, motor torque is transferred to the wheel through the gear.

Let, T = torque exerted by the motor

F_1 = tractive effort at the pinion

F_t = tractive effort at the wheel

γ = gear ratio

Here, d_1, d_2 = diameters of the pinion and gear wheel respectively

D = diameter of the driving wheel

η = efficiency of power transmission from the motor to driving axle

Now, $T = F_1 \times d_1/2$ or $F_1 = 2T/d_1$

Tractive effort transferred to the driving wheel is

$$F_t = \eta F_1\left(\frac{d_2}{D}\right) = \eta \cdot \frac{2T}{d_1}\left(\frac{d_2}{D}\right) = \eta\, T\left(\frac{2}{D}\right)\left(\frac{d_2}{d_1}\right) = 2\,\gamma\eta\,\frac{T}{D}$$

For obtaining motion of the train without slipping, $F_t \le \mu_a W_a$ where μ_a is the coefficient of adhesion (Art. 43.45) and W_a is the adhesive weight.

Example 43.5. *The peripheral speed of a railway traction motor cannot be allowed to exceed 44 m/s. If gear ratio is 18/75, motor armature diameter 42 cm and wheel diameter 91 cm, calculate the limiting value of the train speed.*

Solution. Maximum number of revolutions per second made by armature

$$= \frac{\text{armature velocity}}{\text{armature circumference}} = \frac{44}{0.42\,\pi} = \frac{100}{3} \text{ rps.}$$

Maximum number of revolutions per second made by the driving wheel

$$= \frac{100}{3} \times \frac{18}{75} = 8 \text{ rps.}$$

Maximum distance travelled by the driving wheel in one second

$$= 8 \times 0.91\,\pi \text{ m/s} = 22.88 \text{ m/s}$$

Hence, limiting value of train speed

$$= 22.88 \text{ m/s} = 22.88 \times 18/5 = \mathbf{82 \text{ km/h}}$$

Example 43.6. *A 250-tonne motor coach driven by four motors takes 20 seconds to attain a speed of 42 km/h, starting from rest on an ascending gradient of 1 in 80. The gear ratio is 3.5, gear efficiency 92%, wheel diameter 92 cm train resistance 40 N/t and rotational inertia 10 percent of the dead weight. Find the torque developed by each motor.*

Solution. $F_t = (277.8 \times M_e a + 98 \text{ MG} + \text{Mr})$ newton

Now, $\alpha = V_m/t_1 = 42/20 = 2.1$ km/h/s Since gradient is 1 in 80, it becomes 1.25 in 100. Hence, percentage gradient $G = 1.25$. Also, $M_e = 1.1\,M$. The tractive effort at the driving wheel is

$$F_t = 277.8 \times (1.1 \times 250) \times 2.1 + 98 \times 250 \times 1.25 + 250 \times 40$$
$$= 160{,}430 + 30{,}625 + 10{,}000 = 201{,}055 \text{ N}$$

Now, $F_t = 2\gamma\eta T/D$ or $201{,}055 = 2 \times 3.5 \times 0.92 \times T/0.92$ $\therefore T = 28{,}744$ N–m

Torque developed by each motor = 28,744/4 = **7,186 N–m**

Example 43.7. *A 250-tonne motor coach having 4 motors, each developing a torque of 8000 N-m during acceleration, starts from rest. If up-gradient is 30 in 1000, gear ratio 3.5, gear transmission efficiency 90%, wheel diameter 90 cm, train resistance 50 N/t, rotational inertia effect 10%, compute the time taken by the coach to attain a speed of 80 km/h.*

If supply voltage is 3000 V and motor efficiency 85%, calculate the current taken during the acceleration period.

Solution. Tractive effort (Art. 43.46) at the wheel

$$= 2\gamma\eta T/D = 2 \times 3.5 \times 0.9 \times (8000 \times 4)/0.9 = 224{,}000 \text{ N}$$

Also, $F_t = (277.8\, a\, M_e + 98\, MG + Mr)$ newton

$$= (277.8 \times (1.1 \times 250) \times a + 98 \times 250 \times 3 + 250 \times 50 \text{ N}$$
$$= (76{,}395\, a + 86{,}000) \text{ N}$$

Equating the two expression for tractive effort, we get

$$224{,}000 = 76{,}395\, a + 86{,}000\ ;\ a = 1.8 \text{ km/h/s}$$

Time taken to achieve a speed of 80 km/h is

$$t_1 = V_m/a = 80/1.8 = \mathbf{44.4 \text{ second}}$$

Power taken by motors (Art. 41.36) is

$$= \frac{F_t \times v}{\eta} = \frac{F_t \times V_m}{\eta} = F_t \cdot \left(\frac{1000}{3600}\right) \cdot \frac{V_m}{\eta} \text{ watt}$$
$$= 22{,}000 \times 0.2778 \times 80/0.85 = 58.56 \times 10^5 \text{ W}$$

Total current drawn $= 55.56 \times 10^5/3000 = 1952$ A

Current drawn/motor $= 1952/4 =$ **488 A.**

Example 43.8. *A goods train weighing 500 tonne is to be hauled by a locomotive up an ascending gradient of 2% with an acceleration of 1 km/h/s. If coefficient of adhesion is 0.25, train resistance 40 N/t and effect of rotational inertia 10%, find the weight of locomotive and number of axles if load is not to increase beyond 21 tonne/axle.*

Solution. It should be clearly understood that a train weighing 500 tonne has a mass of 500 (Art. 43.33).

Tractive effort required is

$$F_t = (277.8\, a\, M_e + 98\, MG + Mr) \text{ newton} = M\left(277.8\, a \, . \frac{M_e}{M} + 98\, G + r\right) \text{ newton}$$

$$= M\,(277.8 \times 1 \times 1.1 + 98 \times 2 + 40) = 541.6\, M \text{ newton}$$

If M_L is the mass of the locomotive, then

$$F_t = 541.6\,(M + M_L) = 541.6\,(500 + M_L) \text{ newton}$$

Maximum tractive effort (Art. 43.45) is given by

$$F_t = 1000\, \mu_a\, M_L \, . \, g = 1000 \times 0.25\, M_L \times 9.8 \qquad \text{— } x = 1$$

$\therefore \quad 541.6\,(500 + M_L) = 1000 \times 0.25\, M_L \times 9.8 \qquad \therefore \quad M_L =$ **142 tonne**

Hence, weight of the locomotive is 142 tonne. Since, weight per axle is not to exceed 21 tonne, the number of axles required is = 142/21 = 7.

Example 43.9. *An electric locomotive weighing 100 tonne can just accelerate a train of 500 tonne (trailing weight) with an acceleration of 1 km/h/s on an up-gradient of 0.1%. Train resistance is 45 N/t and rotational inertia is 10%. If this locomotive is helped by another locomotive of weight 120 tonne, find :*

(*i*) *the trailing weight that can now be hauled up the same gradient under the same conditions.*

(*ii*) *the maximum gradient, if the trailing hauled load remains unchanged.*

Assume adhesive weight expressed as percentage of total dead weight as 0.8 for both locomotives. **(Utilization of Elect. Power ; AMIE, Summer)**

Solution. Dead weight of the train and locomotive combined = (100 + 500) = 600 tonne. Same is the value of the dead mass.

$$\therefore \quad F_t = (277.8a\, M_e + 98\, MG + Mr) \text{ newton}$$

$$= 277.8 \times 1 \times (1.1 \times 600) + 98 \times 600 \times 0.1 + 600 \times 45 = 216{,}228 \text{ N}$$

Maximum tractive effort (Art. 43.45) of the first locomotive

$$= 9800x\ \mu_a M_L = 9800 \times 0.8 \times \mu_a \times 1000 = 784{,}000\ \mu_a$$

$$\therefore \quad 784{,}000\mu_a = 216{,}288\ ; \qquad \mu_a = 0.276$$

With two locomotive, $M_L' = (100 + 120) = 220$ tonne

$$\therefore \quad F_t = 9800\ x\ \mu_a M_L' = 9800 \times 0.8 \times 0.276 \times 220 = 476{,}045 \text{ N}$$

(*i*) Let trailing load which the two combined locomotives can haul be M tonne. In that case, total dead mass becomes $M = (100 + 120 + M) = (220 + M)$ tonne. Tractive effort required is

$$= (277.8\, M_e' + 98\, M'G + M'r) \text{ newton}$$

$$= M'\,(277.8 \times 1 \times 1.1 + 98 \times 0.1 + 45) = 360.4\, M' \text{ newton}$$

$\therefore \quad 360.4\, M' = 476{,}045;\ M' = 1321$ tonne $\quad \therefore$ trailing load, $M = 1321 - 220 =$ **1101 tonne**

(*ii*) Total hauled load = 500 + 100 + 120 = 720 tonne

Let G be the value of maximum percentage gradient. Then

$$F_t = (277.8\, a\, M_e + 98MG + Mr) \text{ newton} = M\left(277.8\, a\, \frac{M_e}{M} + 98G + r\right) \text{ newton}$$

$= 720 (277.8 \times 1 \times 1.1 + 98G + 45)$ newton $= (252{,}418 + 70{,}560\ G)$ newton

Equating it with the combined tractive effort of the two locomotive as calculated above, we have,

$476{,}045 = 252{,}418 + 70{,}560\ G \qquad \therefore \quad G =$ **3.17 percent**

Example 43.10. *The average distance between stops on a level section of a railway is 1.25 km. Motor-coach train weighing 200 tonne has a schedule speed of 30 km/h, the duration of stops being 30 seconds. The acceleration is 1.9 km/h/s and the braking retardation is 3.2 km/h/s. Train resistance to traction is 45 N/t. Allowance for rotational inertia is 10%. Calculate the specific energy output in Wh/t-km. Assume a trapezoidal speed/time curve.* **(Elect. Power, Bombay Univ.)**

Solution. $\alpha = 1.9 \times 5/18 = 9.5/18\ \text{m/s}^2$: $\quad \beta = 3.2 \times 5/18 = 8/9\ \text{m/s}^2$

$K = (\alpha + \beta)/2\alpha\beta = 1.5$; $\quad D = 1.25\ \text{km} = 1250\ \text{m}$

Schedule time $= 1.25 \times 3600/30 = 150$ s. Running time $= 150 - 30 = 120$ s

$$V_m = \frac{t - \sqrt{t^2 - 4KD}}{2K} = \frac{120 - \sqrt{120^2 - 4 \times 1.5 \times 1250}}{2 \times 1.5} = 10.4\ \text{m/s} = \mathbf{37.4\ km/h}$$

Braking distance $D = V_m^2/2\beta = 10.42/2 \times (8/9) = 0.06$ km

$\therefore \quad D' = D -$ braking distance $= 1.25 - 0.06 = 1.19$ km

$$\text{Specific energy output} = 0.01072\ \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 0.2778\ r\ \frac{D'}{D}$$

$$= 0.01072 \times \frac{37.4^2}{1.25} \times 1.1 + 0.2778 \times 50 \times \frac{1.19}{1.25}\ \text{Wh/t-km}$$

$= 16.5 + 13.2 =$ **29.7 Wh/t-km**

Example. 43.11. *A 300-tonne EMU is started with a uniform acceleration and reaches a speed of 40 km/h in 24 seconds on a level track. Assuming trapezoidal speed/time curve, find specific energy consumption if rotational inertia is 8%, retardation is 3 km/h/s, distance between stops is 3 km, motor efficiency is 0.9 and train resistance is 40 N/tonne.*

(Elect. Traction, AMIE Summer)

Solution. First of all, let us find D' – the distance upto which energy is consumed from the supply. It is the distance travelled upto the end of free-running period. It is equal to the total distance minus the distance travelled during braking.

Braking time, $\quad t_2 = V_m/\beta = 40/3 = 13.33$ second

Distance travelled during braking period

$$= \frac{1}{2} V_m\, t_3 = \frac{1}{2} \times 40 \times \left(\frac{13.33}{3600}\right) = 0.074\ \text{km}$$

$\therefore \quad D' = D -$ braking distance $= 3 - 0.074 = 2.926$ km

Since, $M_e/M = 1.08$, using the relation derived in Art. 43.43, we get the value of specific energy consumption as

$$= \left(0.01072\ \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 0.2778\ \frac{r}{\eta}\ \frac{D'}{D}\right) \text{Wh/t-km}$$

$$= \left(0.01072 \times \frac{40^2}{0.9 \times 3} \times 1.08 + 0.2778 \times \frac{49}{0.9} \times \frac{2.926}{3}\right) = \mathbf{21.6\ Wh/t\text{-}km.}$$

Example 43.12. *An electric train accelerates uniformly from rest to a speed of 50 km/h in 25 seconds. It then coasts for 70 seconds against a constant resistance of 60 N/t and is then braked to rest with uniform retardation of 3.0 km/h/s in 12 seconds. Compute*

(i) uniform acceleration *(ii) coasting retardation*

(iii) *schedule speed if station stops are of 20-second duration*

Allow 10% for rotational inertia. How will the schedule speed be affected if duration of stops is reduced to 15 seconds, other factors remaining the same ?

Solution. *(i)* As seen from Fig. 43.15, $\alpha = V_1/t_1 = 50/25 =$ **2 km/h/s**

(ii) The speeds at points *B* and *C* are connected by the relation

$$0 = V_2 + \beta t_3 \quad \text{or} \quad 0 = V_2 + (-3) \times 12 \qquad \therefore \quad V_2 = 36 \text{ km/h}$$

Coasting retardation, $\beta_c = (V_2 - V_1)/t_2 = (36 - 50)/70 = -$ **0.2 km/h/s**

(iii) Distance travelled during acceleration

$$= \frac{1}{2} V_1 t_1 = \frac{1}{2} \times 50 \frac{\text{km}}{\text{h}} \times \frac{25}{3600} \text{ h}$$

$$= 0.174 \text{ km}$$

Distance travelled during coasting can be found from the relation

$$V_{22} - V_{12} = 2 \ \beta_c D \qquad \text{or}$$

$$D = (362 - 502)/2 \times -0.2 \times 3600$$

$$= 0.836 \text{ km}$$

Distance covered during braking

$$= \frac{1}{2} V_2 t_3 = \frac{1}{2} \times 36 \frac{\text{km}}{\text{h}} \times \frac{12}{3600} \text{ h}$$

$$= 0.06 \text{ km}$$

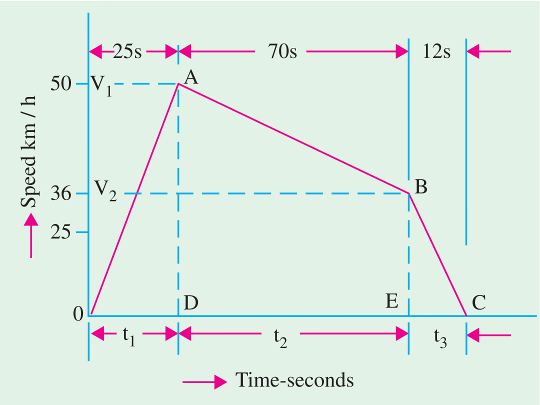


Fig. 43.15

Total distance travelled from start to stop

$$= 0.174 + 0.836 + 0.06 = 1.07 \text{ km}$$

Total time taken including stop time

$$= 25 + 70 + 12 + 20 = 127 \text{ second}$$

Schedule speed $= 1.07 \times 3600/127 =$ **30.3 km/s**

Schedule speed with a stop of 15 s is $= 1.07 \times 3600/122 =$ **31.6 km/h**

Example 43.13. *A 350-tonne electric train runs up an ascending gradient of 1% with the following speed/time curves :*

1. *uniform acceleration of 1.6 km/h/s for 25 seconds*

2. *constant speed for 50 seconds* ***3.*** *coasting for 30 seconds*

4. *braking at 2.56 km/h/s to rest.*

Compute the specific energy consumption if train resistance is 50 N/t, effect of rotational inertia 10%, overall efficiency of transmission gear and motor, 75%.

Solution. As seen from Fig. 43.16, $V_1 = \alpha \, . \, t_1 = 1.6 \times 25 = 40$ km/h

Tractive force during coasting is

$$F_t = (98 \, MG + M.r)$$

$$= M \, (98 \times 1 + 50)$$

$$= 148 \, M \text{ newton}$$

Also, $F_t = 277.8 \, M_e \, \beta_c$ during coasting. Equating the two expressions, we get $277.8 \, M_e$

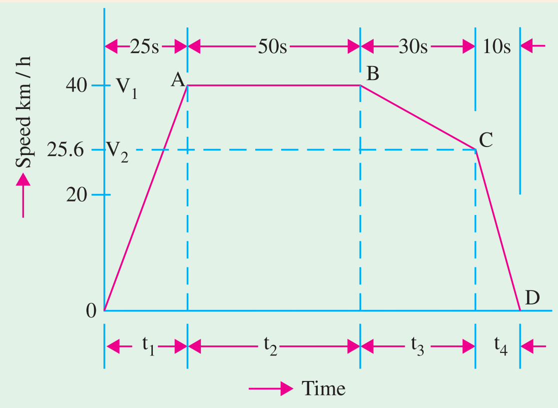


Fig. 43.16

. $\beta_c = 148\,M$

$$\therefore \beta_c = \frac{148}{277.8} \times \frac{M}{M_e} = \frac{148}{277.8} \times \frac{1}{1.1};$$

$$\beta_c = 0.48 \text{ km/h/s}$$

Now, $V_2 = V_1 + \beta_c t_3$

$= 40 + (-0.48) \times 30$

$= 25.6$ km/h

$t_4 = V_2/\beta = 25.6/2.56 = 10$ second

Distance travelled during acceleration period $= \frac{1}{2} \times 40 \frac{\text{km}}{\text{h}} \times \frac{25}{3600}\text{ h} = 0.139$ km

Distance travelled during constant speed period is $= V_1 \times t_2 = 40 \times 50/3600 = 0.555$ km

Distance travelled during coasting $= \left(\frac{V_1 + V_2}{2}\right) \times t_3 = \frac{40 + 25.6}{2} \times \frac{30}{3600} = 0.273$ km

Distance travelled during braking $= \frac{1}{2} V_2 t_4 = \frac{1}{2} \times 25.6 \times \frac{10}{3600} = 0.035$ km

Total distance between stops $= 0.139 + 0.555 + 0.273 + 0.035 = 1.002$ km

Distance travelled during acceleration and free-running period is

$$D' = 0.139 + 0.555 = 0.694 \text{ km}$$

Specific energy consumption (Art. 43.43) is

$$= \left(0.01072 \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 27.25 \frac{G}{\eta} \cdot \frac{D'}{D} + 0.2778 \frac{r}{\eta} \cdot \frac{D'}{D}\right) \text{ Wh/t-km}$$

$$= \left(0.01072 \times \frac{40^2}{0.75 \times 1.002} \times 1.1 + 27.25 \times \frac{1}{0.75} \times \frac{0.694}{1.002} + 0.2778 \times \frac{50}{0.75} \times \frac{0.694}{1.002}\right)$$

$= 25.1 + 25.2 + 12.8 =$ **63.1 Wh/t-km**

Example 43.14. *An ore-carrying train weighing 5000 tonne is to be hauled down a gradient of 1 : 50 at a maximum speed of 30 km/h and started on a level track at an acceleration of 0.29 km/h/s. How many locomotives, each weighing 75 tonne, will have to be employed ?*

Train resistance during starting = 29.4 N/t, Train resistance at 30 km/h = 49 N/t

Coefficient of adhesion = 0.3, Rotational inertia = 10%

(Utilization of Elect. Power, AMIE)

Solution. Downward force due to gravity

$= Mg \sin\theta = (5000 \times 1000) \times 9.8 \times 1/50 = 980{,}000$ N

Train resistance $= 49 \times 5000 = 245{,}000$ N

Braking force to be supplied by brakes $= 980{,}000 - 245{,}000 = 735{,}000$ N

Max. braking force which one locomotive can provide

$= 1000\ \mu_a Mg$ newton —M in tonne

$= 1000 \times 0.3 \times 75 \times 9.8 = 220{,}500$ N

No. of locomotives required for braking $= 735{,}000/220{,}500 = 3.33$

Since fraction is meaningless, it means that 4 locomotives are needed.

Tractive effort required to haul the train on level track

$= (277.8\ \alpha\ M_e + Mr)$ newton

$= 277.8 \times (5000 \times 1.1) \times 0.29 + 5000 \times 29.4 = 590{,}090$ N

No. of locomotives required = 590,090/220,500 = 2.68 ≅ **3**

It means that **4 locomotives** are enough to look after braking as well as starting.

Example 43.15. *A 200-tonne electric train runs according to the following quadrilateral speed/time curve:*

1. *uniform acceleration from rest at 2 km/h/s for 30 seconds*

2. *coasting for 50 seconds*

3. *duration of braking : 15 seconds*

If up-gradient is = 1%, train resistance = 40 N/t, rotational inertia effect = 10%, duration of stops = 15 s and overall efficiency of gear and motor = 75%, find

(i) *schedule speed* ***(ii)*** *specific energy consumption* ***(iii)*** *how will the value of specific energy consumption change if there is a down-gradient of 1% rather than the up-gradient ?*

(Electric Traction Punjab Univ. 1993)

Solution. $V_1 = \alpha.\ t_1 = 2 \times 30 = 60$ km/h/s

During coasting, gravity component and train resistance will cause coasting retardation β_c.

Retarding force = $(98\ MG + Mr)$ newton = $(98 \times 200 \times 1.0 + 200 \times 40) = 27{,}600$ N

As per Art, 43.37, $277.8\ M_e.\ \beta_c = 27{,}600$ or $277.8 \times (200 \times 1.1) \times \beta_c = 27{,}600$

$\therefore \quad \beta_c = 0.45$ km/h/s

Now, $V_2 = V_1 + \beta_c\, t_2$ or $V_2 = 60 + (-0.45) \times 50 = 37.5$ km/h

Braking retardation $\beta = V_2/t_3 = 37.5/15 = 2.5$ km/h/s

Distance travelled during acceleration (area *OAD* in Fig. 43.17)

$$= \frac{1}{2} V_1 t_1 = \frac{1}{2} \times 60 \times \left(\frac{30}{3600}\right) = 0.25 \text{ km}$$

Distance travelled during coasting

$$= \text{area } ABED = \left(\frac{V_1 + V_2}{2}\right) \times t_2 = \left(\frac{60 + 37.5}{2}\right) \times \frac{50}{3600} = 0.677 \text{ km}$$

Distance travelled during braking

$$= \text{area } BCE = \frac{1}{2} V_2 t_3 = \frac{1}{2} \times 37.5 \times \frac{15}{3600} = 0.078 \text{ km}$$

Total distance travelled, $D = 0.25 + 0.677 + 0.078 = 1.005$ km

Total schedule time $= 30 + 50 + 15 + 15 = 110$ s

(i) $\therefore$ Schedule speed $= \dfrac{1.005}{110/3600} =$ **32.9 km/h**

(ii) As per Art. 43.43, specific energy consumption

$$= \left(0.01072 \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 27.25 \frac{G}{\eta} \cdot \frac{D'}{D} + 0.2778 \cdot \frac{r}{\eta} \cdot \frac{D'}{D}\right) \text{Wh/t-km}$$

$$= \left(0.01072 \times \frac{60^2}{0.75 \times 1.005} \times 1.1 + 27.25 \times \frac{1}{0.75} \times \frac{0.25}{1.005} + 0.2778 \times \frac{40}{0.75} \times \frac{0.25}{1.005}\right) \text{Wh/t-km}$$

(iii) the speed/time curve for this case is shown in Fig. 43.18. As before, $V_1 = 60$ km/h. Here, we will take $G = -1\%$

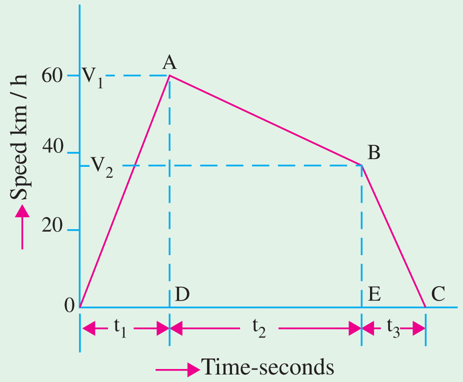


Fig. 43.17

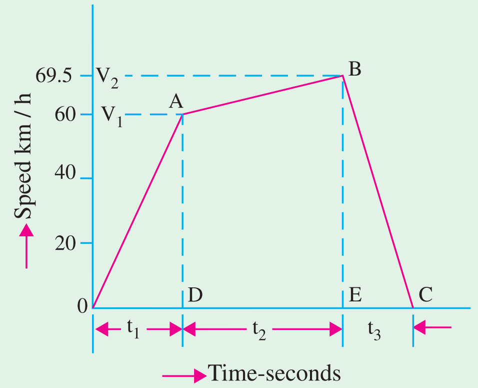


Fig. 43.18

∴ Retarding force = (98 *MG* + *Mr*) newton = 98 × 200 × (–1.0) + 200 × 40 = –11,600 N

The negative sign indicates that instead of being a retarding force, it is, in fact, an accelerating force. If α_c is the acceleration produced, then

$$11{,}600 = 277.8 \times (200 \times 1.1) \times \alpha_c$$

$$\alpha_c = 0.19 \text{ km/h/s}$$

Also, $$V_2 = V_1 + \alpha_c t_2 = 60 + 0.19 \times 50 = 69.5 \text{ km/h}$$

$$\beta = V_2/t_3 = 69.5/15 = 4.63 \text{ km/h/s}$$

Distance travelled during acceleration = 0.25 km —as before

Distance travelled during coasting $= \dfrac{60 + 69.5}{2} \times \dfrac{50}{3600} = 0.9$ km

Distance travelled during braking $= \dfrac{1}{2} \times 69.5 \times \dfrac{15}{3600} = 0.145$ km

∴ $$D = 0.25 + 0.9 + 0.145 = 1.295 \text{ km}$$

Hence, specific energy consumption is

$$= \left(0.01072 \times \frac{60^2}{0.75 \times 1.295} \times 1.1 - 27.25 \times \frac{1}{0.75} \times \frac{0.25}{1.295} + 0.2778 \times \frac{40}{0.75} \times \frac{0.25}{1.295}\right) \text{Wh/t-km}$$

$$= 43.7 - 7.01 + 2.86 = \mathbf{39.55 \text{ Wh/t-km}}$$

As seen, energy consumption has decreased from 69 to 39.55 Wh/t-km.

Example 43.16. *An electric train has an average speed of 45 kmph on a level track between stops 1500 m apart. It is accelerated at 1.8 kmphps and is braked at 3 kmphps. Draw the speed - time curve for the run.*

Solution.

Acceleration α = 1.8 kmphps

Retardation β = 3.0 kmphps

Distance of run S = 1.5 km

Average speed V_a = 45 kmph

Time of run, $$T = \frac{S}{V_a} \times 3600 = \frac{1.5}{45} \times 3600 = 120 \text{ seconds.}$$

Using equation $$V_m = \frac{T}{2K} - \sqrt{\frac{T^2}{4K^2} - \frac{3600\,S}{K}}$$

Where $K = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{1}{2 \times 1.8} + \frac{1}{2 \times 3.0} = 0.4444$

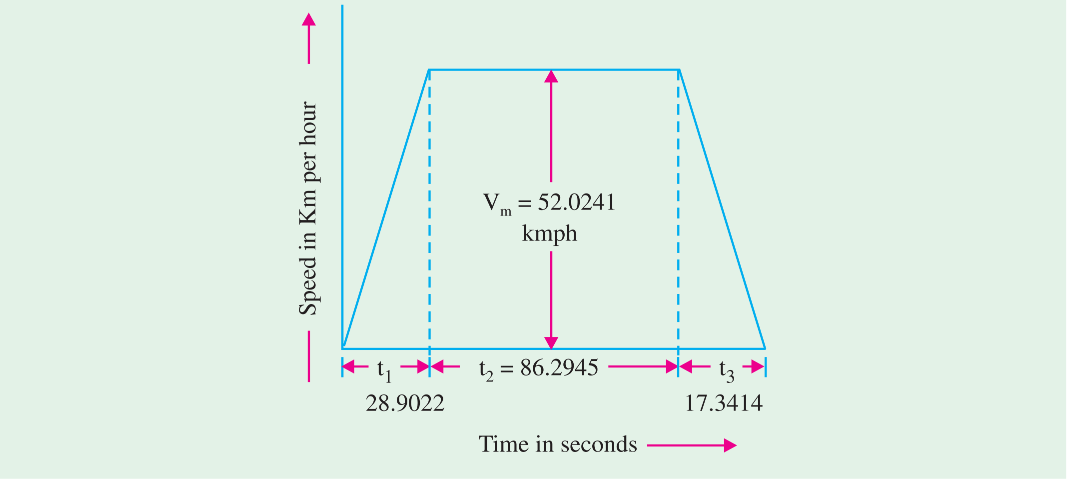


Fig. 43.19

$\therefore$ Maximum speed, $V_m = \frac{120}{2 \times 0.4444} - \sqrt{\frac{(120)^2}{(2 \times 0.4444)^2} - \frac{3600 \times 1.5}{0.4444}}$

$\therefore$ $V_m = 52.0241$ kmph

Acceleration period, $t_1 = \frac{V_m}{\alpha} = \frac{52.0241}{1.8} = 28.9022$ seconds

Braking period, $t_3 = \frac{V_m}{\beta} = \frac{52.0241}{3.0} = 17.3414$ seconds

Free running period, $t_2 = T - (t_1 + t_3) = 120 - (28.9622 + 17.3414) = 86.2945$ seconds

Example 43.17. *A train has schedule speed of 60 km per hour between the stops which are 9 km apart. Determine the crest speed over the run, assuming trapezoidal speed – time curve. The train accelerates at 3 kmphps and retards at 4.5 kmphps. Duration of stops is 75 seconds.*

Solution.

Acceleration α = 3 kmphps

Retardation β = 4.5 kmphps

Distance of run, S = 9 km

Schedule speed, V_s = 60 kmph

Schedule time, $T_s = \frac{S}{V_s} \times 3{,}600$ seconds $= \frac{9}{60} \times 3{,}600 = 540$ seconds

Actual time of run, $T = T_s -$ Time of stop $= 540 - 75 = 465$ seconds

Using the equation

$$V_m = \frac{T}{2K} - \sqrt{\frac{T^2}{4K^2} - \frac{3{,}600\,S}{K}}$$

where $K = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{1}{6} + \frac{1}{9} = 0.2777$

$\therefore$ Maximum speed, $V_m = \frac{465}{2 \times 0.2777} - \sqrt{\frac{(465)^2}{4 \times (0.2777)^2} - \frac{3{,}600 \times 9}{0.2777}}$

$$V_m = 837 - \sqrt{700569 - 116640}$$

$\therefore \quad V_m$ = **72.8475 kmph**

Example 43.18. *An electric train is to have acceleration and braking retardation of 1.2 km/hour/sec and 4.8 km/hour/sec respectively. If the ratio of maximum to average speed is 1.6 and time for stops 35 seconds, find schedule speed for a run of 3 km. Assume simplified trapezoidal speed-time curve.*

Solution.

Acceleration α = 1.2 kmphps

Retardation β = 4.8 kmphps

Distance of run, S = 3 km

Let the actual time of run be T seconds

Average speed, $V_a = \dfrac{3,600\ S}{T} = \dfrac{3,600 \times 3}{T} = \dfrac{10800}{T}$ kmph

Maximum speed, $V_m = 1.6 \times \dfrac{10800}{T} = \dfrac{17280}{T}$ kmph

Since $V_m^2 \left(\dfrac{1}{2\alpha} + \dfrac{1}{2\beta}\right) - V_m\, T + 3,600\ S = 0$

$$\therefore \quad V_m^2 = \frac{V_m\, T - 3,600\ S}{\dfrac{1}{2\alpha} + \dfrac{1}{2\beta}} = \frac{\dfrac{17280}{T} T - 3600 \times 3}{\dfrac{1}{2 \times 1.2} + \dfrac{1}{2 \times 4.8}}$$

or V_m = 111.5419 kmph

and $V_a = \dfrac{V_m}{1.5} = \dfrac{111.5419}{1.5}$ = 74.3613 kmph

Actual time of run $T = \dfrac{3,600\ S}{V_a} = \dfrac{3600 \times 3}{74.3613}$ = 145.2369 seconds

Schedule time, T_s = Actual time of run + Time of stop = 145.2369 + 35 ≈ 180 seconds

$\therefore$ Schedule speed, $V_s = \dfrac{S \times 3,600}{T_s} = \dfrac{3 \times 3600}{180}$ = **60 kmph**

Example 43.19. *An electric train has a schedule speed of 25 kmph between stations 800 m apart. The duration of stop is 20 seconds, the maximum speed is 20 percent higher than the average running speed and the braking retardation is 3 kmphps. Calculate the rate of acceleration required to operate this service.*

Solution.

Schedule speed, V_s = 25 kmph

Distance of run, S = 800 metres = 0.8 km

Retardation, β = 3 kmphps

Schedule time of run, $T_s = \dfrac{3,600 \times S}{T} = \dfrac{3600 \times 0.8}{25}$ = 115.2 seconds

Actual time of run, $T = T_s$ – duration of stop = 115.2 – 20 = 95.2 seconds

Average speed, $V_a = \dfrac{3,600 \times S}{T} = \dfrac{3600 \times 0.8}{95.2}$ = 30.25 kmphs

Maximum speed, $V_m = 1.2\ V_a = 1.2 \times 30.25$ = 36.3 kmph

Since $V_m^2\left(\frac{1}{2\alpha}+\frac{1}{2\beta}\right)-V_m\,T+3{,}600\,S = 0$

$$\therefore \quad \frac{1}{2\alpha}+\frac{1}{2\beta} = \frac{V_m\,T-3{,}600\,S}{V_m^2}$$

$$\text{or} \quad \frac{1}{2\alpha}+\frac{1}{2\times 3} = \frac{36.3\times 95.2-3{,}600\times 0.8}{(36.3)^2} = \mathbf{0.4369}$$

$$\text{or} \quad \alpha = 1.85 \text{ kmphps}$$

Example 43.20. *A suburban electric train has a maximum speed of 80 kmph. The schedule speed including a station stop of 35 seconds is 50 kmph. If the acceleration is 1.5 kmphps, find the value of retardation when the average distance between stops is 5 km.*

Solution.

Schedule speed, V_s = 50 kmph

Distance of run, S = 5 km

Acceleration, α = 1.5 kmphps

Maximum speed, V_m = 80 kmph

Duration of stop = 35 seconds

Schedule time of run, $T_s = \frac{3{,}600\times S}{V_s} = \frac{3{,}600\times 5}{50} = 360$ seconds

Actual time of run, $T = \frac{T_s - \text{duration of stop}}{V_m^2} = 360 - 30 = 330$ seconds

Since $V_m^2\left(\frac{1}{2\alpha}+\frac{1}{2\beta}\right)-V_m\,T+3{,}600\,S = 0$

$$\text{or} \quad \frac{1}{2\alpha}+\frac{1}{2\beta} = V_m\,T-3{,}600\,S = \frac{80\times 330-3{,}600\times 5}{(80)^2} = 1.3125$$

$$\text{or} \quad \frac{1}{2\beta} = 1.3125-\frac{1}{2\alpha} = 1.3125-\frac{1}{2\times 1.5} = 0.9792$$

$$\text{or} \quad \beta = \frac{1}{2\times 0.9792} = \mathbf{0.51064\ kmphps}$$

Example 43.21. *A train is required to run between two stations 1.6 km apart at the average speed of 40 kmph. The run is to be made to a simplified quadrilateral speed-time curve. If the maximum speed is to be limited to 64 kmph, acceleration to 2.0 kmphps and coasting and braking retardation to 0.16 kmphps and 3.2 kmphps respectively, determine the duration of acceleration, coasting and braking periods.*

Solution.

Distance of run, S = 1.6 km

Average speed, V_a = 40 kmph

Maximum speed, V_m = 64 kmph

Acceleration, α = 2.0 kmphps

Coasting retardation, β_C = 0.16 kmphps

Braking retardation, β = 3.2 kmphps

Duration of acceleration, $t_1 = \frac{V_m}{\alpha} = \frac{64}{2.0} =$ **32 seconds**

Actual time of run, $T = \dfrac{3,600\ S}{V_a} = \dfrac{3,600 \times 1.6}{40} = 144$ seconds

Let the speed before applying brakes be V_2

then duration of coasting, $t_2 = \dfrac{V_m - V_2}{\beta_c} = \dfrac{64 - V_2}{0.16}$ seconds

Duration of braking, $t_3 = \dfrac{V_2}{\beta} = \dfrac{V_2}{3.2}$ seconds

Since actual time of run, $T = t_1 + t_2 + t_3$

$$\therefore \quad 144 = 32 + \frac{64 - V_2}{0.16} + \frac{V_2}{3.2}$$

$$\text{or} \quad V_2\left(\frac{1}{0.16} - \frac{1}{3.2}\right) = 32 + 400 - 144$$

$$\text{or} \quad V_2 = \frac{288}{6.25 - 0.3125} = 48.5 \text{ kmph}$$

Duration of coasting, $t_2 = \dfrac{V_m - V_2}{\beta_c} = \dfrac{64 - 48.5}{0.16} =$ **96.85 seconds**

Duration of braking, $t_3 = \dfrac{V_2}{\beta} = \dfrac{48.5}{3.2} =$ **15.15 seconds**

43.47. General Features of Traction Motor

Electric Features

- High starting torque
- Series Speed - Torque characteristic
- Simple speed control
- Possibility of dynamic/ regenerative braking
- Good commutation under rapid fluctuations of supply voltage.

Mechanical Features

- Robustness and ability to withstand continuous vibrations.
- Minimum weight and overall dimensions
- Protection against dirt and dust

No type of motor completely fulfills all these requirements. Motors, which have been found satisfactory are D.C. series for D.C. systems and A.C. series for A.C. systems. While using A.C. three phase motors are used. With the advent of Power Electronics it is very easy to convert single phase A.C. supply drawn from pantograph to three phase A.C.

43.48. Speed - Torque Characteristic of D.C. Motor

$$V = E_b + I_a R_a$$

$$V \cdot I_a = E_b \cdot I_a + I_a^2 R_a$$

where $E_b\ I_a$ = Power input to armature = Electrical power converted into mechanical power at the shaft of motor.

Mechanical Power $= T \cdot \omega = T \times \dfrac{2\pi N}{60}$

$$\therefore \quad \frac{2\pi NT}{60} = E_b \cdot I_a \quad \therefore \quad T = \frac{60\ E_b\ I_a}{2\ \pi\ N} = 9.55\ \frac{E_b\ I_a}{N}$$

But $E_b = \dfrac{\phi\ ZNP}{60\ A}$

$$\therefore \quad T = 9.55 \frac{\phi ZNP}{60\ A} \frac{I_a}{N} = 9.55 \frac{\phi ZP}{60} \frac{I_a}{A}$$

$$= 0.1592 \times \phi \times \left[Z \frac{I_a}{A}\right] P \ \text{Nw-m}$$

$\therefore$ Torque $T = 0.1592 \times$ flux per pole $\times$ armature amp. conductors $\times$ Number of poles

Also speed 'N' can be calculated as:

$$E_b = \frac{\phi ZNP}{60\ A} \qquad \therefore \quad N = \frac{(E_b)}{\phi ZP} 60\ A$$

$$N = \frac{(V - I_a R_a)\ 60\ A}{\phi ZP} \qquad \therefore \quad N \propto \frac{V - I_a R_a}{\phi}$$

But $T = 9.55 \dfrac{\phi ZP}{60} \dfrac{I_a}{A}$ from the equation of torque

$\therefore \quad \dfrac{T}{I_a} = \dfrac{9.55\ \phi\ ZP}{60\ A} \Rightarrow \dfrac{9.55\ I_a}{T} = \dfrac{60\ A}{\phi\ ZP}$ Put this value in the above equation of N

$$\therefore \quad N = \frac{(V - I_a R_a) \times 9.55\ I_a}{T}$$

Speed $$\boxed{N = \frac{9.55\ (V - I_a R_a)}{T / I_a}}$$

The torque - current and speed - torque curves for D.C. motors are shown in Fig. 43.20 (*a*) and (*b*) respectively.

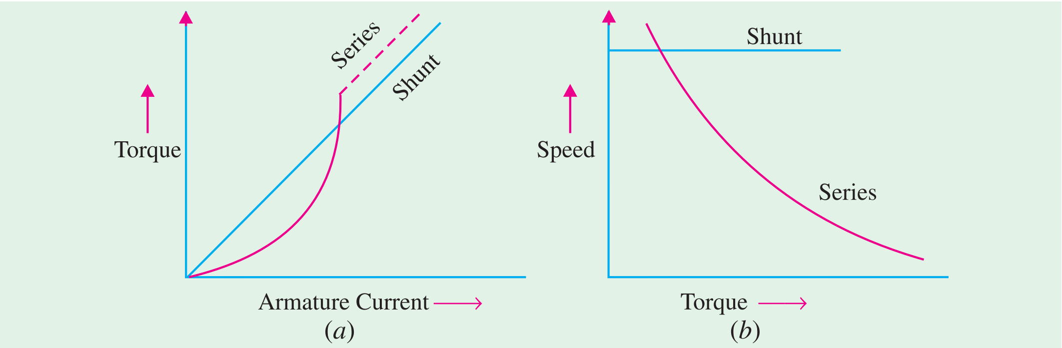


Fig. 43.20

43.49. Parallel Operation of Series Motors with Unequal Wheel Diameter

An electric locomotive uses more than one motor. Each motor drives different set of axles and wheels. Due to wear and tear the diameter of wheels become different, after a long service. But the linear speed of locomotive and wheels will be the same. Therefore, motor speeds will be different due to difference in diameter of wheels driven by them as shown in Fig. 43.21. Therefore, when the motors are connected in parallel they will not share the torques equally, as the current shared by them will be different.

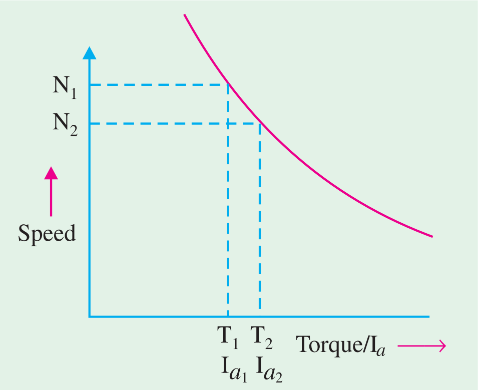


Fig. 43.21

Let the motor wheels ratio is 1.04 : 1 *i.e.* speed of rotation of motor-1 is 1.04 times that of motor-2, as shown in Fig. 43.21.

Let motor 1 drives wheel with 100 c.m. dia. and motor 2 drives wheel with 104 c.m. dia. Then speed of rotation of motor -1 will be $\frac{104}{100}$ = 1.04 times that of 2 *i.e.* $N_2 = 1.04\, N_1$, for a given speed of locomotive.

43.50. Series operation of Series Motor with unequal wheel diameter

Let the motors '*A*' and '*B*' be identical having armature resistance *R* in series, as shown in Fig. 43.22.

Since they are in series, the same current '*I*' will flow through both. But due to unequal wheel diameter; they deliver different loads *i.e.* voltage across each will be different

$$V = V_A + V_B \quad \text{and} \quad N \propto V - IR$$

$$\therefore \quad \frac{N_A}{N_B} = \frac{V_A - IR}{V_B - IR}$$

$$\therefore \quad V_A - IR = \frac{N_A}{N_B}(V_B - IR)$$

$$V_A = \frac{N_A}{N_B}(V_B - IR) + IR$$

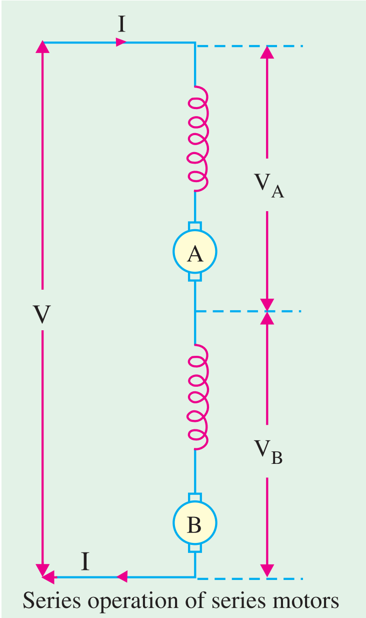
$$V_A = \frac{N_A}{N_B}(V - V_A - IR) + IR$$

$$V_A = \frac{N_A}{N_B}(V - IR) + IR - \frac{N_A}{N_B}V_A$$

$$V_A + \frac{N_A}{N_B}V_A = \frac{N_A}{N_B}(V - I\,R) + IR$$

$$V_A\left(1 + \frac{N_A}{N_B}\right) = \frac{N_A}{N_B}(V - I\,R) + IR$$

$$V_A = \frac{\frac{N_A}{N_B}(V - IR) + IR}{1 + \frac{N_A}{N_B}}$$



Series operation of series motors

Fig. 43.22

Similarly,

$$V_B = \frac{\frac{N_B}{N_A}(V - IR) + IR}{1 + \frac{N_B}{N_A}}$$

43.51. Series Operation of Shunt Motors with Unequal Wheel Diameter

It is similar to the case of series operation of series motors, and hence the same equation holds good.

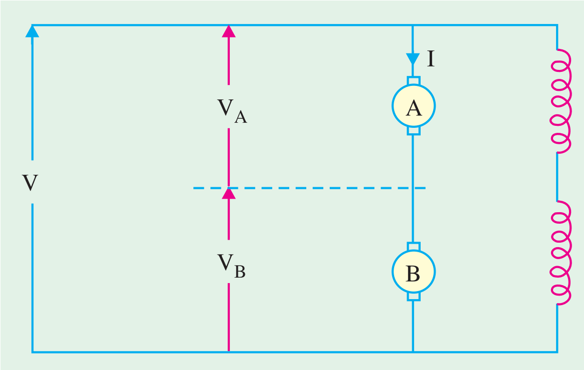


Fig. 43.23

43.52. Parallel Operation of Shunt Motors with Unequal Wheel Diameter

As seen from the Fig. 43.24, a small difference in speeds of two motors, causes motors to be loaded very unequally due to flat speed current curve of D.C. shunt motor.

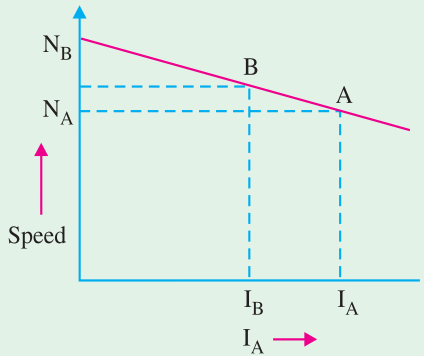


Fig. 43.24

Example 43.22. *The torque-armature current characteristics of a series traction motor are given as:*

Armature Current (amp) :	*5*	*10*	*15*	*20*	*25*	*30*	*35*	*40*
Torque (N-m) :	*20*	*50*	*100*	*155*	*215*	*290*	*360*	*430*

The motor resistance is 0.3Ω. If this motor is connected across 230 V, deduce the speed armature current characteristics.

Solution.

Supply voltage, $V = 230$ V.

Total Resistance of series motor $R_m = R_a + R_{se} = 0.3\ \Omega$.

Armature current, I_a in amperes	5	10	15	20	25	30	35	40
Torque, T in N-m	20	50	100	155	215	290	360	430
Back e.m.f., E_b = $(V - I_aR_m)$ in volts	228.5	227.0	225.5	224.0	222.5	221.0	219.5	218.0
Speed, N_a 9ਓ55 (ट − ट. ट उ) ट. / उ in R.P.M.	545	434	323	276	247	218	204	194

The deduced speed-armature current characteristic is shown in Fig. 43.25.

Example 43.23. *The following figures give the magnetization curve of d.c. series motor when working as a separately excited generator at 600 rpm.:*

Field Current (amperes) :	*20*	*40*	*60*	*80*
E.M.F. (volts) :	*215*	*381*	*485*	*550*

The total resistance of the motor is 0.8 ohm. Deduce the speed – torque curve for this motor when operating at a constant voltage of 600 volts.

Solution.

Voltage applied across the motor, $V = 600$ volts

Resistance of the motor, $R_m = (R_a + R_{se}) = 0.8\ \Omega$

Speed, $N_1 = 600$ r.p.m.

Field Current (amperes)	20	40	60	80
Back e.m.f., E_1 (Volts) at speed N_1 (600 r.p.m.) = e.m.f. generated by the armature (given)	215	381	485	550
Back e.m.f., E (Volts) at speed N (to be determined) = $V - I_a R_m$	584	568	552	536
Speed, N (to be determined in r.p.m.) = $N_1/E_1 \times (V - I_a R_m)$	1,630	895	683	585
Torque, $T = \dfrac{9.55\,(V - I_a R_m)\,I_a}{N}$ N-m	68.4	240	462	700

The deduced speed-torque curve for the motor is shown in Fig. 43.26.

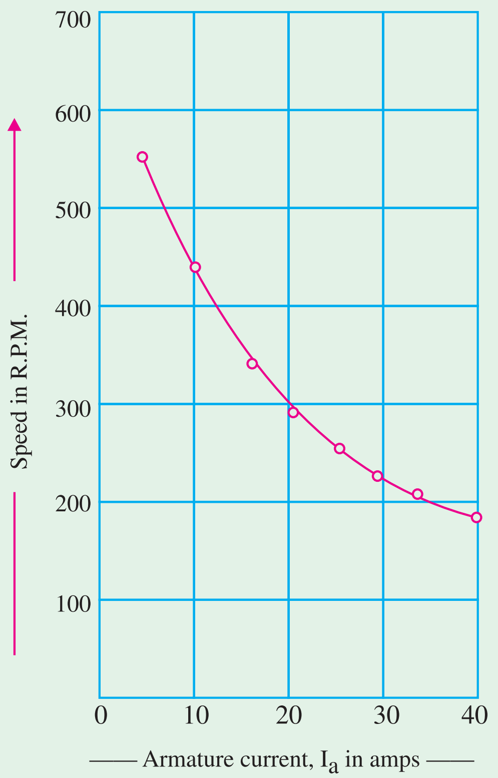


Fig. 43.25

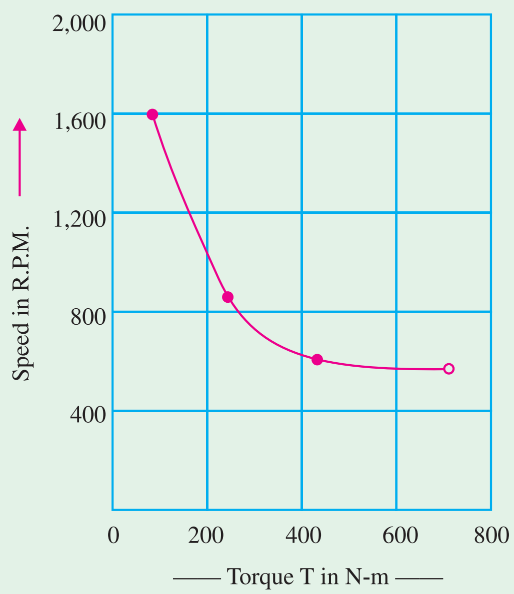


Fig. 43.26

Example 43.24. *Two d.c. traction motors run at a speed of 900 r.p.m. and 950 r.p.m. respectively when each takes a current of 50 A from 500 V mains. Each motor has an effective resistance of 0.3 W. Calculate the speed and voltage across each machine when mechanically coupled and electrically connected in series and taking a current of 50 A from 500 V mains, the resistance of each motor being unchanged.*

Solution.

Let the two motors be A and B of speed N_A = 900 rpm. And N_B = 950 r.p.m. respectively.

Resistance of each motor R_m = 0.3 Ω

Applied voltage across each motor, $V = 500$ V.

Back e.m.f. of motor A when taking a current of 50 A

$$E_{b_A} = V - I\,R_m = 500 - 50 \times 0.3 = 485 \text{ V}$$

Back e.m.f. of motor B when taking a current of 50 A

$$E_{b_B} = V - I\,R_m = 500 - 50 \times 0.3 = 485 \text{ V}$$

When the machines are mechanically coupled and connected in series, the speed of each motor will be same, say N, current will be same and equal to 50 A (given) and the sum of voltage across the two motors will be equal to 500 V.

Let the voltage across motors A and B be V_A and V_B respectively

Now $$V_A + V_B = 500 \qquad ...(i)$$

Back e.m.f. of motor A, $$E_{b'_A} = E_{b_A} \times \frac{N}{N_A} = \frac{485}{900} \times N$$

Voltage across motor A, $$V_A = E_{b'_A} + I\,R_m = \frac{485}{900} \times N + 50 \times 0.3 = \frac{485}{900} \times N + 15$$

Back e.m.f. of motor B, $$E_{b'_B} = E_{b_B} \times \frac{N}{N_B} = \frac{485}{950} N$$

Voltage across motor B, $$V_B = E_{b'_B} + I\,R_m = \frac{485}{950} N + 15$$

Substituting $V_A = \frac{485}{900} N + 15$ and $V_B = \frac{485}{950} N + 15$ in expression (i) we get

$$\frac{485}{900} N + 15 + \frac{485}{950} N + 15 = 500$$

or $$\left(\frac{485}{900} + \frac{485}{950}\right) N = 470$$

or $$N\left(\frac{1}{900} + \frac{1}{950}\right) = \frac{470}{485}$$

$\therefore$ $$N = \mathbf{447.87 \text{ r.p.m.}}$$

P.D. across machine A, $$V_A = \frac{485\,N}{900} + 15 = \mathbf{256.35 \text{ V}}$$

P.D. across machine B, $$V_B = \frac{485\,N}{950} + 15 = \mathbf{243.65 \text{ V}}$$

Example 43.25. *A tram car is equipped with two motors which are operating in parallel. Calculate the current drawn from the supply main at 500 volts when the car is running at a steady speed of 50 kmph and each motor is developing a tractive effort of 2100 N. The resistance of each motor is 0.4 ohm. The friction, windage and other losses may be assumed as 3500 watts per motor.*

Solution.

Voltage across each motor, V = 500 volts

Maximum speed, V_m = 50 kmph

Tractive effort, F_t = 2100 Newtons

Motor resistance, R_m = 0.4 W

Losses per motor = 3500 watts

Power output of each motor $$= \frac{F_t \times V_m \times 1000}{3600} \text{ watts}$$

$$= \frac{2100 \times 50 \times 1000}{3600} \text{ watts} = 29166.67 \text{ watts.}$$

Constant losses = 3500 watts

Copper losses = $I^2 R_m = 0.4\, I^2$

Where I is the current drawn from supply mains

$$\text{Input to motor} = \text{Motor output} + \text{constant losses} + \text{copper losses}$$

$$VI = 29166.67 + 3500 + 0.4\, I^2$$

$$0.4\, I^2 - 500\, I + 32666.67 = 0$$

$$I = \frac{500 \pm \sqrt{(500)^2 - 4 \times 0.4 \times 32666.67}}{2 \times 0.4}$$

$$I = 69.16 \text{ A} \quad \text{or} \quad 1180.84 \text{ A}$$

Current drawn by each motor = 69.16A

Œ 1180.84 A being unreasonably high can not be accepted

Total current drawn from supply mains = 69.16×2 = **138.32 A**

Example 43.26. *A motor coach is being driven by two identical d.c. series motors. First motor is geared to driving wheel having diameter of 90 cm and other motor to driving wheel having diameter of 86 cm. The speed of the first motor is 500 r.p.m. when connected in parallel with the other across 600 V supply. Find the motor speeds when connected in series across the same supply. Assume armature current to remain same and armature voltage drop of 10% at this current.*

Solution.

Speed of first motor, $N_1 = 500$ r.p.m.

Back e.m.f., $E_{b_1} = 600 - \frac{10}{100} \times 600$

$= 540$ volts.

When the motors are connected in series across 600 V supply, as shown in Fig. 43.27.

Let the supply voltage across motors I and II be V_1 and V_2 volts and speed N_1' and N_2' respectively.

Since speed, $N \propto \frac{V - IR}{\phi}$

Current through the motors remains the same, therefore flux produced by it also remains the same and $N \propto (V - IR)$

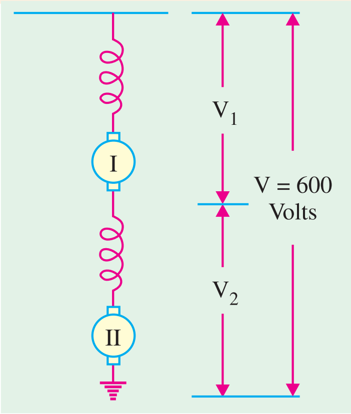


Fig. 43.27

$$\therefore \quad \frac{N_1'}{N_2'} = \frac{V_1 - I\,R}{V_2 - I\,R} = \frac{V_1 - \frac{10}{100} \times 600}{V_2 - \frac{10}{100} \times 600} = \frac{V_1 - 60}{V_2 - 60} \qquad \text{...}(i)$$

And also $\quad N'_1\, D_1 = N'_2 D_1 = N'_2\, D_2$

$$\therefore \quad \frac{N_1'}{N_2'} = \frac{D_2}{D_1} = \frac{86}{90}$$

$$\frac{V_1 - 60}{V_2 - 60} = \frac{86}{90} \qquad (ii) \text{ Since peripheral speed is equal}$$

or $\quad 90\, V_1 - 5{,}400 = 86\, V_2 - 5{,}160$

or $\quad 90\, V_1 - 86\, V_2 = 5{,}400 - 5{,}160 = 240 \qquad$ ...*(iii)*

and also $\quad V_1 + V_2 = 600$ V $\qquad$ ...*(iv)*

Solving expressions (*iii*) and (*iv*)

$$V_1 = 294.55 \text{ V}$$

And $$V_2 = 305.45 \text{ V}$$

Now the speeds of the motors can be calculated as follows :

$$\frac{N_1'}{N_2'} = \frac{E_{b_1}'}{E_b}$$

or $$N'_1 = N_1 \times \frac{E_{b_1}'}{E_b} = 500 \times \frac{294.55 - 60}{600 - 60} = 217 \text{ r.p.m}$$

and $$N'_2 = N_1' \times \frac{D_1}{D_2} = 217 \times \frac{90}{86} = \textbf{277 r.p.m}$$

Example 43.27. *Two similar series type motors are used to drive a locomotive. The supply fed to their parallel connection is 650 V. If the first motor 'A' is geared to drive wheels of radius 45 cms. and other motor 'B' to 43 cms. And if the speed of first motor 'A' when connected in parallel to 2nd motor 'B' across the main supply lines is 400 rpm., find voltages and speeds of motors when connected in series. Assume I_a to be constant and armature voltage drop of 10% at this current.*

Solution.

$N \propto V - IR$ as flux ϕ is constant, since I_a is constant

$N_A = V_A - IR \qquad N_B = V_B - IR \qquad$ Also $V = V_A + V_B$

Assume $$\frac{N_A}{N_B} = \rho$$

$$V_A = \frac{\rho\,(V - IR) + IR}{1 + \rho}$$

Armature voltage drop = 10% of 650 V $\qquad \therefore \quad IR = 65$

But $$\frac{N_A}{N_B} = \frac{r_B}{r_A} = \frac{43}{45} = \rho$$

$$V_A = \frac{43/45\,(650 - 65) + 65}{1 + 43/45} = 320 \text{ V}$$

$$V_B = V - V_A = 650 - 320 = 330 \text{ V}$$

Speed N_A of motor *A* is 400 rpm with a supply of 650 V.

$\therefore$ Speed N'_A of motor *A* with supply voltage of 320 V will be

$$\frac{N_A'}{N_A} = \frac{320 - IR}{650 - IR} = \frac{320 - 65}{650 - 65} = \frac{255}{585}$$

$\therefore$ $$N'_A = \frac{255}{585} N_A = \frac{255}{585} \times 400 = \textbf{175 r.p.m.}$$

$$\frac{N_A}{N_B} = \frac{r_B}{r_A} = \frac{N_A'}{N_B'} = \frac{43}{45}$$

$$N'_B = \frac{45}{43} N_A' = \frac{45}{43} \times 175 = \textbf{184 r.p.m.}$$

43.53. Control of D.C. Motors

The starting current of motor is limited to its normal rated current by starter during starting. At the instant of switching on the motor, back e.m.f. $E_b = 0$

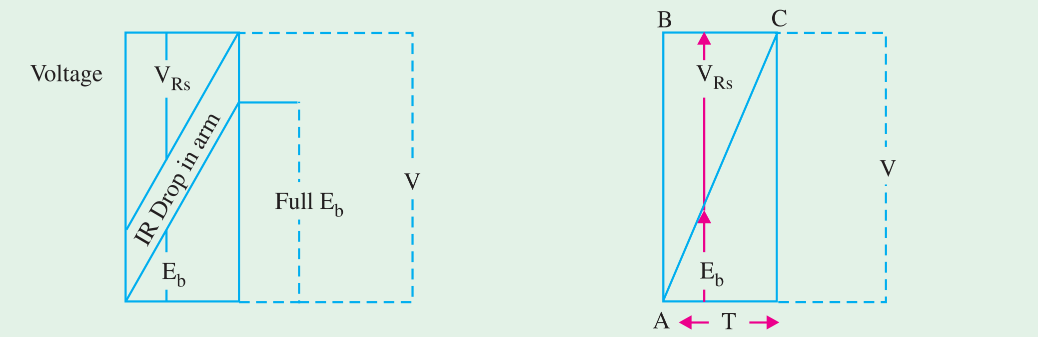
$\therefore$ Supply voltage = V = IR + Voltage drop across R_s.

At any other instant during starting

$$V = IR + \text{Voltage across } R_s + E_b$$

At the end of accelerating period, when total R_s is cut-off

$$V = E_b + IR$$



(*a*) Voltage during starting of D.C. motor (*b*) Voltage during starting neglecting IR drop

Fig. 43.28

If T is the time in sec. for starting and neglecting IR drop, total energy supplied = $V.I.T.$ watt-sec

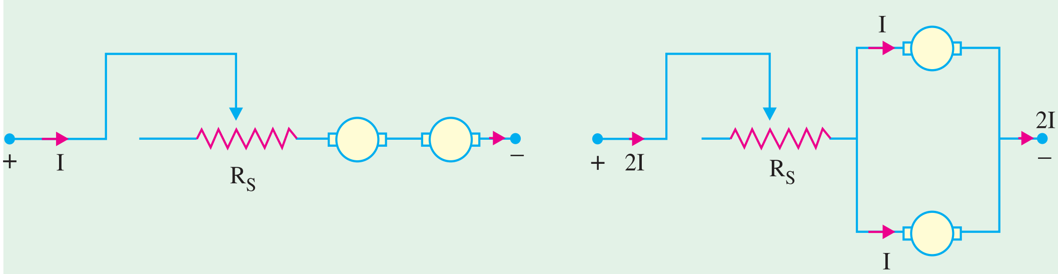
From Fig. 43.28 (*b*) Energy wasted in R_s = Area of triangle $ABC \times I$ = ½. $T.V.I.$ watt - sec. = ½ VIT watt - sec. But total energy supplied = $V.I.T$ watt - sec.

$\therefore$ Half the energy is wasted in starting

$\therefore$ $\eta_{starting} = 50\%$

43.54. Series - Parallel Starting

With a 2 motor equipment ½ the normal voltage will be applied to each motor at starting as shown in Fig. 43.29 (*a*) (Series connection) and they will run upto approximate ½ speed, at which instant they are switched on to parallel and full voltage is applied to each motor. R_s is gradually cut-out, with motors in series connection and then reinserted when the motors are connected in parallel, and again gradually cut-out.



(*a*) Motors in series (*b*) Motors in parallel

Fig. 43.29

In traction work, 2 or more similar motors are employed. Consider 2 series motors started by series parallel method, which results in saving of energy.

(*a*) Series operation. The 2 motors, are started in series with the help of R_s. The current during starting is limited to normal rated current '*I*' per motor. During series operation, current '*I*' is drawn from supply. At the instant of starting $OA = AB = IR$ drop in each motor. OK = Supply voltage 'V'. The back e.m.fs. of 2 motors jointly develop along OM as shown in Fig. 43.30 (*a*). At point. E, supply voltage V = Back e.m.fs of 2 motors + IR drops of 2 motor. Any point on the line BC represents the sum of Back e.m.fs. of 2 motors + IR drops of 2 motors + Voltage across resistance R_s of 2 motors

$$OE = \text{time taken for series running.}$$

At pt 'E' at the end of series running period, each motor has developed a back e.m.f.

$$= \frac{V}{2} - IR$$

$$EL = ED - LD$$

(*b*) Parallel operation. The motors are switched on in parallel at the instant 'E', with R_s reinserted as shown in Fig. 43.29 (*b*). Current drawn is $2I$ from supply. Back e.m.f. across each motor = EL. So the back e.m.f. now develops along LG. At point 'H' when the motors are in full parallel, ($R_s = 0$ and both the motors are running at rated speed)

$$\text{Supply voltage} = V = HF = HG + GF$$

$$= \text{Normal Back e.m.f. of each motor} + IR \text{ drop in each motor.}$$

43.55. To find t_s, t_p and η of starting

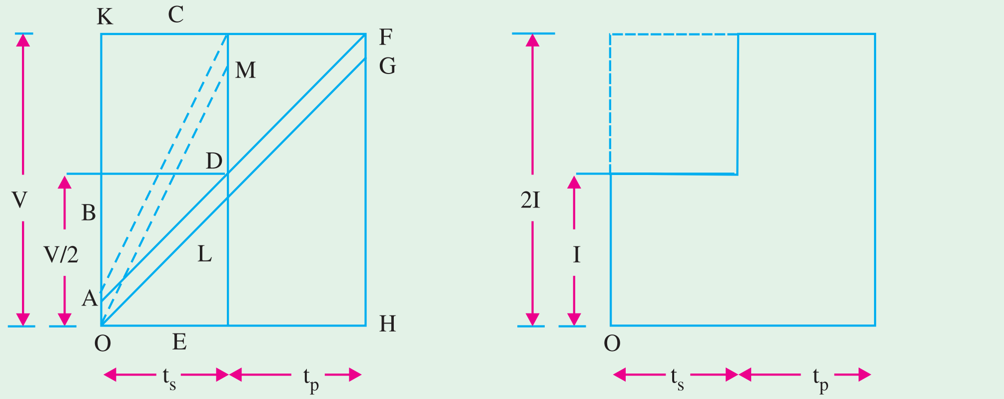
The values of time t_s during which the motors remain in series and t_p during which they are in parallel can be determined from Fig. 43.30 (*a*), (*c*). From Fig. 43.30 (*a*), triangles OLE and OGH are similar

$$\therefore \qquad \frac{OE}{OH} = \frac{LE}{GH} \quad \therefore \quad \frac{t_s}{T} = \frac{DE - DL}{FH - FG} = \frac{V/2 - IR}{V - IR}$$

$$\therefore \qquad \boxed{t_s = \frac{1}{2}\left(\frac{V - 2IR}{V - IR}\right)T}$$

$$t_p = T - t_s = T - \left\{\frac{1}{2}\left(\frac{V - 2IR}{V - IR}\right)T\right\}$$

$$\boxed{t_p = T\left\{1 - \frac{1}{2}\left(\frac{V - 2IR}{V - IR}\right)T\right\}}$$



(*a*) Voltage built-up in series-parallel starting (*b*) Variation of current in series-parallel starting

Fig. 43.30

To calculate η of starting, neglect IR drop in armature circuit.

This modifies Fig. 43.30 (a) to Fig. 43.30 (c). 'D' is midpoint of CE and back e.m.f. develops along DF in parallel combination. $KC = CF$ *i.e.* time for series combination = time for parallel combination

i.e. $t_s = t_p = t$ and average starting current = I per motor.

Energy lost in R_s = Area under triangle OKC + Area under triangle CDF

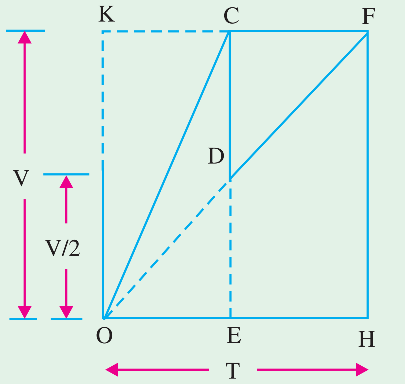
$$= \left(\frac{1}{2} V I\right) \times t + \left(\frac{1}{2}\frac{V}{2} 2I\right) \times t = VIt$$

But total energy supplied

$$= I V t \quad + \quad 2 I V t$$
$$\text{(Series)} \quad \text{(Parallel)}$$
$$= 3 V I t$$

$$\therefore \eta \text{ of starting} = \frac{3VIt - VIt}{3VIt}$$
$$= \frac{2}{3} = 66.6\%$$

∴ η is increased by 16.66% as compared to pervious case. If there are 4 motors then $\eta_{starting}$ = 73%. So there is saving of energy lost in R_s, during starting period as compared with starting by both motors in parallel.



(c) Efficiency of starting by series-parallel method

Fig. 43.30

Example 43.28. *Two motors of a motor coach are started on series - parallel system, the current per motor being 350 A (Considered as being maintained constant) during the starting period which is 18 sec. If the acceleration during starting period is uniform, the line voltage is 600 V and resistance of each motor is 0.1 W. Find* ***(a)*** *the time during which the motors are operated in series.* ***(b)*** *the energy loss in the rheostat during starting period.* **[Nagpur University, Summer 2002]**

Solution.

Time during which motors are in series is given by

$$t_s = \frac{1}{2}\left(\frac{V - 2IR}{V - IR}\right) T = \frac{1}{2}\left(\frac{600 - 2 \times 350 \times 0.1}{600 - 350 \times 0.1}\right) 18$$

$$t_s = \mathbf{8.44 \text{ sec.}}$$

Time during which motors are in parallel.

$$t_p = T - T_s = 18 - 8.44 = 9.56 \text{ sec.}$$

Back e.m.f. E_b of each motor, in series operation (from Fig. 43.30a)

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 350\ (0.1) = 265 \text{ V.}$$

When 2 motors are in series,

Total $\quad E_b = 265 + 265 = 530$ V

$$E_{b_p} = V - IR = 600 - 350\ (0.1) = 565 \text{ V}$$

Energy lost when motors are connected in series

$$= \frac{1}{2} E_b\ I\ t_s = \frac{1}{2} \times 530 \times 350 \times \frac{8.44}{3600} = 217 \text{ watt - hours}$$

Energy lost when motors are connected in parallel

$$\frac{1}{2}\frac{E_b}{2} 2 I t_p = \frac{1}{2} \times \frac{565}{2} \times 2 \times 350 \times \frac{9.56}{3600} = 262.5 \text{ watt - hour}$$

j 263 watt - hours

∴ Total energy lost = (217 + 263) watt - hours = **480 watt - hours**

43.56. Series Parallel Control by Shunt Transition Method

The various stages involved in this method of series – parallel control are shown in Fig. 43.31

In steps 1, 2, 3, 4 the motors are in series and are accelerated by cutting out the R_s in steps. In step 4, motors are in full series. During transition from series to parallel, R_s is reinserted in circuit– step 5. One of the motors is bypassed -step 6 and disconnected from main circuit – step 7. It is then connected in parallel with other motor -step 8, giving 1st parallel position. R_s is again cut-out in steps completely and the motors are placed in full parallel.

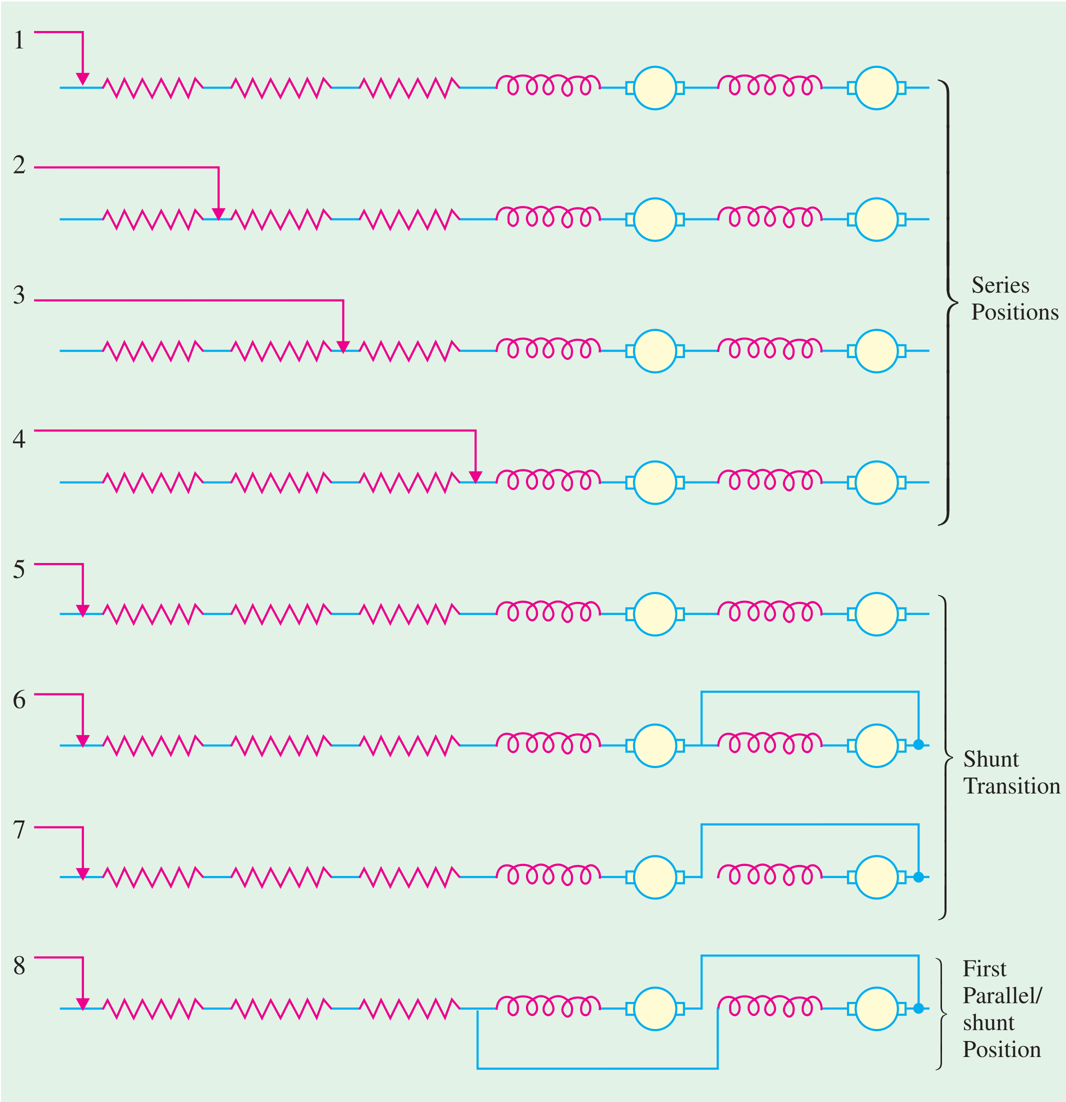


Fig. 43.31

The main difficulty with series parallel control is to obtain a satisfactory method of transition from series to parallel without interrupting the torque or allowing any heavy rushes of current.

In shunt transition method, one motor is short circuited and the total torque is reduced by about 50% during transition period, causing a noticeable jerk in the motion of vehicle.

The Bridge transition is more complicated, but the resistances which are connected in parallel with or 'bridged' across the motors are of such a value that current through the motors is not altered in magnitude and the total torque is therefore held constant and hence it is normally used for railways. So in this method it is seen that, both motors remain in circuit through-out the transition. Thus the jerks will not be experienced if this method is employed.

43.57. Series Parallel Control by Bridge Transition

(*a*) At starting, motors are in series with R_s *i.e.* link *P* in position = *AA′*

(*b*) Motors in full series with link *P* in position = *BB′* (No R_s in the circuit)

The motor and R_s are connected in the form of Wheatstone Bridge. Initially motors are in series with full R_s as shown in Fig. 43.32 (*a*). *A* and *A′* are moved in direction of arrow heads. In position *BB′* motors are in full series, as shown in Fig. 43.32 (*b*), with no R_s present in the circuit.

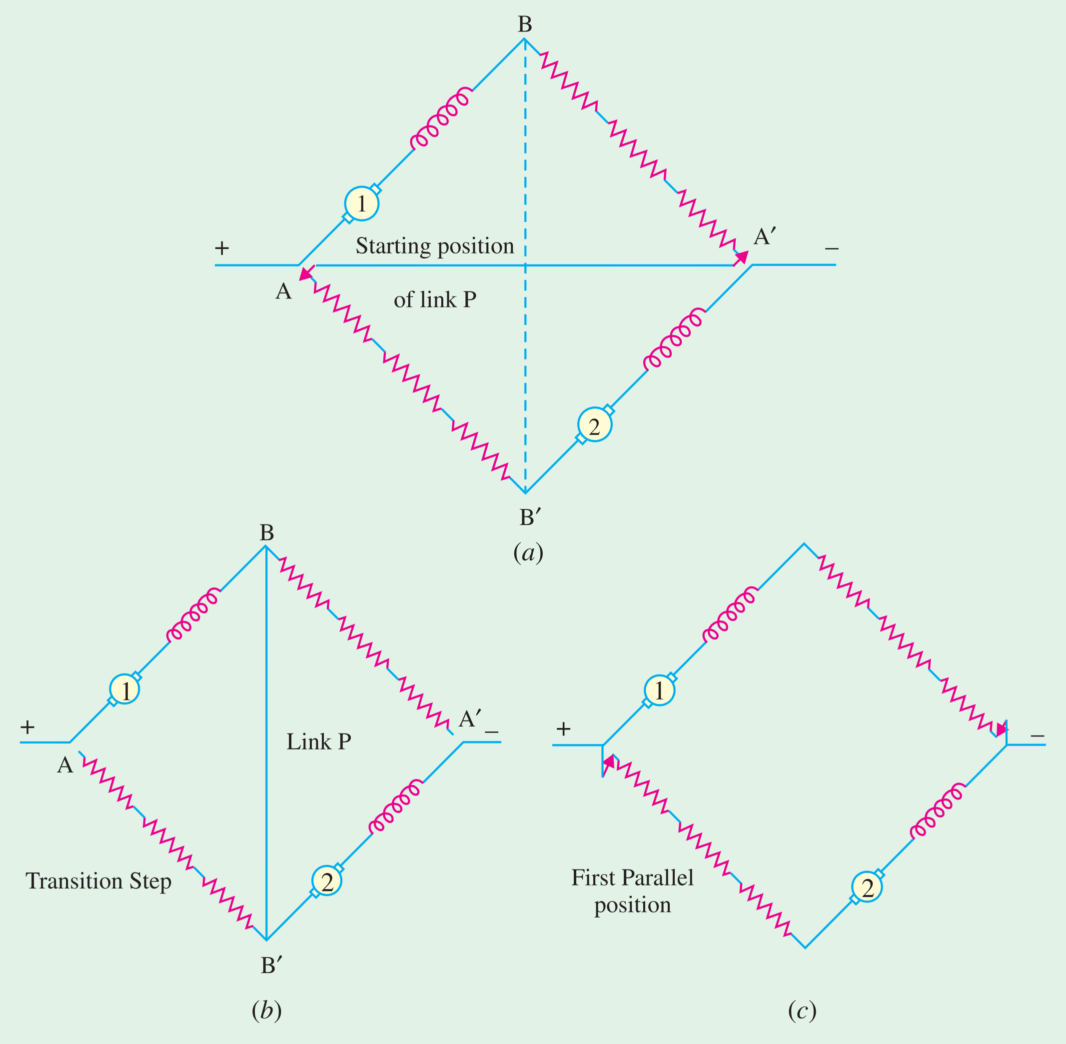


Fig. 43.32

In transition step the R_s is reinserted.

In Ist parallel step, link *P* is removed and motors are connected in parallel with full R_s as shown in Fig. 43.32 (*c*). Advantage of this method is that the normal acceleration torque is available from both the motors, through - out starting period. Therefore acceleration is smoother, without any jerks, which is very much desirable for traction motors.

43.58. Braking in Traction

Both electrical and mechanical braking is used. Mechanical braking provides holding torque. Electric Braking reduces wear on mechanical brakes, provides higher retardation, thus bringing a vehicle quickly to rest. Different types of electrical braking used in traction are discussed.

43.59. Rheostatic Braking

(*a*) Equalizer Connection (*b*) Cross Connection

(*a*) Equalizer Connection

For traction work, where 2 or more motors are employed, these are connected in parallel for braking, because series connection would produce too high voltage. K.E. of the vehicle is utilized in driving the machines as generators, which is dissipated in braking resistance in the form of heat.

To ensure that the 2 machines share the load equally, an equalizer connection is used as shown in Fig. 43.33 (*a*). If it is not used, the machine whose acceleration builts-up first would send a current through the 2nd machine in opposite direction, causing it to excite with reverse voltage. So that the 2 machines would be short circuited on themselves. The current would be dangerously high. Equalizer prevents such conditions. Hence Equalizer connection is important during braking in traction.

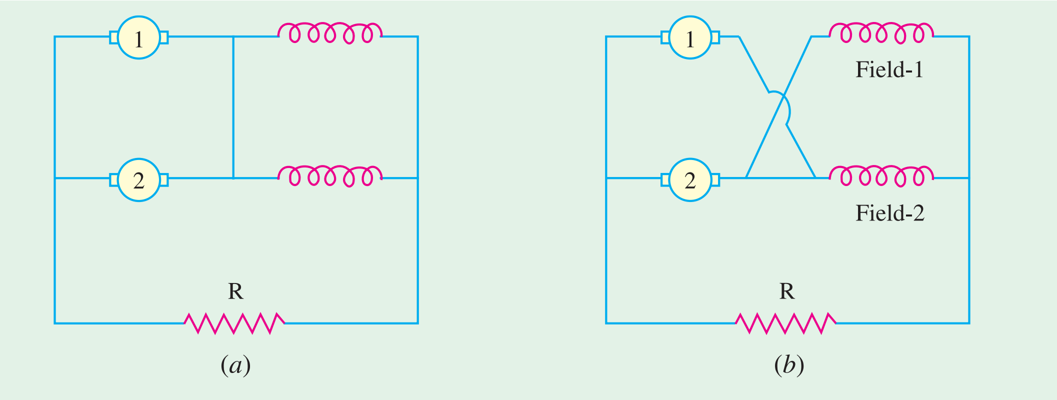


Fig. 43.33

(*b*) Cross Connection

In cross connection the field of machine 2 is connected in series with armature of machine 1 and the field of machine 1 is connected in series with armature of machine 2 as shown in Fig. 43.33 (*b*). Suppose the voltage of machine 1 is greater than that of 2. So it will send greater current through field of machine 2, causing it to excite to higher voltage. At the same time machine 1 excitation is low, because of lower voltage of machine 2. Hence machine 2 will produce more voltage and machine 1 voltage will be reduced. Thus automatic compensation is provided and the 2 machines operate satisfactorily.

Because of cross - connection during braking of traction motors, current in any of the motor will not go to a very high value.

43.60. Regenerative Braking with D.C. Motors

In order to achieve the regenerative braking, it is essential that **(*i*)** the voltage generated by the machine should exceed the supply voltage and **(*ii*)** the voltage should bez kept at this value, irrespective of machine speed. Fig. 43.34 (*a*) shows the case of 4 series motors connected in parallel during normal running *i.e.* motoring.

One method of connection during regenerative barking, is to arrange the machines as shunt machines, with series fields of 3 machines connected across the supply in series with suitable resistance. One of the field winding is still kept in series across the 4 parallel armatures as shown in figure 43.34 (*b*).

The machine acts as a compound generator. (with slight differential compounding) Such an arrangement is quiet stable; any change in line voltage produces a change in excitation which produces corresponding change in e.m.f. of motors, so that inherent compensation is provided *e.g.* let the line voltage tends to increase beyond the e.m.f. of generators. The increased voltage across the shunt circuit increases the excitation thereby increasing the generated voltage. Vice-versa is also true. The arrangement is therefore self compensating.

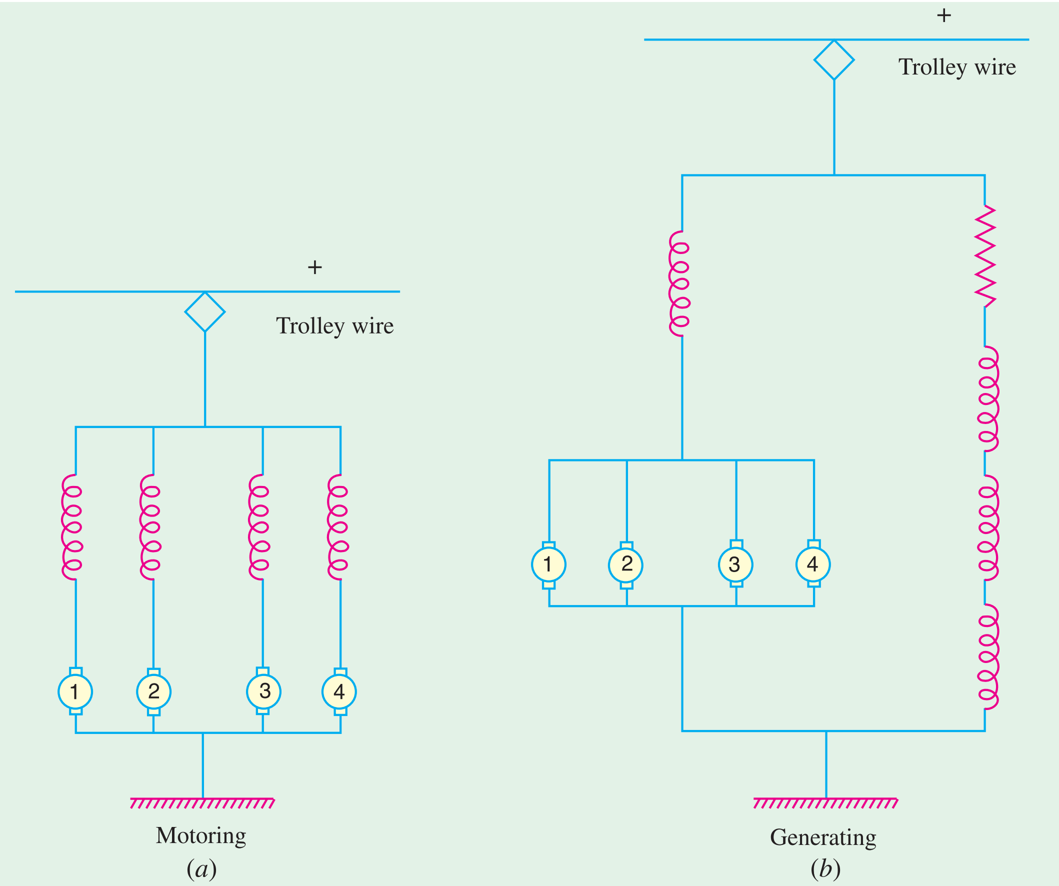


Fig. 43.34

D.C. series motor can't be used for regenerative braking without modification for obvious reasons. During regeneration current through armature reverses; and excitation has to be maintained. Hence field connection must be reversed.

Example 43.29. *Two 750 V D.C. series motors each having a resistance of 0.1 W are started on series - parallel system. Mean current through - out the starting period is 300 A. Starting period is 15 sec. and train speed at the end of this period is 25 km/hr. Calculate*

(i) *Rheostatic losses during series and parallel combination of motors*
(ii) *Energy lost in motor*
(iii) *Motor output*
(iv) *Starting η*
(v) *Train speed at which transition from series to parallel must be made.*

[Nagpur University, Summer 2000]

Solution.

(i)
$$t_s = \frac{1}{2}\left[\frac{V - 2IR}{V - IR}\right]T$$
$$t_s = \frac{1}{2}\left[\frac{750 - 2(300)\,0.1}{750 - (300)\,0.1}\right]15 = 7.1875 \text{ rec.}$$
$$\therefore \quad t_p = T - t_s = 7.8125 \text{ sec.}$$

$$\text{Energy lost in Rheostat} = \frac{1}{2}\,E_{b_s}\,I\,t_s + \frac{1}{2}\,\frac{E_{b_p}}{2}\,2\,I.t_p$$
$$= \frac{1}{2}\left[2\times\left[\frac{V}{2} - IR\right]\right]I\,.\,t_s + \frac{1}{2}\,[[V - IR]/2]\,2I\,.\,t_p$$
$$= \frac{1}{2}\left[2\times\left[\frac{750}{2} - 300\,(0.1)\right]\right]300\times 7.1875 + \frac{1}{2}\left[\frac{750 - 300\,(0.1)}{2}\right]\times 2(300)\times 7.1825$$
$$= 743906.25 + 843750$$
$$= 1587656.25 \text{ watt} - \text{sec.}$$
$$= \frac{1587656.25}{3600} = \mathbf{441.00 \text{ watt - hrs.}}$$

(ii) Total Energy supplied $= V.I.\,t_s + 2\,I\,.\,V\,.\,t_p$
$$= 750\times 300\,(7.1875) + 2\,(300)\,750\,(7.8125)$$
$$= 1617187.5 + 3515625$$
$$= 5132812.5 \text{ watt-sec} = 1425.7812 \text{ watt} - \text{hrs.}$$

Energy lost in 2 Motors $= (I_a^2 \times R_a)\times 2\times 15$
$$= (300^2\times 0.1)\times 2\times 15 = 270000 \text{ watt - sec.} = \mathbf{75 \text{ watt - hrs.}}$$

(iii) Motor O/P = Total Energy supplied – Energy lost in Rheostat – Energy lost in armature
$$= 1425.7812 - 441 - 75$$
$$= \mathbf{909.7812 \text{ watt} - \text{hrs.}}$$

(iv)
$$\eta \text{ starting} = \frac{\text{Total Energy Supplied} - \text{Energy lost in Rheostat}}{\text{Total Energy Supplied}}$$
$$= \frac{1425.7812 - 441.00}{1425.7812}\times 100$$
$$= \mathbf{69.0605\%}$$

(v) Acceleration is uniform during starting period of 15 sec. Therefore speed after which series to parallel transition must be made is given as –
$$= \frac{\text{Speed after starting period}}{\text{Total starting period}}\times t_s$$

$$= \frac{25}{15} \times 7.1875$$

$= \mathbf{11.9791\ \ km/hr.}$

Example 43.30. *Two 600-V motors each having a resistance of 0.1Ω are started on the series-parallel system, the mean current per motor throughtout the starting period being 300A. The starting period is 20 seconds and the train speed at the end of this period is 30 km per hour. Calculate* ***(i)*** *the rheostatic losses (in kwh) during* ***(a)*** *the series and* ***(b)*** *the parallel combinations of motors* ***(ii)*** *the train speed at which transition from series to parallel must be made.*

Solution.

Number of motors operating = 2

Line voltage, V = 600 volts

Current per motor, I = 300 amperes

Starting period, T_s = 20 seconds

Motor resistance, R = 0.1 Ω

Maximum speed, V_m = 30 kmph.

Back e.m.f. of each motor in full series position,

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 300 \times 0.1 = 270 \text{ volts.}$$

Back e.m.f. of each motor in full parallel position,

$$E_{b_p} = V - IR - 300 \times 0.1 = 570 \text{ volts}$$

Assuming smooth acceleration, back e.m.f. will be built up at constant rate.

Since motors take 20 seconds to build up 570 volts, therefore time taken to build up 270 volts e.m.f. will be :

$$T_{\text{series}} = 20 \times \frac{270}{570} = 9.4737 \text{ seconds}$$

$$T_{\text{parallel}} = 20 - 9.4737 = 10.5263 \text{ seconds}$$

(i) ***(a)*** Voltage drop in the starting rheostat in series combination at the starting instant

$$= V - 2IR = 600 - 2 \times 300 \times 0.1 = 540 \text{ volts,}$$

which reduces to zero in full series position

Energy dissipated in starting resistance during series combination

$$= \frac{(V - 2\,I\,R) + 0}{2} \times I \times \frac{T_{\text{series}}}{3600} = \frac{540 + 0}{2} \times 300 \times \frac{9.4737}{3600}$$

$= \mathbf{213.1579\ watt - hours}$

(b) Voltage drop across the starting resistance in first parallel position is equal to $V/2$ *i.e.* 300 volts which gradually reduces to zero.

Energy dissipated in starting resistance during parallel combination

$$= \frac{\frac{V}{2} + 0}{2} \times 21 \times \frac{T_{\text{parallel}}}{3600} = \frac{\frac{600}{2} + 0}{2} \times 2 \times 300 \times \frac{10.5263}{3600}$$

$= \mathbf{263.1579\ watt - hours}$

(ii) Acceleration, $\alpha = \dfrac{\text{Maximum speed}}{\text{Starting period}} = \dfrac{V_m}{T_s} = \dfrac{30}{20} = 1.5$ kmphps.

Speed at the end of series period $= \alpha\ T_{\text{series}} = 1.5 \times 9.4737 =$ **14.21 km/hour**

Example 43.31. *Two d.c. series motors of a motor coach have resistance of 0.1 W each. These motors draw a current of 500 A from 600 V mains during series – parallel starting period of 25 seconds. If the acceleration during starting period remains uniform, determine:*

(i) time during which the motors operate in (a) series (b) parallel.
(ii) the speed at which the series connections are to be changed if the speed just after starting period is 80 kmph.

Solution.

Number of motors operating = 2
Line voltage, V = 600 V
Current per motor, I = 500 A
Motor resistance, R = 0.1 Ω
Maximum speed, V_m = 80 kmph.

Back e.m.f. of each motor in full series position.

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 500 \times 0.1 = 250 \text{ V}$$

Back e.m.f. each motor in full parallel operation,

$$E_{b_p} = V - IR = 600 - 500 \times 0.1 = 550 \text{ V}$$

Since motors take 25 seconds to build up 550 V, therefore, time taken to build up 250 V, will be: (assuming smooth acceleration and building up of e.m.f. at constant rate.)

(i) Period of series operation, $T_{\text{series}} = 25 \times \frac{255}{550}$ = **11.3636 seconds**

Period of parallel operation, $T_{\text{parallel}} = T - T_{se} = 25 - 11.3636$ = **13.6363 seconds**

(iii) Speed at which the series connections are to be changed

$$= \alpha\, T_{\text{series}} = \frac{V_m}{T} \,.\, T_{\text{series}} = \frac{80}{25} \times 11.3636 = \textbf{36.3636 kmph}$$

Example 43.32. *The following figures refer to the speed-current and torque – current characteristics of a 600 V d.c. series traction motor.*

Current, amperes :	*50*	*100*	*150*	*200*	*250*
Speed, kmph :	*73.6*	*48*	*41.1*	*37.3*	*35.2*
Torque, N-m :	*150*	*525*	*930*	*1,335*	*1,750*

Determine the braking torque at a speed of 48 kmph when operating as self excited d.c. generator. Assume resistance of motor and braking rheostat to be 0.6Ω and 3.0 Ω respectively.

Solution.

As motor :

Terminal voltage, V = 600 volts.

The motor current at a speed of 48 kmph (from speed-current characteristic curve),

I = 100 A

Back e.m.f. developed by the motor, $E_b = V - IR_m = 600 - 100 \times 0.6$ = **540 V**

As Generator:

At the instant of applying rheostatic braking at speed of 48 kmph, the terminal voltage of machine will be equal to e.m.f. developed by the machine *i.e.* 540 volts.

Total resistance in the circuit = $R_m + R_{\text{rheostat}} = 0.6 + 3 = 3.6\ \Omega$

Current delivered by the machine, $I = \frac{540}{3.6}$ = **150 amps**

The braking torque (the torque corresponding to 150 amperes from torque-current curve)

= 930 N-m

Tutorial Problem No. 43.1

1. A train weighs 500 tonnes. What is its mass in (*i*) tonnes and (*ii*) kilograms.
[(*i*) 500 t (*ii*) 500,000 kg]
2. A train has a mass of 200 tonnes. What is its weight in (*i*) newtons and (*ii*) kg-wt (*iii*) tonnes-wt.
[(*i*) 19.6×10^5 N (*ii*) 200,000 kg. wt (*iii*) 200 t-wt)]
3. A train has a speed of 100 km/h. What is its value in m/s ? **[27.78 m/s]**
4. A certain express train has an acceleration of 3.6 km/h/s. What is its value in m/s^2 ? **[1.0 m/s^2]**
5. If there is an ascending gradient of 15 m in a track length of 1 km, what is the value of percentage gradient ? **[1.5%]**
6. A train runs at an average speed of 45 km per hour between stations 2.5 km apart. The train accelerates at 2 km/h/s and retards at 3 km/h/s. Find its maximum speed assuming a trapezoidal speed/time curve. Calculate also the distance travelled by it before the brakes are applied.
[50.263 km/h, 2.383 km] (*Elect. Traction and Utilization B.H.U.*)
7. The schedule speed with a 200 tonne train on an electric railway with stations 777 metres apart is 27.3 km/h and the maximum speed is 20 percent higher than the average running speed. The braking rate is 3.22 km/h/s and the duration of stops is 20 seconds. Find the acceleration required. Assume a simplified speed-time curve with free running at the maximum speed.
[2.73 km/h/s] (*Traction and Utilization of Elect. Power, Agra Univ.*)
8. A suburban electric train has a maximum speed of 65 km/h.. The schedule speed including a station stop of 30 seconds is 43.5 km/h. If the acceleration is 1.3 km/h/s, find the value of retardation when the average distance between stops is 3 km.
[β = 1.21 km/h/s] (*Utilization of Elect. Power and Traction, Gorakhpur Univ.,*)
9. An electric train is accelerated uniformly from rest to a speed of 40 km/h, the period of acceleration being 20 seconds. If it coasts for 60 seconds against a constant resistance of 50 N/t and is brought to rest in a further period of 10 seconds by braking, determine :
(*i*) the acceleration (*ii*) the coasting retardation (*iii*) the braking retardation (*iv*) distance travelled and (*v*) schedule speed with station stops of 10 seconds duration.
Allow 10 percent for rotational inertia. **(*Elect. Traction, Punjab Univ.*)**
[α = 2 km/h/s, β_c = 0.1636 km/h/s, β = 3 km/h/s., D = 0.736 km., V = 27.5 km/h]
10. The speed-time curve of an electric train on a uniform rising gradient of 1 in 100 comprises :
(*i*) uniform acceleration from rest at 2 km/h/s/ for 30 seconds.
(*ii*) coasting with power off for 70 seconds.
(*iii*) braking at 3 km/h/s to a standstill.
The weight of the train is 250 tonnes, the train resistance on level track being 49 N/tonne and allowance for rotary inertia 1%.
Calculate the maximum power developed by traction motors and total distance travelled by the train. Assume transmission efficiency as 97%.
[3,3258 kW, 1.12 km] (*Traction and Utilization of Elect. Power, Agra Univ.*)
11. A 400-tonne goods train is to be hauled by a locomotive up a gradient of 2% with an acceleration of 1 km/h/s. Co-efficient of adhesion is 20%, track resistance 40 N/tonne and effective rotating masses 10% of the dead weight. Find the weight of the locomotive and number of axles if the axle load is not increased beyond 22 tonnes.**[152.6 tonnes, 7] (*Traction and Utilization of Elect. Power, Agra Univ.*)**
12. A 500-tonne goods train is to be hauled by a locomotive up a gradient of 20% with an acceleration of 1.2 km/h/s. Co-efficient of adhesion is 25%, track resistance 40 N/ tonne and effective rotating masses 10% of dead weight. Find the weight of the locomotive and number of axles if axle load is not to exceed 20 tonnes. **[160 tonnes, 8] (*Utilization of Elect. Power, A.M.I.E. Winter*)**

13. Determine the maximum adhesive weight of a loco required to start a 2340 tonne weight (inclusive of loco) on 1 : 150 gradient and accelerate it at 0.1 km/h/s. Assume co-efficient of adhesion as 0.25, train resistance 39.2 N/tonne and rotary inertia as 8%. **[128.5 tonnes]**

(Elect. Traction, A.M.I.E., May)

14. Ore carrying trains weighing 5000 tonne each are to be hauled down a gradient of 1 in 60 at a maximum speed of 40 km/h and started on a level track at an acceleration of 0.1 m/s^2. How many locomotives, each weighing 75 tonne, will have to be employed ?

Train resistance during starting	=	29.4 N/tonne
Train resistance at 40 km/h	=	56.1 kg/tonne
Co-efficient of adhesion	=	1/3 ; Rotational inertia = 1/10

[3 Loco] (*Engg. Service Examination U.P.S.C.*)

15. A locomotive accelerates a 400-tonne train up a gradient of 1 in 100 at 0.8 km/h/s. Assuming the coefficient of adhesion to be 0.25, determine the minimum adhesive weight of the locomotive. Assume train resistance of 60 N/tonne and allow 10% for the effect of rotational inertia.

[65.7 t] (*Elect. Traction and Utilization, Nagpur Univ.*)

16. Calculate the specific energy consumption if a maximum speed of 12.20 metres/sec and for a given run of 1525 m an acceleration of 0.366 m/s^2 are desired. Train resistance during acceleration is 52.6 N/1000 kg and during coasting is 6.12 N/1000 kg, 10% being allowable for rotational inertia. The efficiency of the equipment during the acceleration period is 50%. Assume a quadrilateral speed-time curve. **[3.38 Wh/kg-m] (*Util. of Elect. Power, A.M.I.E. Sec. B*)**

17. An electric locomotive of 100 tonne can just accelerate a train of 500 tonne (trailing weight) with an acceleration of 1 km/h/s on an upgradient of 1/1000. Tractive resistance of the track is 45 N per tonne and the rotational inertia is 10%. If this locomotive is helpled by another locomotive of 120 tonnes, find, (*i*) the trailing weight that can be hauled up the same gradient under the same conditions and (*ii*) the maximum gradient, the trailing weight hauled remaining unchanged.

Assume adhesive weight expressed as percentage of total dead weight to be the same for both the locomotive. **[(*i*) 1120 t (*ii*) 3.15%] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)**

18. An electric train has quadrilateral speed-time curve as follows :

(*i*) uniform acceleration from rest at 2 km/h/s for 30 sec,

(*ii*) coasting for 50 sec.

(*iii*) uniform braking to rest for 20 seconds.

If the train is moving uniform upgradient of a 10/1000, train resistance is 40 N/tonne, rotational inertia effect 10% of dead weight and duration of stop 30 seconds, find the schedule speed.

[28.4 km/h] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)

19. The schedule speed with a 200 tonne train on an electric railway with stations 777 metres apart is 27.3 km/h and the maximum speed is 20% higher than the average running speed. The braking rate is 3.22 km/h/s and the duration of stops is 20 seconds. Find the acceleration required. Assume a simplified speed-time curve with the free running at the maximum speed.

[2.73 km/h/s] (*Traction & Util. of Elect. Power, Agra Univ.*)

20. An electric train has an average speed of 42 km/h on a level track between stops 1,400 metre apart. It is accelerated at 1.7 km/h/s and is braked at 3.3 km/h/s. Draw the speed-time curve for the run. Estimate the sp. energy consumption. Assume tractive resistance as 50 N/t and allow 10% for rotational inertia. **[39.48 Wh/t-km] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)**

21. An electric train weighing 200 tonne has eight motors geared to driving wheels, each wheel is 90 cm diameter. Determine the torque developed by each motor to accelerate the train to a speed of 48 km/h in 30 seconds up a gradient of 1 in 200. The tractive resistance is 50 N/t, the effect of rotational inertia is 10% of the train weight, the gear ratio is 4 to 1 and gearing efficiency is 80%.

[2,067 N-m] (*Traction & Util. of Elect. Power, Agra Univ.*)

22. An electric train accelerates uniformly from rest to a speed of 48 km/h in 24 seconds. It then coasts for 69 seconds against a constant resistance of 58 N/t and is braked to rest at 3.3 km/h/s in 11 seconds.

Calculate (*i*) the acceleration (*ii*) coasting retardation and (*iii*) the schedule speed, if the station stops are of 20 seconds duration. What would be the effect on schedule speed of reducing the station stops to 15 second duration, other conditions remaining the same? Allow 10% for the rotational inertia.

[(*i*) 2 km/h/s (*ii*) 0.19 km/h/s (*iii*) 30.25 km/h]
(Util. of Elect. Power, A.M.I.E. Sec. B.)

23. An electric train accelerates uniformly from rest to a speed of 50 km/h in 25 seconds. It then coasts for 1 minute 10 seconds against a constant resistance of 70 N/t and is braked to rest at 4 km/h/s in 10 seconds. Calculate the schedule speed, if the station stops are of 15 second duration.

[31.125 km/h] (*Util. of Elect. Power, A.M.I.E. Sec. B*)

24. An electric train has a quadrilatural speed-time curve as follows :

(*i*) uniform acceleration from rest at 2.5 km/h/s for 25 second

(*ii*) coasting for 50 second (*iii*) duration of braking 25 second.

If the train is moving along a uniform upgradient of 1 in 100 with a tractive resistance of 45 N/t, rotational inertia 10% of dead weight, duration of stops at stations 20 second and overall efficiency of transmission gear and motor 80%, calculate the schedule speed and specific energy consumption of run. **[69 km/h, 26.61 Wh/t-km] (*Util. of Elect. Power, A.M.I.E. Sec. B*)**

25. An ore-carrying train weighing 5000 tonne is to be hauled down a gradient of 1 in 50 at a maximum speed of 30 km/h and started on a level track at an acceleration of 0.08 m/s^2. How many locomotives, each weighing 75 tonne, will have to be employed ?

Train resistance during starting =3 kg/t

Train resistance at 30 km/h = 5 kg/t

Co-efficient of adhesion = 0.3, Rotational inertia = 10%.

[4 loco] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)

26. A train with an electric locomotive weighing 300 tonne is is to be accelerated up a gradient of 1 in 33 at an acceleration of 1 km/h/s. If the train resistance, co-efficient of adhesion and effect of rotational inertia are 80 N/t, 0.25 and 12.5% of the dead weight respectively, determine the minimum adhesive weight of the locomotive. **[88 t] *(Util. of Elect. Power, A.M.I.E. Sec. B.)***

27. A train weighing 400 tonne has speed reduced by regenerative braking from 40 to 20 km/h over a distance of 2 km at a down gradient of 20%. Calculate the electrical energy and average power returned to the line. Tractive resistance is 40 N/t and allow rotational inertia of 10% and efficiency of conversion 75%. **[324 kW/h, 4860 kW] (*Util. & Traction Power, Agra Univ.*)**

28. A 250-tonne motor coach having 4 motors, each developing 5,000 N-m torque acceleration, starts from rest. If upgradient is 25 in 1000, gear ratio 5, gear transmission efficiency 88%, wheel radius 44 cm, train resistance 50 N/t addition of rotational inertia 10%, calculate the time taken to reach a speed of 45 km/h.

If the supply voltage were 1500 V d.c. and efficiency of motor is 83.4%, determine the current drawn per motor during notching period. **[27.25 s, 500 A] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)**

29. An electric train weighing 100 tonne has a rotational inertia of 10%. This train while running between two stations which are 2.5 km apart has an average speed of 50 km/h. The acceleration and retardation during braking are respectively 1 km/h/s and 2 km/h/s. The percentage gradient between these two stations is 1% and the train is to move up the incline. The track resistance is 40 N/t. If the combined efficiency of the electric train is 60%, determine (*i*) maximum power at driving axle (*ii*) total energy consumption and (*iii*) specific energy consumption. Assume that journey estimation is being made in simplified trapezoidal speed-time curve.

[(*i*) 875 kW (*ii*) 23.65 kW/h (*iii*) 94.6 Wh/t-km] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)

30. A 500-tonne goods train is to be hauled by a locomotive up a gradient of 1 in 40 with an acceleration of 1.5 km/h/s. Determine the weight of the locomotive and number of axles, if axle load is not to exceed 24 tonne. Co-efficient of adhesion is 0.31, track resistance 45 N/t and effective rotating masses 10% of dead weight. **[7] *(Util. of Elect. Power, A.M.I.E. Sec. B.)***

31. Two d.c. series motors of a motor coach have resistance of 0.1 W each. These motors draw a current of 500 A from 600V mains during series-parallel starting period of 20 seconds. If the acceleration during starting period remains uniform, determine :

(*i*) time during which motor operates in (*a*) series, (*b*) parallel

(*ii*) the speed at which the series connections are to be changed if the speed just after starting period is 70 km/h.

[(*i*) 9.098, 10.971 Sec. (*ii*) 31.82 km/h] *(Utili. of Elect. Power and Traction, Agra Univ.)*

32. Explain how series motors are ideally suited for traction service. ***(Nagpur University, Summer 2004)***

33. Explain any one method for regenerative braking of D.C. motor for traction. ***(Nagpur University, Summer 2004)***

34. Discuss the effect of unequal wheel diameters on the parallel operation of traction motors. ***(Nagpur University, Summer 2004)***

35. Explain the various modes of operation in traction services with neat speed-time curve. ***(Nagpur University, Summer 2004)***

36. A 100 tonne motor coach is driven by 4 motors, each developing a torque of 5000 N-m during acceleration. If up-gradient is 50 in 1000, gear ratio a = 0.25, gear transmission efficiency 98%, wheel radius 0.54 M, train resistance 25 N/tonne, effective mass on account of rotational inertia is 10% higher, calculate the time taken to attain a speed of 100 kmph. ***(Nagpur University, Summer 2004)***

37. What are the requirements of an ideal traction system? ***(J.N. University, Hyderabad, November 2003)***

38. What are the advantages and disadvantages of electric traction? ***(J.N. University, Hyderabad, November 2003)***

39. Write a brief note on the single phase a.c. series motor and comment upon it's suitability for traction services. How does it compare in performance with the d.c. Services motor. ***(J.N. University, Hyderabad, November 2003)***

40. Draw the speed-time curve of a main line service and explain. ***(J.N. University, Hyderabad, November 2003)***

41. A train hs a scheduled speed of 40 km/hr between two stops, which are 4 kms apart. etermine the crest speed over the run, if the duration of stops is 60 sec and acceleration and retardation both are 2 km/hr/sec each. Assume simplified trapezoidal speed-time curve. ***(J.N. University, Hyderabad, November 2003)***

42. What are the various electric traction systems in India? Compare them. ***(J.N. University, Hyderabad, November 2003)***

43. Give the features of various motors used in electric traction. ***(J.N. University, Hyderabad, November 2003)***

44. Draw the speed-time curve of a suburban service train and explain. ***(J.N. University, Hyderabad, November 2003)***

45. A train accelerates to a speed of 48 km/hr in 24 sec. then it coasts for 69 sec under a constant resistance of 58 newton/tonne and brakes are applied at 3.3 km/hr/sec in 11 sec. Calculate (*i*) the acceleration (*ii*) the coasting retardation (*iii*) the scheduled speed if station stoppage is 20 secs. What is theeffect of scheduled speed if station stoppage is reduced to 15 sec duration, other conditions remaining same. Allow 10% for rotational inertia. ***(J.N. University, Hyderabad, November 2003)***

46. Derive an expression for specific energy output on level track using a simplified speed-time curve. What purpose is achieved by this quantity? *(J.N. University, Hyderabad, November 2003)*

47. A 400 tonne goods train is to be hauled by a locomotive up a gradient of 2% with acceleration of 1 km/hr/sec, coefficient of adhesion 20%, track resistance 40 newtons/tonne and effective rotating masses 10% of the dead weight. Find the weight of the locomotive and the number of axles if the axle load is not to increase beyond 22 tonnes. *(J.N. University, Hyderabad, November 2003)*

48. A motor has the following load cycle :
Accelerating period 0-15 sec Load rising uniformly from 0 to 1000 h.p.
Full speed period 15-85 sec Load constant at 600 h.p.
Decelerating period 85-100 sec h.p. returned to line falls uniformly from 200 to zero
Decking period 100-120 sec Motor stationary. Estimate the size of the motor.
(J.N. University, Hyderabad, November 2003)

49. Explain the characteristics of series motors and also explain how they are suitable of electric traction work? *(J.N. University, Hyderabad, November 2003)*

50. For a trapezoidal speed-time curve of a electric train, derive expression for maximum speed and distance between stops. *(J.N. University, Hyderabad, November 2003)*

51. A mail is to be run between two stations 5 kms apart at an average speed of 50 km/hr. If the maximum speed is to be limited to 70 km/hr, acceleration to 2 km/hr/sec, braking retardation to 4 km/hr/sec and coasting retardation to 0.1 km/hr/sec, determine the speed at the end of coasting, duration of coasting period and braking period. *(J.N. University, Hyderabad, November 2003)*

52. Discuss the merits and demerits of the D.C. and 1–ϕ A.C. systems for the main and suburban line electrification of the railways. *(J.N. University, Hyderabad, April 2003)*

53. Which system do you consider to be the best for the suburban railways in the vicinity of large cities? Given reasons for your answer. *(J.N. University, Hyderabad, April 2003)*

54. Derive expression for the tractive effort for a train on a level track. *(J.N. University, Hyderabad, April 2003)*

55. The maximum speed of a suburbanelectric train is 60 km/hr. Its scheduled speed is 40 km/hr and duration of stops is 30 sec. If the acceleration is 2 km/hr/sec and distance between stops is 2 kms, determine the retardation. *(J.N. University, Hyderabad, April 2003)*

56. What are various types of traction motors? *(J.N. University, Hyderabad, April 2003)*

57. What are the advantages of series parallel control of D.C. motors? *(J.N. University, Hyderabad, April 2003)*

58. Describe about duplication of railway transmission lines. *(J.N. University, Hyderabad, April 2003)*

59. Write a note on feeding and distributing system on A.C. Traction and for d.c. tram ways. *(J.N. University, Hyderabad, April 2003)*

60. For a quadrilateral speed-time curve of a electric train, derive expression for the distance between stops and speed at the end of the coasting period. *(J.N. University, Hyderabad, April 2003)*

61. A train is required to run between stations 1.6 kms apart at an average speed of 40 km/hr. The runis to be made from a quadilateral speed-time curve. The acceleration is 2 km/hr/sec. The coasting and braking retardations are 0.16 km/hr/sec and 3.2 km/hr/sec respectively. Determine the duration of acceleration, coasting and braking and the distance covered in each period. *(J.N. University, Hyderabad, April 2003)*

62. Explain the characteristics of D.C. compound motors and explain its advantage over the series motor. *(J.N. University, Hyderabad, April 2003)*

63. What are the requirements to be satisfied by an ideal traction system? *(J.N. University, Hyderabad, April 2003)*

64. What are the advantages and disdvantages of electrification of track? *(J.N. University, Hyderabad, April 2003)*

65. Discuss why a D.C. series motor is ideally suited for traction services.
(J.N. University, Hyderabad, April 2003)

66. An electric locomotive of 100 tonnes can just accelerate a train of 500 tonnes (trailing weight) with an acceleration of 1 km/hr/sec on an up gradient 1 in 1000. Tractive resistance of the track is 45 newtons/tonne and the rotational inertia is 10%. If this locomotive is helped by another locomotive of 120 tonnes, find (i) the trailing weight that can be hauled up the same gradient, under the same condition (ii) the maximum gradient, the trailing hauled load remaining unchanged. Assume adhesive weight expressed as percentage of total dead weight to be same for both the locomotives. *(J.N. University, Hyderabad, April 2003)*

67. Explain how electric regeneration braking is obtained with a D.C. locomotive. How is the braking torque varied? *(J.N. University, Hyderabad, April 2003)*

68. Explain why a series motor is preferred for the electric traction.
(J.N. University, Hyderabad, April 2003)

69. The characteristics of a series motor at 525 – V are as follows :

Current (A)	50	100	150	200
Speed (RPM)	1200	952	840	745

Determine the current when working as a generator at 1000 R.P.M. and loaded with a resistance of 3 ohms. The resistance of the motor is 0.5 ohms. *(J.N. University, Hyderabad, April 2003)*

70. Briefly explain the a.c. motors used in traction. *(J.N. University, Hyderabad, April 2003)*

71. The scheduled speed of a trolley service is to be 53 km/hr. The distance between stops is 2.8 km. The track is level and each stop is of 30 sec duration. Using simplified speed-time curve, calculate the maximum speed, assuming the acceleration to be 2 km/hr/sec, retardation 3.2 km/hr/sec, the dead weight of the car as 16 tonnes, rotational inertia as 10% of the dead weight and track resistance as 40 newtons/tonne. If the overall efficiency is 80%, calculate (*i*) the maximum power output from the driving axles (*ii*) the specific energy consumption in watt-hr/tonne-km.
(J.N. University, Hyderabad, April 2003)

72. Discuss various traction systems you know of?
(J.N. University, Hyderabad, December 2002/January 2003)

73. Explain the requirements for ideal traction and show which drive satisfies almost all the requirements.
(J.N. University, Hyderabad, December 2002/January 2003)

74. Define the adhesive weight of a locomotive which accelerates up a gradient of 1 in 100 at 0.8 kmphps. The self weight of locomotive is 350 Tonnes. Coefficient of adhesion is 0.25. Assume a trainresistance of 45 N–m/Tonne and allow 10% for the effect of rotational inertia.
(J.N. University, Hyderabad, December 2002/January 2003)

75. State Factors affecting specific energy consumption.
(J.N. University, Hyderabad, December 2002/January 2003)

76. Explain with the help of a diagram, the four quadrant speed-torque characteristic of an induction motor when running in (*i*) forward direction (*ii*) reverse direction.
(J.N. University, Hyderabad, December 2002/January 2003)

77. Explain the general features of traction motors.
(J.N. University, Hyderabad, December 2002/January 2003)

78. A 250 tonne electric train maintains a scheduled speed of 30 kmph between stations situated 5 km apart, with station stops of 30 sec. The acceleration is 1.8 kmph ps and the braking retardation is 3 kmph ps. Assuming a trapezoidal speed-time curve, calculate (*i*) maximum speed of the train (*ii*) energy output of the motors if the tractive resistance is 40 NW per tonne.
(J.N. University, Hyderabad, December 2002/January 2003)

79. Discuss the relative merits of electric traction and the factors on which the choice of traction system depends. *(J.N. University, Hyderabad, December 2002/January 2003)*

80. Explain the terms (*i*) tractiveeffort (*ii*) coefficient of adhesion (*iii*) specific energy consumption of train (*iv*) tractive resistance. *(J.N. University, Hyderabad, December 2002/January 2003)*

81. Existing traction systems in India.*(J.N. University, Hyderabad, December 2002/January 2003)*

82. Explain the terms tractive effort, coefficient of adhesion, train resistance and specific energy consumption of train. *(J.N. University, Hyderabad, December 2002/January 2003)*

83. An electric train maintains a scheduled speed of 40 kmph between stations situated at 1.5 km apart. If is accelerated at 1.7 kmph.ps and is braked at 3.2 kmph.ps. Draw the speed-time curve for the run. Estimate the energy consumption at the axle of the train. Assume tractive resistance constants at 50 NW per tonne and allow 10% for the effect of rotation inertia. *(J.N. University, Hyderabad, December 2002/January 2003)*

84. Explain the advantages of series parallel control of starting as compared to the rheostatic starting for a pair of dc traction motors. *(J.N. University, Hyderabad, December 2002/January 2003)*

85. Discuss the main features of various train services. What type of services correspond to trapezoidal and quadrilateral speed-time curves.*(J.N. University, Hyderabad, December 2002/January 2003)*

86. Existing electric traction system in India. *(J.N. University, Hyderabad, December 2002/January 2003)*

87. Briefly explain the controlling of D.C. Motor. *(Anna Univ., Chennai 2003)*

OBJECTIVE TESTS – 43

1. Diesel electric traction has comparatively limited overload capacity because
(*a*) diesel electric locomotive is heavier than a plain electric locomotive
(*b*) diesel engine has shorter life span
(*c*) diesel engine is a constant-kW output prime mover
(*d*) regenerative braking cannot be employed.

2. The most vital factor against electric traction is the
(*a*) necessity of providing a negative booster
(*b*) possibility of electric supply failure
(*c*) high cost of its maintenance
(*d*) high initial cost of laying out overhead electric supply system.

3. The direct current system used for tramways has a voltage of about ....volt.
(*a*) 750
(*b*) 1500
(*c*) 3000
(*d*) 2400

4. In electric traction if contact voltage exceeds 1500 V, current collection is invariably via a
(*a*) contact rail
(*b*) overhead wire
(*c*) third rail
(*d*) conductor rail.

5. For the single-phase ac system of track electrification, low frequency is desirable because of the following advantages
(*a*) it improves commutation properties of ac motors
(*b*) it increases ac motor efficiency
(*c*) it increases ac motor power factor
(*d*) all of the above.

6. In Kando system of track electrification, ...is converted into ....
(*a*) 1-phase ac, dc
(*b*) 3-phase ac, 1-phase ac
(*c*) 1-phase ac, 3-phase ac
(*d*) 3-phase ac, dc.

7. The main reason for choosing the composite 1-phase ac-to-dc system for all future track electrification in India is that it
(*a*) needs less number of sub-stations
(*b*) combines the advantages of high-voltage ac distribution at 50 Hz with dc series traction motors
(*c*) provides flexibility in the location of sub-stations
(*d*) requires light overhead catenary.

8. Ordinary, tramway is the most economical means of transport for
(*a*) very dense traffic of large city
(*b*) medium traffic densities
(*c*) rural services
(*d*) suburban services.

9. Unlike a tramway, a trolleybus requires no
(*a*) overhead contact wire
(*b*) driving axles
(*c*) hand brakes
(*d*) running rail.

10. The current collector which can be used at different speeds under all wind conditions and stiffness of OHE is called .... collector.
(*a*) trolley
(*b*) bow
(*c*) pantograph
(*d*) messenger.

11. The speed/time curve for city service has no........................ period.
(*a*) coasting
(*b*) free-running
(*c*) acceleration
(*d*) braking.

12. For the same value of average speed, increase in the duration of stops............. speed.
(*a*) increases the schedule
(*b*) increases the crest
(*c*) decreases the crest
(*d*) decreases the schedule.

13. A train weighing 490 tonne and running at 90 km/h has a mass of .... kg and a speed of .... m/s.
(*a*) 50,000, 25
(*b*) 490,000, 25
(*c*) 490, 25
(*d*) 50, 324.

14. A train has a mass of 500 tonne. Its weight is
(*a*) 500 t.wt
(*b*) 500,000 kg-wt
(*c*) 4,900,000 newton
(*d*) all of the above
(*e*) none of the above.

15. The free-running speed of a train does NOT depend on the
(*a*) duration of stops
(*b*) distance between stops
(*c*) running time
(*d*) acceleration.

16. A motor coach weighing 100 tonnes is to be given an acceleration of 1.0 km/h/s on an ascending gradient of 1 percent. Neglecting rotational inertia and train resistance, the tractive force required is .... newton.
(*a*) 109,800
(*b*) 37,580
(*c*) 28,760
(*d*) 125,780.

17. In a train, the energy output of the driving axles in used for
(*a*) accelerating the train
(*b*) overcoming the gradient
(*c*) overcoming train resistance
(*d*) all of the above.

18. Longer coasting period for a train results in
(*a*) higher acceleration
(*b*) higher retardation
(*c*) lower specific energy consumption
(*d*) higher schedule speed.

19. Tractive effort of an electric locomotive can be increased by
(*a*) increasing the supply voltage
(*b*) using high kW motors
(*c*) increasing dead weight over the driving axles
(*d*) both (*b*) and (*c*) (*e*)both (*a*) and (*b*).

20. Skidding of a vehicle always occurs when
(*a*) braking effort exceeds its adhesive weight
(*b*) it negotiates a curve
(*c*) it passes over points and crossings
(*d*) brake is applied suddenly.

21. Which of the following is an advantage of electric traction over other methods of traction?
(*a*) Faster acceleration
(*b*) No pollution problems
(*c*) Better braking action
(*d*) All of the above

22. Which of the following is the voltage for single phase A.C. system?
(*a*) 22 V
(*b*) 440 V
(*c*) 5 kV
(*d*) 15 kV
(*e*) None of the above

23. Long distance railways use which of the following?
(*a*) 200 V D.C.
(*b*) 25 kV single phase A.C.
(*c*) 25 kV two phase A.C.
(*d*) 25 kV three phase A.C.

24. The speed of a locomotive is controlled by
(*a*) flywheel
(*b*) gear box
(*c*) applying brakes
(*d*) regulating steam flow to engine

25. Main traction system used in India are, those using
 (*a*) electric locomotives
 (*b*) diesel engine locomotives
 (*c*) steam engine locomotives
 (*d*) diesel electric locomotives
 (*e*) all of the above
26. In India diesel locomotives are manufactured at
 (*a*) Ajmer
 (*b*) Varanasi
 (*c*) Bangalore
 (*d*) Jamalpur
27. For diesel locomotives the range of horsepower is
 (*a*) 50 to 200
 (*b*) 500 to 1000
 (*c*) 1500 to 2500
 (*d*) 3000 to 5000
28. .... locomotive has the highest operational availability.
 (*a*) Electric
 (*b*) Diesel
 (*c*) Steam
29. The horsepower of steam locomotives is
 (*a*) upto 1500
 (*b*) 1500 to 2000
 (*c*) 2000 to 3000
 (*d*) 3000 to 4000
30. The overall efficiency of steam locomotive is around
 (*a*) 5 to 10 percent
 (*b*) 15 to 20 percent
 (*c*) 25 to 35 percent
 (*d*) 35 to 45 percent
31. In tramways which of the following motors is used?
 (*a*) D.C. shunt motor
 (*b*) D.C. series motor
 (*c*) A.C. three phase motor
 (*d*) A.C. single phase capacitor start motor
32. In a steam locomotive electric power is provided through
 (*a*) overhead wire
 (*b*) battery system
 (*c*) small turbo-generator
 (*d*) diesel engine generator
33. Which of the following drives is suitable for mines where explosive gas exists?
 (*a*) Steam engine
 (*b*) Diesel engine
 (*c*) Battery locomotive
 (*d*) Any of the above
34. In case of locomotives the tractive power is provided by
 (*a*) single cylinder double acting steam engine
 (*b*) double cylinder, single acting steam engine
 (*c*) double cylinder, double acting steam engine
 (*d*) single stage steam turbine
35. Overload capacity of diesel engines is usually restricted to
 (*a*) 2 percent
 (*b*) 10 percent
 (*c*) 20 percent
 (*d*) 40 percent
36. In case of steam engines the steam pressure is
 (*a*) 1 to 4 kgf/cm^2
 (*b*) 5 to 8 kgf/cm^2
 (*c*) 10 to 15 kgf/cm^2
 (*d*) 25 to 35 kgf/cm^2
37. The steam engine provided on steam locomotives is
 (*a*) single acting condensing type
 (*b*) single acting non-condensing type
 (*c*) double acting condensing type
 (*d*) double acting non-condensing type
38. Electric locomotives in India are manufactured at
 (*a*) Jamalpur
 (*b*) Bangalore
 (*c*) Chittranjan
 (*d*) Gorakhpur
39. The wheels of a train, engine as well as bogies, are slightly tapered to
 (*a*) reduce friction
 (*b*) increase friction
 (*c*) facilitate braking
 (*d*) facilitate in taking turns
40. Automatic signalling is used for which of the following trains?

(a) Mail and express trains
(b) Superfast trains
(c) Suburban and Urban electric trains
(c) All trains

41. The efficiency of diesel locomotives is nearly
(a) 20 to 25 percent
(b) 30 to 40 percent
(c) 45 to 55 percent
(d) 60 to 70 percent

42. The speed of a superfast train is
(a) 60 kmph
(b) 75 kmph
(c) 100 kmph
(d) more than 100 kmph

43. The number of passenger coaches that can be attached to a diesel engine locomotive on broad gauge is usually restricted to
(a) 5
(b) 10
(c) 14
(d) 17

44. Which of the following state capitals is not on broad gauge track?
(a) lucknow
(b) Bhopal
(c) Jaipur
(d) Chandigarh

45. Which of the following is the advantage of electric braking?
(a) It avoids wear of track
(b) Motor continues to remain loaded during braking
(c) It is instantaneous
(d) More heat is generated during braking

46. Which of the following braking systems on the locomotives in costly?
(a) Regenerative braking on electric locomotives
(b) Vacuum braking on diesel locomotives
(c) Vacuum braking on steam locomotives
(d) All braking systems are equally costly

47. Tractive effort is required to
(a) overcome the gravity component of train mass
(b) overcome friction, windage and curve resistance
(c) accelerate the train mass
(d) do all of the above

48. For given maximum axle load tractive efforts of A.C. locomotive will be
(a) less than that of D.C. locomotive
(b) more than that of D.C. locomotive
(c) equal to that of D.C. locomotive
(d) none of the above

49. Co-efficient of adhesion reduces due to the presence of which of the following?
(a) Sand on rails
(b) Dew on rails
(c) Oil on the rails
(d) both (b) and (c)

50. Due to which of the following co-efficient of adhesion improves?
(a) Rust on the rails
(b) Dust on the rails
(c) Sand on the rails
(d) All of the above

51. Quadrilateral speed-time curve pertains to which of the following services?
(a) Main line service
(b) Urban service
(c) Sub-urban service
(d) Urban and sub-urban service

52. Which of the following is the disadvantage of electric traction over other systems of traction?
(a) Corrosion problems in the underground pip work
(b) Short time power failure interrupts traffic for hours
(c) High capital outlay in fixed installations beside route limitation
(d) Interference with communication lines
(e) All of the above

53. Co-efficient of adhesion is
(a) high in case of D.C. traction than in the case of A.C. traction
(b) low in case of D.C. traction that in the case of A.C. traction
(c) equal in both A.C. and D.C. traction
(d) any of the above

54. Speed-time curve of main line service differs from thoseof urban and suburban services on following account
(a) it has longer free running period
(b) it has longer coasting period

(c) accelerating and braking periods are comparatively smaller
(d) all of the above

55. The rate of acceleration on suburban or urban services is restricted by the consideration of
(a) engine power
(b) track curves
(c) passanger discomfort
(d) track size

56. The specific energy consumption of a train depends on which of the following?
(a) Acceleration and retardation
(b) Gradient
(c) Distance covered
(d) all of the above

57. The friction at the track is proportional to
(a) 1/speed
(b) $1/(\text{speed})^2$
(c) speed
(d) none of the above

58. The air resistance to the movement of the train is proportional to
(a) speed
(b) $(\text{speed})^2$
(c) $(\text{speed})^3$
(d) 1/speed

59. The normal value of adhesion friction is
(a) 0.12
(b) 0.25
(c) 0.40
(d) 0.75

60. The pulsating torque exerted by steam locomotives causes which of the following?
(a) Jolting and skidding
(b) Hammer blow
(c) Pitching
(d) All of the above

61. Which of the following braking systems is used on steam locomotives?
(a) Hydraulic system
(b) Pneumatic system
(c) Vacuum system
(d) None of the above

62. Vacuum is created by which of the following?
(a) Vacuum pump
(b) Ejector
(c) Any of the above
(d) None of the above

63. The resistance encountered by a train in motion is on account of
(a) resistance offered by air
(b) friction at the track
(c) friction at various parts of the rolling stock
(d) all of the above

64. Battery operated trucks are used in
(a) steel mills
(b) power stations
(c) narrow gauge traction
(d) factories for material transportation

65. .... method can bring the locomotive todead stop.
(a) Plugging braking
(b) Rheostatic braking
(c) Regenerative braking
(d) None of the above

66. The value of co-efficient of adhesion will be high when rails are
(a) greased
(b) wet
(c) sprayed with oil
(d) cleaned with sand

67. The voltage used for suburban trains in D.C. system is usually
(a) 12 V
(b) 24 V
(c) 220 V
(d) 600 to 750 V

68. For three-phase induction motors which of the following is the least efficient method of speed control?
(a) Cascade control
(b) Pole changing
(c) Rheostatic control
(d) Combination of cascade and pole changing

69. Specific energy consumption becomes
(a) more on steeper gradient
(b) more with high train resistance
(c) less if distance between stops is more
(d) all of the above

70. In main line service as compared to urban and suburban service

(*a*) distance between the stops is more
(*b*) maximum speed reached is high
(*c*) acceleration and retardation rates are low
(*d*) all of the above

71. Locomotive having monomotor bogies
(*a*) has better co-efficient of adhesion
(*b*) are suited both for passanger as well as freight service
(*c*) has better riding qualities due to the reduction of lateral forces
(*d*) has all above qualities

72. Series motor is not suited for traction duty due to which of the following account?
(*a*) Less current drain on the heavy load torque
(*b*) Current surges after temporary switching off supply
(*c*) self relieving property
(*d*) Commutating property at heavy load

73. When a bogie negotiates a curve, reduction in adhesion occurs resulting in sliding. Thus sliding is acute when
(*a*) wheel base of axles is more
(*b*) degree of curvature is more
(*c*) both (a) and (b)
(*c*) none of the above

74. Energy consumption in propelling the train is required for which of the following?
(*a*) Work against the resistance to motion
(*b*) Work against gravity while moving up the gradient
(*c*) Acceleration
(*d*) All of the above

75. An ideal traction system should have ....
(*a*) easy speed control
(*b*) high starting tractive effort
(*c*) equipment capable of with standing large temporary loads
(*d*) all of the above

76. .... have maximum unbalanced forces
(*a*) Diesel shunters
(*b*) Steam locomotives
(*c*) Electric locomotives
(*d*) Diesel locomotives

77. Specific energy consumption is affected by which of the following factors?
(*a*) Regardation and acceleration values
(*b*) Gradient
(*c*) Distance between stops
(*d*) All of the above

78. In case of .... free running and coasting periods are generally long.
(*a*) main-line service
(*b*) urban wervice
(*c*) sub-urban service
(*d*) all of the above

79. Overhead lines for power supply to tramcars are at a minimum height of
(*a*) 3 m
(b) 6 m
(*c*) 10 m
(*d*) 20 m

80. The return circuit for tram cars is through ....
(*a*) neutral wire
(*b*) rails
(*c*) cables
(*d*) common earthing

81. Specific energy consumption is least in .... service.
(*a*) main line
(*b*) urban
(*c*) suburban

82. Locomotives with monometer bogies have
(*a*) uneven distribution of tractive effect
(*b*) suitability for passanger as well as freight service
(*c*) lot of skidding
(*d*) low co-efficient of adhesion

83. .... was the first city in India to adopt electric traction.
(*a*) Delhi
(*b*) Madras
(*c*) Calcutta
(*d*) Bombay

84. .... frequency is not common in low frequency traction system
(*a*) 40 Hz
(*b*) 25 Hz
(*c*) 16Hz

85. For 25 kV single phase system power supply frequency is ....
(*a*) 60 Hz
(*b*) 50 Hz
(*c*) 25 Hz
(*d*) $16\frac{2}{3}$ Hz

86. Power for lighting in passenger coach, in a long distance electric train, is provided
(*a*) directly through overhead electric
(*b*) through individual generator of bogie and batteries
(*c*) through rails
(*d*) through locomotive

87. In India, electrification of railway track was done for the first time in which of the following years?
(*a*) 1820–1825
(*b*) 1880–1885
(*c*) 1925–1932
(*d*) 1947–1954

88. Suri transmission is ....
(*a*) electrical-pneumatic
(*b*) mechanical-electrical
(*c*) hydro-mechanical
(*d*) hydro-pneumatic

89. In case of a steam engine an average coat consumption per km is nearly
(*a*) 150 to 175 kg
(*b*) 100 to 120 kg
(*c*) 60 to 80 kg
(*d*) 28 to 30 kg

90. Which of the following happens in Kando system?
(*a*) Three phase A.C. is converted into D.C.
(*b*) Single phase A.C. is converted into D.C.
(*c*) Single phase supply is converted into three phase system
(*d*) None of the above

91. For which of the following locomotives the maintenance requirements are the least?
(*a*) Steam locomotives
(*b*) Diesel locomotives
(*c*) Electric locomotives
(*d*) Equal in all of the above

92. Which of the following methods is used to control speed of 25 kV, 50 Hz single phase traction?
(*a*) Reduced current method
(*b*) Tapchanging control of transformer
(*c*) Series parallel operation of motors
(*d*) All of the above

93. If the co-efficient of adhesion on dry rails is 0.26, which of the following could be the value for wet rails?
(*a*) 0.3
(*b*) 0.26
(*c*) 0.225
(*d*) 0.16

94. .... watt-hours per tonne km is usually the specific energy consumption for suburban services.
(*a*) 15–20
(*b*) 50–75
(*c*) 120–150
(*d*) 160–200

95. The braking retardation is usually in the range
(*a*) 0.15 to 0.30 km phps
(*b*) 0.30 to 0.6 km phps
(*c*) 0.6 to 2.4 km phps
(*d*) 3 to 5 km phps
(e) 10 to 15 km phps

96. The rate of acceleration on suburban or urban service is in the range
(*a*) 0.2 to 0.5 km phps
(*b*) 1.6 to 4.0 km phps
(*c*) 5 to 10 km phps
(*d*) 15 to 25 km phps

97. The coasting retardation is around
(*a*) 0.16 km phps
(b) 1.6 km phps
(*c*) 16 km phps
(*d*) 40 km phps

98. which of the following track is electrified
(*a*) Delhi–Bombay
(*b*) Delhi–Madras
(*c*) Delhi–Howrah
(*d*) Delhi–Ahmedabad

99. .... is the method of braking in which motor armature remains connected to the supply and draws power from it producing torque opposite to the direction of motion.
(*a*) Rheostatic braking
(*b*) Regerative braking
(*c*) Plugging

100. For 600 V D.C. line for tramcars, brack is connected to ....
(*a*) positive of the supply
(*b*) negative of the supply
(*c*) mid voltage of 300 V
(*d*) none of the above

ANSWERS

1. (*c*)	**2.** (*d*)	**3.** (*a*)	**4.** (*b*)	**5.** (*d*)	**6.** (*c*)	**7.** (*b*)	**8.** (*a*)	**9.** (*d*)	**10.** (*c*)
11. (*b*)	**12.** (*d*)	**13.** (*b*)	**14.** (*d*)	**15.** (*a*)	**16.** (*b*)	**17.** (*d*)	**18.** (*c*)	**19.** (*d*)	**20.** (*a*)
21. (*d*)	**22.** (*d*)	**23.** (*b*)	**24.** (*d*)	**25.** (*e*)	**26.** (*b*)	**27.** (*c*)	**28.** (*a*)	**29.** (*a*)	**30.** (*a*)
31. (*b*)	**32.** (*c*)	**33.** (*c*)	**34.** (*c*)	**35.** (*b*)	**36.** (*c*)	**37.** (*d*)	**38.** (*c*)	**39.** (*d*)	**40.** (*c*)
41. (*a*)	**42.** (*d*)	**43.** (*d*)	**44.** (*c*)	**45.** (*a*)	**46.** (*a*)	**47.** (*d*)	**48.** (*b*)	**49.** (*d*)	**50.** (*d*)
51. (*d*)	**52.** (*e*)	**53.** (*b*)	**54.** (*d*)	**55.** (*c*)	**56.** (*d*)	**57.** (*c*)	**58.** (*b*)	**59.** (*b*)	**60.** (*a*)
61. (*c*)	**62.** (*c*)	**63.** (*d*)	**64.** (*d*)	**65.** (*a*)	**66.** (*d*)	**67.** (*d*)	**68.** (*c*)	**69.** (*d*)	**70.** (*d*)
71. (*d*)	**72.** (*b*)	**73.** (*c*)	**74.** (*d*)	**75.** (*d*)	**76.** (*b*)	**77.** (*d*)	**78.** (*a*)	**79.** (*c*)	**80.** (*b*)
81. (*a*)	**82.** (*b*)	**83.** (*d*)	**84.** (*a*)	**85.** (*b*)	**86.** (*b*)	**87.** (*c*)	**88.** (*c*)	**89.** (*d*)	**90.** (*c*)
91. (*c*)	**92.** (*b*)	**93.** (*d*)	**94.** (*b*)	**95.** (*d*)	**96.** (*b*)	**97.** (*a*)	**98.** (*c*)	**99.** (*c*)	**100.** (*b*)

ROUGH WORK

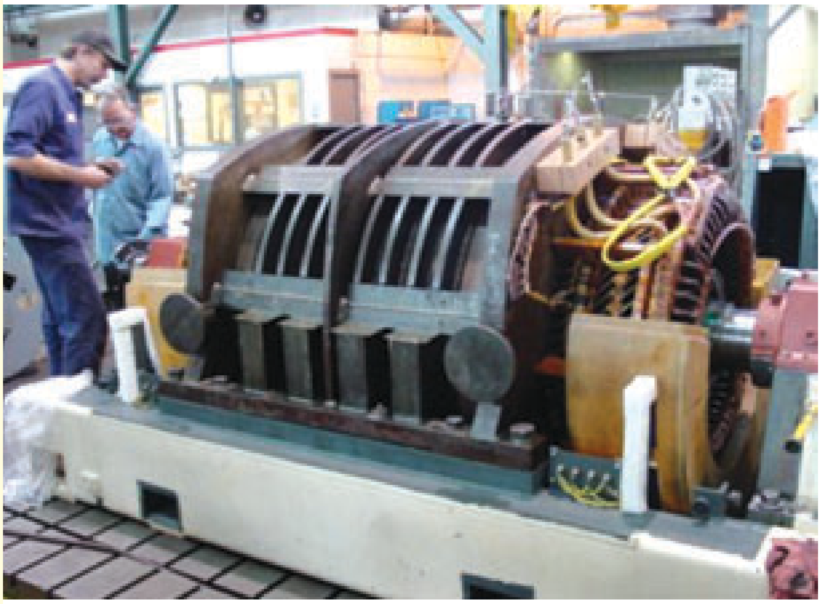
CONTENTS

CHAPTER 44

INDUSTRIAL APPLICATIONS OF ELECTRIC MOTORS

Learning Objectives

- Advantages of Electric Drive
- Classification of Electric Drives
- Advantages of Individual Drives
- Selection Drive
- Electric Characteristics
- Types of Enclosures
- Bearings Transmission of Power
- Noise
- Size and Rating
- Estimation of Motor Rating
- Different Types of Industrial Loads
- Motors for Different Industrial Drives
- Types of Electric Braking
- Plugging Applied to DC Motors
- Plugging of Induction Motors
- Rheostatic Braking
- Rheostatic Braking of DC Motors
- Rheostatic Braking Torque
- Rheostatic Braking of Induction Motors
- Regenerative Braking
- Energy Saving in Regenerative Braking



The above figure shows a squirrel cage motor. In industries electric drive is preferred over mechanical drive, because electric drive has the advantages of quick start, high torques and comparatively hassle free operation

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44.1. Advantages of Electric Drive

Almost all modern industrial and commercial undertakings employ electric drive in preference to mechanical drive because it possesses the following advantages :

1. It is simple in construction and has less maintenance cost
2. Its speed control is easy and smooth
3. It is neat, clean and free from any smoke or flue gases
4. It can be installed at any desired convenient place thus affording more flexibility in the layout
5. It can be remotely controlled
6. Being compact, it requires less space
7. It can be started immediately without any loss of time
8. It has comparatively longer life.

However, electric drive system has two inherent disadvantages :

1. It comes to stop as soon as there is failure of electric supply and
2. It cannot be used at far off places which are not served by electric supply.

However, the above two disadvantages can be overcome by installing diesel-driven dc generators and turbine-driven 3-phase alternators which can be used either in the absence of or on the failure of normal electric supply.

44.2. Classification of Electric Drives

Electric drives may be grouped into three categories : group drive, individual drive and multimotor drive.

In group drive, a single motor drives a number of machines through belts from a common shaft. It is also called line shaft drive. In the case of an indvidual drive, each machine is driven by its own separate motor with the help of gears, pulley etc. In multi-motor drives separate motors are provided for actuating different parts of the driven mechanism. For example, in travelling cranes, three motors are used : one for hoisting, another for long travel motion and the third for cross travel motion. Multimotor drives are commonly used in paper mills, rolling mills, rotary printing presses and metal working machines etc.

Each type of electric drive has its own advantages and disadvantages. The group drive has following advantages :

1. It leads to saving in initial cost because one 150-kW motor costs much less than ten 15-kW motors needed for driving 10 separate machines.
2. Since all ten motors will seldomly be required to work simultaneously, a single motor of even 100-kW will be sufficient to drive the main shaft. This diversity in load reduces the initial cost still further.
3. Since a single large motor will always run at full-load, it will have higher efficiency and power factor in case it is an induction motor.
4. Group drive can be used with advantage in those industrial processes where there is a sequence of continuity in the operation and where it is desirable to stop these processes simultaneously as in a flour mill.

However, group drive is seldom used these days due to the following disadvantages :

1. Any fault in the driving motor renders all the driven equipment idle. Hence, this system is unreliable.
2. If all the machines driven by the line shaft do not work together, the main motor runs at reduced load. Consequently, it runs with low efficiency and with poor power factor.

3. Considerable amount of power is lost in the energy transmitting mechanism.
4. Dlexibility of layout of different machines is lost since they have to be so located as to suit the position of the line shaft.
5. The use of line shaft, pulleys and belts etc. makes the drive look quite untidy and less safe to operate.
6. It cannot be used where constant speed is required as in paper and textile industry.
7. Noise level at the worksite is quite high.

44.3. Advantages of Individual Drive

It has the following advantages :

1. Since each machine is driven by a separate motor, it can be run and stopped as desired.
2. Machines not required can be shut down and also replaced with a minimum of dislocation.
3. There is flexibility in the installation of different machines.
4. In the case of motor fault, only its connected machine will stop whereas others will continue working undisturbed.
5. The absence of belts and line shafts greatly reduces the risk of accidents to the operating personnel.
6. Ach operator has full control of the machine which can be quickly stopped if an accident occurs.
7. Maintenance of line shafts, bearings, pulleys and belts etc. is eliminated. Similarly there is no danger of oil falling on articles being manufactured–something very important in textile industry.

The only disadvantage of individual drive is its initial high cost (Ex 44.1). However, the use of individual drives and multimotor drives has led to the introduction of automation in production processes which, apart from increasing the productivity of various undertakings, has increased the reliability and safety of operation.

Example 44.1. *A motor costing Rs. 10,000/- is used for group drive in a certain installation. How will its total annual cost compare with the case where four individual motors each costing Rs. 4000/- were used ? With group drive, the energy consumption is 50 MWh whereas it is 45 MWh for individual drive. The cost of electric energy is 20 paise/kWh. Assume depreciation, maintenance and other fixed charges at 10% in the case of group drive and 15 per cent in the case of individual drive.*

Solution. Group Drive

Capital cost = Rs. 10,000/-

Annual depreciation, maintenance and other fixed charges = 10% of Rs. 10,000 = Rs. 1,000/-

Annual cost of energy = Rs. $50 \times 10^3 \times (20/100)$ = Rs. 10,000/-

Total annual cost = Rs. 1,000 + Rs. 10,000 = **Rs. 11,000/-**

Individual Drive

Capital cost = $4 \times$ Rs. 4000 = Rs. 16,000/-

Annual depreciation, maintenance and other fixed charges = 15% of Rs. 16,000 = Rs. 2400/-

Annual cost of energy = Rs. $45 \times 10^3 \times (20/100)$ = Rs. 9000/-

Total annual cost = Rs. 9000 + Rs. 2400= **Rs. 11,400/-**

It is seen from the above example that individual drive is costlier than the group drive.

44.4. Selection of a Motor

The selection of a driving motor depends primarily on the conditions under which it has to operate and the type of load it has to handle. Main guiding factors for such a selection are as follows :

(*a*) Electrical characteristics

1. Starting characteristics
2. Running characteristics
3. Speed control
4. Braking

(*b*) Mechanical considerations

1. Type of enclosure
2. Type of bearings
3. Method of power transmission
4. Type of cooling
5. Noise level

(*c*) Size and rating of motors

1. Requirement for continuous, intermittent or variable load cycle
2. Overload capacity

(*d*) Cost

1. Capital cost
2. Running cost

In addition to the above factors, one has to take into consideration the type of current available whether alternating or direct. However, the basic problem is one of matching the mechanical output of the motor with the load requirement *i.e.* to select a motor with the correct speed/torque characteristics as demanded by the load. In fact, the complete selection process requires the analysis and synthesis of not only the load and the proposed motor but the complete drive assembly and the control equipment which may include rectification or frequency changing.

44.5. Electrical Characteristics

Electrical characteristics of different electric drives have been discussed in Vol. II of this book entitled "A.C. and D.C. Machines".

44.6. Types of Enclosures

The main function of an enclosure is to provide protection not only to the working personnel but also to the motor itself against the harmful ingress of dirt, abrasive dust, vapours and liquids and solid foreign bodies such as a spanner or screw driver etc. At the same time, it should not adversely affect the proper cooling of the motor. Hence, different types of enclosures are used for different motors depending upon the environmental conditions. Some of the commonly used motor enclosures are as under :

1. Open Type. In this case, the machine is open at both ends with its rotor being supported on pedestal bearings or end brackets. There is free ventilation since the stator and rotor ends are in free contact with the surrounding air. Such, machines are housed in a separate neat and clean room. This type of enclosure is used for large machines such as d.c. motors and generators.

2. Screen Protected Type. In this case, the enclosure has large openings for free ventilation. However, these openings are fitted with screen covers which safeguard against accidental contacts and rats entering the machine but afford no protection from dirt, dust and falling water. Screen-protected type motors are installed where dry and neat conditions prevail without any gases or fumes.

3. Drip Proof Type. This enclosure is used in very damp conditions. *i.e.* for pumping sets. Since motor openings are protected by over-hanging cowls, vertically falling water and dust are not able to enter the machine.

4. Splash-proof Type. In such machines, the ventilating openings are so designed that liquid or dust particles at an angle between vertical and 100° from it cannot enter the machine. Such type of motors can be safely used in rain.

5. Totally Enclosed (TE) Type. In this case, the motor is completely enclosed and no openings are left for ventilation. All the heat generated due to losses is dissipated from the outer surface which is finned to increase the cooling area. Such motors are used for dusty atmosphere *i.e.* sawmills, coal-handling plants and stone-crushing quarries etc.

Fig. 44.1. A three-phase motor

6. Totally-enclosed Fan-cooled (TEFC) Type. In this case, a fan is mounted on the shaft external to the totally enclosed casing and air is blown over the ribbed outer surfaces of the stator and endshields (Fig. 44.1). Such motors are commonly used in flour mills, cement works and sawmills etc. They require little maintenance apart from lubrication and are capable of giving years of useful service without any interruption of production.

7. Pipe-ventilated Type. Such an enclosure is used for very dusty surroundings. The motor is totally enclosed but is cooled by neat and clean air brought through a separate pipe from outside the dust-laden area. The extra cost of the piping is offset by the use of a smaller size motor on account of better cooling.

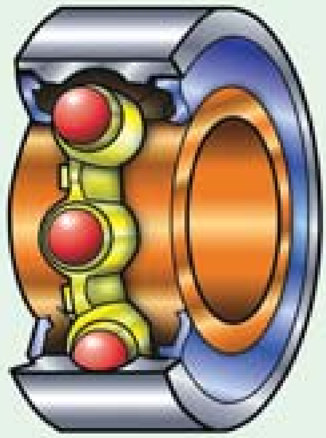
8. Flame-proof (FLP) Type. Such motors are employed in atmospheres which contain inflammable gases and vapours *i.e.* in coal mines and chemical plants. They are totally enclosed but their enclosures are so constructed that any explosion within the motor due to any spark does not ignite the gases outside. The maximum operating temperature at the surface of the motor is much less than the ignition temperature of the surrounding gases.

44.7. Bearings

These are used for supporting the rotating parts of the machines and are of two types :

1. Ball or roller bearings **2.** Sleeve or bush bearings

(*a*) Ball Bearings



A ball-bearing

Upto about 75kW motors, ball bearings are preferred to other bearings because of their following advantages :

1. They have low friction loss
2. They occupy less space
3. They require less maintenance
4. Their use allows much smaller air-gap between the stator and rotor of an induction motor
5. Their life is long.

Their main disadvantages are with regard to cost and noise particularly at high motor speeds.

(*b*) Sleeve Bearings

These are in the form of self-aligning pourous bronze bushes for fractional kW motors and in the form of journal bearings for larger motors. Since they run very silently, they are fitted on super-silent motors used for driving fans and lifts in offices or other applications where noise must be reduced to the absolute minimum.


A sleeve bearing

44.8. Transmission of Power

There are many ways of transmitting mechanical power developed by a motor to the driven machine.

1. Direct Drive. In this case, motor is coupled directly to the driven machine with the help of solid or flexible coupling. Flexible coupling helps in protecting the motor from sudden jerks. Direct drive is nearly 100% efficient and requires minimum space but is used only when speed of the driven machine equals the motor speed.

2. Belt Drive. Flat belts are extensively used for line-shaft drives and can transmit a maximum power of about 250 kW. Where possible, the minimum distance between the pulley centres should be 4 times the diameter of the larger pulley with a maximum ratio between pulley diameters of 6 : 1. The power transmitted by a flat belt increases in proportion to its width and varies greatly with its quality and thickness. There is a slip of 3 to 4 per cent in the belt drive.

3. Rope Drive. In this drive, a number of ropes are run in V-grooves over the pulleys. It has negligible slip and is used when the power to be transmitted is beyond the scope of belt drive.


Fig. 44.2. Geared Motor Unit

4. Chain Drive. Though somewhat more expensive, it is more efficient and is capable of transmitting larger amounts of power. It is noiseless, slipless and smooth in operation.

5. Gear Drive. It is used when a high-speed motor is to drive a low-speed machine. The coupling between the two is through a suitable ratio gear box. In fact motors for low-speed drives are manufactured with the reduction gear incorporated in the unit itself. Fig. 44.2 shows such a unit consisting of a flange motor bolted to a high-efficiency gear box which is usually equipped with feet, the motor being overhung.

44.9. Noise

The noise produced by a motor could be magnetic noise, windage noise and mechanical noise. Noise level must be kept to the minimum in order to avoid fatigue to the workers in a workshop. Similarly, motors used for domestic and hospital appliances and in offices and theatres must be almost noiseless. Tranmission of noise from the building where the motor is installed to another building can be reduced if motor foundation is flexible *i.e.* has rubber pads and springs.

44.10. Motors for Different Industrial Drives

1. D.C. Series Motor. Since it has high starting torque and variable speed, it is used for heavy duty applications such as electric locomotives, steel rolling mills, hoists, lifts and cranes.

2. D.C. Shunt Motor. It has medium starting torque and a nearly constant speed. Hence, it is used for driving constant-speed line shafts, lathes, vacuum cleaners, wood-working machines, laundry washing machines, elevators, conveyors, grinders and small printing presses etc.

3. Cumulative Compound Motor. It is a varying-speed motor with high starting torque and is used for driving compressors, variable-head centifugal pumps, rotary presses, circular saws, shearing machines, elevators and continuous conveyors etc.

4. Three-phase Synchronous Motor. Because its speed remains constant under varying loads, it is used for driving continuously-operating equipment at constant speed such as ammonia and air compressors, motor-generator sets, continuous rolling mills, paper and cement industries.

Heavy duty hydraulic motor for high torque and low speeds

5. Squirrel Cage Induction Motor. This motor is quite simple but rugged and possesses high over-load capacity. It has a nearly constant speed and poor starting torque. Hence, it is used for low and medium power drives where speed control is not required as for water pumps, tube wells, lathes, drills, grinders, polishers, wood planers, fans, blowers, laundary washing machines and compressors etc.

6. Double Squirrel Cage Motor. It has high starting torque, large overload capacity and a nearly constant speed. Hence, it is used for driving loads which require high starting torque such as compressor pumps, reciprocating pumps, large refrigerators, crushers, boring mills, textile machinery, cranes, punches and lathes etc.

7. Slip-ring Induction Motor. It has high starting torque and large overload capacity. Its speed can be changed upto 50% of its normal speed. Hence, it is used for those industrial drives which require high starting torque and speed control such as lifts, pumps, winding machines, printing presses, line shafts, elevators and compressors etc.

8. Single-phase Synchronous Motor. Because of its constant speed, it is used in teleprinters, clocks, all kinds of timing devices, recording instruments, sound recording and reproducing systems.

9. Single-phase Series Motor. It possesses high starting torque and its speed can be controlled over a wide range. It is used for driving small domestic appliances like refrigerators and vacuum cleaners etc.

10. Repulsion Motor. It has high starting torque and is capable of wide speed control. Moreover, it has high speed at high loads. Hence, it is used for drives which require large starting torque and adjustable but constant speed as in coil winding machines.

11. Capacitor-start Induction-run Motor. It has fairly constant speed and moderately high starting torque. Speed control is not possible. It is used for compressors, refrigerators and small portable hoists.

12. Capacitor-start-and-run Motor. Its operating characteristics are similar to the above motor except that it has better power factor and higher efficiency. Hence, it is used for drives requiring quiet operations.

44.11. Advantages of Electrical Braking Over Mechanical Braking

1. In mechanical braking; due to excessive wear on brake drum, liner etc. it needs frequent and costly replacement. This is not needed in electrical braking and so electrical braking is more economical than mechanical braking.
2. Due to wear and tear of brake liner frequent adjustments are needed thereby making the maintenance costly.
3. Mechanical braking produces metal dust, which can damage bearings. Electrical braking has no such problems.
4. If mechanical brakes are not correctly adjusted it may result in shock loading of machine or machine parts in case of lift, trains which may result in discomfort to the occupants.
5. Electrical braking is smooth.

6. In mechanical braking the heat is produced at brake liner or brake drum, which may be a source of failure of the brake. In electric braking the heat is produced at convenient place, which in no way is harmful to a braking system.
7. In regenerative braking electrical energy can be returned back to the supply which is not possible in mechanical braking.
8. Noise produced is very high in mechanical braking.

Only disadvantage in electrical braking is that it is ineffective in applying holding torque.

44.12. Types of Electric Braking

There are three types of electric braking as applicable to electric motors in addition to eddy-current braking. These have already been discussed briefly in Art. 44.7.

1. Plugging or reverse-current braking.
2. Rheostatic or dynamic braking.
3. Regenerative braking.

In many cases, provision of an arrangement for stopping a motor and its driven load is as important as starting it. For example, a planing machine must be quickly stopped at the end of its stroke in order to achieve a high rate of production. In other cases, rapid stops are essential for preventing any danger to operator or damage to the product being manufactured. Similarly, in the case of lifts and hoists, effective braking must be provided for their proper functioning.

44.13. Plugging Applied to D.C. Motors

As discussed earlier in Art. 42.7, in this case, armature connections are reversed whereas ***field winding connections remains unchanged***. With reversed armature connections, the motor develops a torque in the ***opposite*** direction. When speed reduces to zero, motor will accelerate in the opposite direction. Hence, the arrangement is made to disconnect the motor from the supply as soon as it comes to rest. Fig. 44.3 shows running and reversed connections for shunt motors whereas Fig. 44.4 shows similar conditions for series motors.

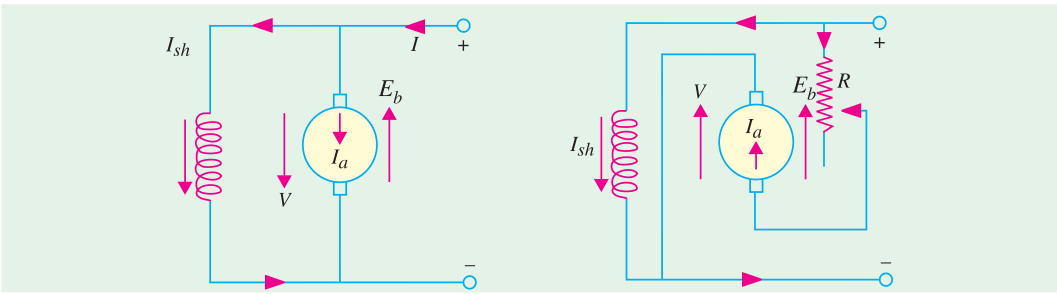


Fig. 44.3

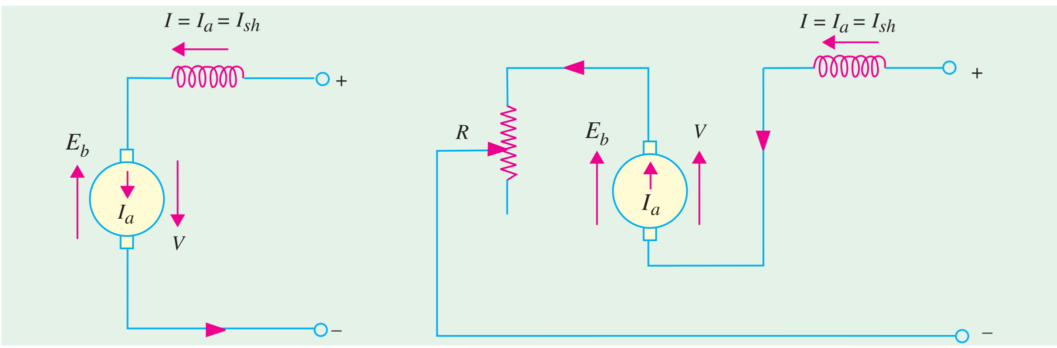


Fig. 44.4

Since with reversed connection, V and E_b are in the same direction, voltage across the armature is almost double of its normal value. In order to avoid excessive current through the armature, additional resistance R is connected in series with armature.

This method of braking is wasteful because in addition to wasting kinetic energy of the moving parts, it draws additional energy from the supply during braking.

Braking Torque. The electric braking torque is given by

$$T_B \propto \Phi I_a = k_1 \Phi I_a; \text{ Now, } I_a = (V + E_b)/R$$

$$\therefore \quad T_B = K_1 \Phi \cdot \frac{V + E_b}{R} = k_1 \Phi \frac{V + k_2 \Phi N}{R} \qquad (\because E_b \propto \Phi N)$$

$$= \frac{K_1 \Phi V}{R} + \frac{k_1 k_2 \Phi^2 N}{R} = k_3 \Phi + k_4 \Phi^2 N$$

Shunt Motor

Since in the case, Φ is practically constant, $T_B = k_5 + k_6 N$.

Series Motor

$$T_B = k_3 \Phi + k_4 \Phi^2 N = k_5 I_a + k_6 N I_a^2 \qquad (\because \phi \propto I_a)$$

The value of braking torque can be found with the help of magnetisation curve of a series motor.

Example 44.2. *A 40-kW, 440-V, d.c. shunt motor is braked by plugging. Calculate* ***(i)*** *the value of resistance that must be placed in series with the armature circuit to limit the initial braking current to 150 A* ***(ii)*** *the braking torque and* ***(iii)*** *the torque when motor speed falls to 360 rpm.*

Armature resistance R_a = 0.1W, full-load I_a = 100 A, full-load speed = 600 rpm.

(Electric Drives & Util. Punjab Univ. : 1994)

Solution. Full-load $E_b = 440 - 100 \times 0.1 = 430$ V

Voltage across the armature at the start of braking $= V + E_b = 440 + 430 = 870$ V

(i) Since initial braking current is limited to 150 A, total armature circuit resistance required is

$$R_t = 870 / 150 = 5.8\ \Omega \quad \therefore \quad R = R_t - R_a = 5.8 - 0.1 = \mathbf{5.7\ \Omega}$$

(ii) For a shunt motor, $T_B \propto \Phi I_a \propto I_a$ $\qquad \therefore$ Φ is constant

Now, $\dfrac{\text{initial braking torque}}{\text{full-load toruqe}} = \dfrac{\text{initial braking current}}{\text{full-load current}}$

Full-load torque $= 40 \times 103 / 2\pi\ (600/60) = 636.6$ N-m

$\therefore$ initial braking torque $= 636.6 \times 150 / 100 =$ **955 N-m**

(iii) The decrease in E_b is directly proportional to the decrease in motor speed.

$\therefore$ E_b at 360 rpm $= 430 \times 360/600 = 258$ V

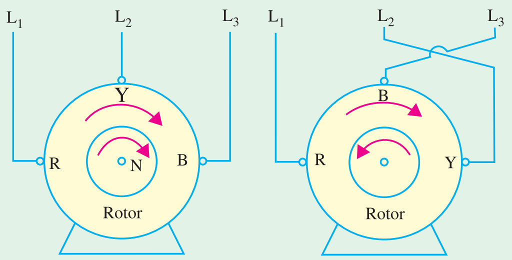
I_a at 360 rpm $= (440 + 258) / 5.8 = 120$ A

T_B at 360 rpm $= 636.6 \times 120/100 =$ **764 N-m**

44.14. Plugging of Induction Motors

This method of braking is applied to an induction motor by transposing any of its two line leads as shown in Fig. 44.5. It reverses the direction of rotation of the synchronously-rotating magnetic field which produces a torque in the reverse direction, thus applying braking on the motor. Hence, at the ***first instant*** after plugging, the rotor is running in a direction opposite to that of the stator field. It means that speed of the rotor relative to the magnetic field is $(N_s + N) \cong 2N_s$ as shown in Fig. 44.6.

In Fig 44.6. ordinate BC represents the braking torque at the instant of plugging. As seen, this torque gradually increases as motor approaches standstill condition after which motor is disconnected from the supply (otherwise it will start up again in the reverse direction).



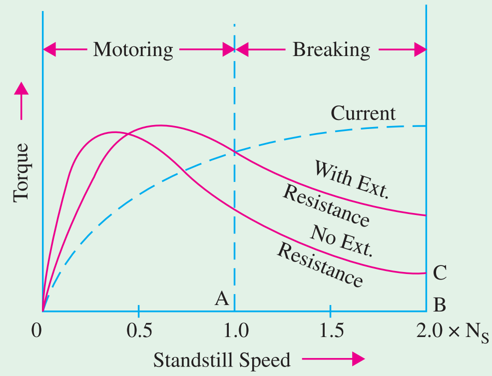


Fig. 44.5 **Fig. 44.6**

As compared to squirrel cage motors, slip-ring motors are more suitable for plugging because, in their case, external resistance can be added to get the desired braking torque.

Example 44.3. *A 30-kW, 400-V, 3-phase, 4-pole, 50-Hz induction motor has full-load slip of 5%. If the ratio of standstill reactance to resistance per motor phase is 4, estimate the plugging torque at full speed.* **(Utilisation of Elect. Energy, Punjab Univ.)**

Solution. $N_s = 120\,f/P = 120 \times 50/4 = 1500$ rpm

Full-load speed, $N_f = N_s\,(1-s) = 1500\,(1 - 0.05) = 1425$ rpm

Full-load torque, $T_f = \dfrac{30 \times 10^3}{2\pi \times 1425/60} = 200$ N-m

Since, $T \propto \dfrac{sR_2\,E_2^2}{R_2^2 + s^2\,X_2^2} \quad \therefore \quad \dfrac{T_2}{T_1} = \dfrac{s_2\,R_2\,E_2^2/(R_2^2 + s_2^2\,X_2^2)}{s_1 R_2 E_2^2/(R_2^2 + s_1^2\,X_2^2)}$

$$= \frac{s_2(R_2^2 + s_1^2\,X_2^2)}{s_1\,(R_2^2 + s_2^2\,X_2^2)} = \frac{s_2}{s_1} \cdot \frac{1 + s_1^2\,(X_2/R_2)^2}{1 + s_2^2\,(X_2/R_2)^2}$$

$$= \frac{s_1}{s_2} \cdot \frac{1 + 16 s_1^2}{1 + 16 s_2^2} \left(\frac{X_2}{R_2} = 4\right)$$

Slip, $S_p = 2 - 0.05 = 1.95$

$\therefore$ plugging torque, $T_p = \dfrac{1.95}{0.05} \cdot \dfrac{1 + 16 \times (0.05)^2}{1 + 16\,(1.95)^2} \times T_f$

$$= 39 \times \frac{1.04}{61.84} \times 200 = \mathbf{131\ N\text{-}m.}$$

44.15. Rheostatic Braking

In this method of electric braking, motor is disconnected from the supply though its field continues to be energised in ***the same direction***. The motor starts working as a generator and all the kinetic energy of the equipment to be braked is converted into electrical energy and is further dissipated in the variable external resistance *R* connected across the motor during the braking period. This external resistance must be less than the critical resistance otherwise there will not be enough current for generator excitation (Art. 44.3).

Rheostat

D.C. and synchronous motors can be braked this way but induction motors require separate d.c. source for field excitation.

This method has advantage over plugging because, in this case, no power is drawn from the supply during braking.

44.16. Rheostatic Braking of D.C. Motors

Fig. 44.7 shows connections for a d.c. shunt motor. For applying rheostatic braking armature is disconnected from the supply and connected to a variable external resistance R while the field remains on the supply. The motor starts working as a generator whose induced emf E_b depends upon its speed. At the start of braking, when speed is high, E_b is large, hence I_a is large. As speed decreases, E_b decreases, hence I_a decreases. Since $T_b \propto \Phi\, I_a$, it will be high at high speeds but low at low speeds. By gradually cutting out R, I_a and, hence, T_B can be kept constant throughout. Value of $I_a = E_b\,(R + R_a)$.

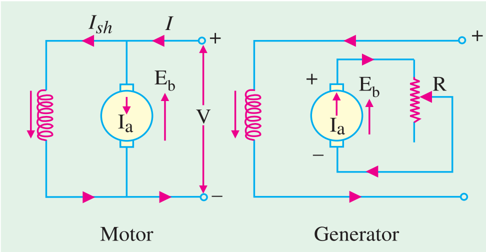


Fig. 44.7

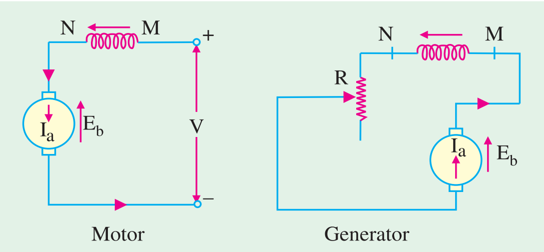


Fig. 44.8

Fig. 44.8 shows running and braking conditions for a d.c. series motor. In this case also, for rheostatic braking, the armature is disconnected from the supply and, at the same time, is connected across R. However, connections are so made that current keeps flowing through the series field ***in the same direction*** otherwise no braking torque would be produced. The motor starts working as a series generator provided R is less than the critical resistance.

44.17. Rheostatic Braking Torque

$T_B \propto \Phi\, I_a$. Now, $I_a = E_b/(R + R_a) = E_b/Rt$

Since $E_b \propto \Phi N$, $I_a \propto \Phi N/R_t$ $\therefore$ $\boxed{T_B \propto \Phi^2 N/R_t = k_1 \Phi^2 N}$

1. For D.C. shunt motors and synchronous motors, Φ is constant. Hence

$$\boxed{T_B = k_1 N}$$

2. In the case of series motors, flux depends on current. Hence, braking torque can be found from its magnetisation curve.

When rheostatic braking is to be applied to the series motors used for traction work, they are connected in parallel (Fig. 44.9) rather than in series because series connection produces excessive voltage across the loading rheostats.

However, it is essential to achieve electrical stability in parallel operation of two series generators. It can be achieved either by equalizing the exciting currents *i.e.* by connecting the two fields in parallel [Fig. 44.9 (*a*)] or by cross-connection [Fig. 44.9 (*b*)] where field of one machine is excited by the armature current of the other. If equalizer is not used, then the machine which happens to build up first will send current through the other ***in the opposite direction*** thereby exciting it with reverse voltage. Consequently, the two machines would be short-circuited upon themselves and may burn out on account of excessive voltage and, hence, current.

In the cross-connection of Fig. 44.9 (*b*), suppose the voltage of machine No. 1 is greater than that of No. 2. It would send a larger current through F_2 , thereby exciting it to a higher voltage. This results in stability of their parallel operation because stronger machine always helps the weaker one.

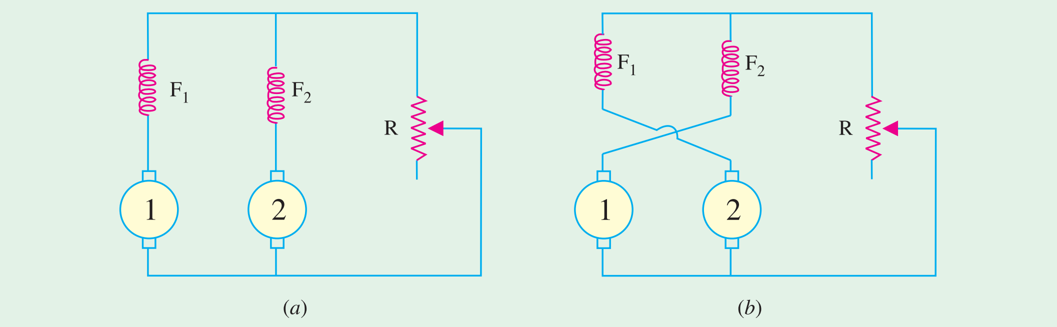


Fig. 44.9

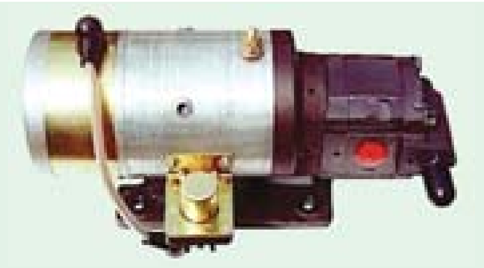
The cross-connection method has one special advantage over equalizer-connection method. If due to any reason (say, a run-back on a gradient) direction of rotation of the generators is reversed, no braking effect would be produced with connections of Fig. 44.9 (*a*) since the machines will fail to excite. However, with cross-excited fields, the machines will build up in series and being short-circuited upon themselves, will provide an emergency braking and would not allow the coach/car to run back on a gradient.

44.18. Rheostatic Braking of Induction Motors

If an induction motor is disconnected from the supply for rheostatic braking, there would be no magnetic flux and, hence, no generated emf in the rotor and no braking torque. However, if after disconnection, direct current is passed through the stator, steady flux would be set up in the air-gap which will induce current, in the short-circuited rotor. This current which is proportional to the rotor speed, will produce the required braking torque whose value can be regulated by either controlling d.c. excitation or varying the rotor resistance.

44.19. Regenerative Braking

In this method of braking, motor is not disconnected from the supply but is made to run as a generator by utilizing the kinetic energy of the moving train. Electrical energy is fed back to the supply. The magnetic drag produced on account of generator action offers the braking torque. It is the most efficient method of braking. Take the case of a shunt motor. It will run as a generator whenever its E_b becomes greater than V. Now, E_b can exceed V in two ways :



DC Shunt Motor

1. by increasing field excitation
2. by increasing motor speed beyond its normal value, field current remaining the same. It happens when load on the motor has overhauling characteristics as in the lowering of the cage or a hoist or the down-gradient movement of an electric train.

Regenerative braking can be easily applied to d.c. shunt motors though not down to very low speeds because it is not possible to increase field current sufficiently.

In the case of d.c. series motors, reversal of current necessary to produce regeneration would cause reversal of the field and hence of E_b. Consequently, modifications are necessary if regenerative braking is to be employed with d.c. series motors used in electric traction.

It may, however, be clearly understood that regenerative braking cannot be used for stopping a motor. Its main advantages are (*i*) reduced energy consumption particularly on main-line railways having long gradients and mountain railways (*ii*) reduced wear of brake shoes and wheel tyres and (*iii*) lower maintenance cost for these items.

44.20. Energy Saving in Regenerative Braking

We will now compute the amount of energy recuperated between any two points on a level track during which regenerative braking is employed. The amount of energy thus recovered and then returned to the supply lines depends on :

(*i*) initial and final velocities of the train during braking

(*ii*) efficiency of the system and (*iii*) train resistance.

Suppose regenerative braking is applied when train velocity is V_1 km/h and ceases when it is V_2 km/h. If M_e tonne is the effective mass of the train, then

$$\text{K.E. of the train at } V_1 = \frac{1}{2} M_e V_1^2 = \frac{1}{2}(1000\, M_e) \times \left(\frac{1000\, V_1}{3600}\right)^2 \text{ joules}$$

$$= \frac{1}{2}(1000\, M_e)\left(\frac{1000\, V_1}{3600}\right)^2 \times \frac{1}{3600} \text{ Wh}$$

$$= 0.01072\, M_e V_1^2 \text{ Wh} = 0.01072\, \frac{M_e}{M} V_1^2 \text{ Wh/tonne}$$

$$\text{K.E. at } V_2 = 0.01072\, \frac{M_e}{M} V_2^2 \text{ Wh/tonne}$$

Hence, energy available for recovery is $= 0.01072\, \dfrac{M_e}{M}\,(V_1^2 - V_2^2)$ Wh/tonne

If r N/t is the specific resistance of the train, then total resistance = rM newton.

If d km is the distance travelled during braking, then

energy spent $= rM \times (1000\, d)$ joules $= rMd \propto \dfrac{1000}{3600}$ Wh $= 0.2778\, rd$ Wh/tonne

Hence, net energy recuperated during regenerative braking is

$$= 0.01072\, \frac{M_e}{M}\,(V_1^2 - V_2^2) - 0.2778\, rd \text{ Wh/tonne}$$

Gradient. If there is a *descending* gradient of G per cent over the same distance of d km, then downward force is = 98 MG newton

Energy provided during braking

$$= 98\, MG \times (1000\, d) \text{ joules} = 98\, MG\, d\,(1000 / 3600) \text{ Wh} = 27.25\, Gd \text{ Wh/tonne}$$

Hence, net energy recuperated in this case is

$$= \left[0.01072\, \frac{M_e}{M}(V_1^2 - V_2^2) - 0.2778\, rd + 2725\, Gd\right] \text{ Wh/tonne}$$

$$= 0.01072\, \frac{M_e}{M}(V_1^2 - V_2^2) + d\,(27.75\, G - 0.2778\, r) \text{ Wh/tonne}$$

If η is the system efficiency, net energy returned to the line is

(*i*) **level track**

$$= \eta\left[0.01072\, \frac{M_e}{M}(V_1^2 - V_2^2) - 0.2778\, rd\right] \text{ Wh/tonne}$$

(*ii*) **descending gradient**

$$= \eta\left[0.01072\,\frac{M_e}{M}\,(V_1^2 - V_2^2) + d\,(27.25\,G - 0.2778\,r)\right] \text{ Wh/tonne}$$

Example 44.4. *A 500-t electric trains travels down a descending gradient of 1 in 80 for 90 seconds during which period its speed is reduced from 100 km/h to 60 km/h by regenerative braking. Compute the energy returned to the lines of kWh if tractive resistance = 50 N/t; allowance for rotational inertia = 10% ; overall efficiency of the system = 75 %.*

Solution. Here $G = 1 \times 100 / 80 = 1.25\%$ $\quad M_e/M = 1.1$

$$d = \left(\frac{V_1 + V_2}{2}\right) \times t = \left(\frac{100 + 60}{2}\right) \times \frac{90}{3600} = 2 \text{ km}$$

Hence, energy returned to the supply line

$$= 0.75\ [(0.01072 \times 1.1\ (1002 - 602) + 2\ (27.25 \times 1.25 - 0.2778 \times 50)]\ \text{Wh/t}$$

$$= 0.75\ [75.5 + 2\ (34 - 13.9)] = 86.77 \text{ Wh/t}$$

$$= 86.77 \times 500 \text{ Wh} = 86.77 \times 500 \times 10^{-3} \text{ kWh} = \mathbf{43.4\ kWh}$$

Example 44.5. *A 350-t electric train has its speed reduced by regenerative braking from 60 to 40 km/h over a distance of 2 km along down gradient of 1.5%. Caculate* **(i)** *electrical energy and* **(ii)** *average power returned to the line. Assume specific train resistance = 50 N/t ; rotational inertia effect = 10% ; conversion efficiency of the system = 75%.* **(Elect. Power, Bombay Univ.)**

Solution . (*i*) Energy returned to the line is

$$= 0.75\ [0.01072 \times 1.1\ (60^2 - 40^2) + 2\ (27.25 \times 1.5 - 0.2778 \times 50)]\ \text{Wh/t}$$

$$= 58.2 \text{ Wh/t} = 58.2 \times 350 \times 10^{-3} = \mathbf{20.4\ kWh}$$

(*ii*) Average speed $= (60 + 40)/2 = 50$ km/h ; time taken $= 2/50$ h $= 1/25$ h

$$\therefore \quad \text{power returned} = \frac{20.4 \text{ kWh}}{1/25\text{h}} = \mathbf{510\ kW}$$

Example 44.6. *If in Example 42.4, regenerative braking is applied in such a way that train speed on down gradient remains constant at 60 km/h, what would be the power fed into the line?*

Solution : Since no acceleration is involved, the down-gradient tractive effort which drives the motors as generators is

$$F_t = (98\,MG - Mr) \text{ newton} = (98 \times 350 \times 1.5 - 350 \times 50) = 33{,}950 \text{ N}$$

Power that can be recuperated is

$$= F_t \times \left(\frac{1000}{3600}\right) V = 0.2778\, F_t\, V \text{ watt} = 0.2778 \times 33{,}950 \times 60 = 565{,}878 \text{ W}$$

Since $\eta = 0.75$, the power that is actually returned to the line is

$$= 0.75 \times 565{,}878 \times 10^{-3} = \mathbf{424.4\ kW}$$

Example 44.7. *A train weighing 500 tonne is going down a gradient of 20 in 1000. It is desired to maintain train speed at 40 km/h by regenerative braking. Calculate the power fed into the line. Tractive resistance is 40 N/t and allow rotational intertia of 10% and efficiency of conversion of 75%.* **(Util. of Elect. Power, A.M.I.E. Sec. B.)**

Solution. Down-gradient tractive effort which drives the motors as generators is

$$F_t = (98\,MG - Mr) = (98{,}000 \times 500 \times 2 - 500 \times 40) = 78{,}000 \text{ N}$$

Power that can be recuperated is $= 0.2778\, F_t V = 0.2778 \times 78{,}000 \times 40 = 866{,}736$ W

Since $\eta = 0.75$, the power that is actually fed into the lines is

$$= 0.75 \times 866{,}736 \times 10^{-3} = \mathbf{650\ kW.}$$

Example 44.8. *A 250-V d.c. shunt motor, taking an armature current of 150 A and running at 550 r.p.m. is braked by reversing the connections to the armature and inserting additional resistance in series with it. Calculate :*

(*a*) *the value of series resistance required to limit the initial current to 240 A.*

(*b*) *the initial value of braking torque.*

(*c*) *the value of braking torque when the speed has fallen to 200 r.p.m.*

The armature resistance is 0.09 Ω. Neglect winding friction and iron losses.

(Traction and Util. of Elect. Power, Agra Univ.)

Solution. Induced emf at full-load, $E_b = 250 - 150 \times 0.09 = 236.5$ V

Voltage across the armature at braking instant = $V + E_b = 250 + 236.5 = 486.5$ V

(*a*) Resistance required in the armature circuit to limit the initial current to 240 A

$$= \frac{486.5}{240} = \mathbf{2.027\ \Omega}$$

Resistance to be added in the armature circuit = 2.027 – 0.09 = **1.937 Ω**

(*b*) F.L. Torque, $T_f = VI/2\pi\,(N/60) = 250 \times (550/60) = 650$ N-m

$$\text{Initial braking torque} = T_f\,\frac{\text{initial braking current}}{\text{full-load current}} = \frac{650 \times 240}{150} = \mathbf{1040\ N\text{-}m}$$

(*c*) When speed falls to 200 r.p.m., back emf also falls in the same proportion as the speed.

∴ $E_b = E_b \times 200/550 = 236.5 \times 200.550 = 94.6$ V

∴ current drawn = (250 + 94.6)/2.027 = 170 A

∴ braking torque = 650 × 170/150= **737 N-m**

Example 44.9. *A 400 V 3-ph squirrel cage induction motor has a full load slip of 4%. A stand-still impedance of 1.54Ω and the full load current = 30A. The maximum starting current which may be taken from line is 75A. What taping must be provided on an auto-transformer starter to limit the current to this value and what would be the starting torque available in terms of full load torque ?*

[Nagpur University, Winter 1994]

Solution.
$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = X$$

or
$$V_1 I_1 = V_2 I_2 = X$$

$$I_1 = 75\text{A}$$

$$V_1 = \frac{400}{\sqrt{3}} = 231 \text{ V and } I_1 = I_2 X$$

$$I_1 = \frac{V_2}{Z}X$$

$$I_1 = \frac{XV_1}{1.54}X$$

$$I_1 = \frac{X^2 \times 231}{1.54}$$

$$\Rightarrow \quad 75 = \frac{X^2 \times 231}{1.54}$$

$$\Rightarrow \quad X^2 = \frac{75 \times 1.54}{231}$$

$$X = 0.7071$$

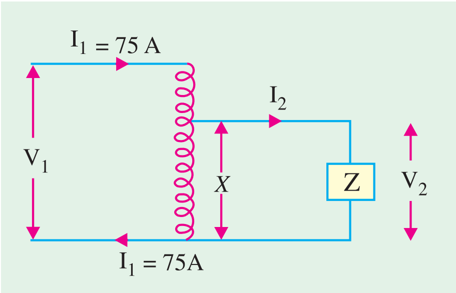


Fig. 44.10

$$\frac{T_s}{T_{FL}} = X^2\left(\frac{I_S}{I_{FL}}\right)^2 \times \text{Slip}_{(FL)}$$

Now $\quad I_2 = I_s = \dfrac{I_1}{X} = \dfrac{75}{0.708} = 106\ A$

$$s_{FL} = 0.04. \quad I_s = 106\text{ A} \quad I_{FL} = 30\text{ A.}$$

$\therefore \quad \dfrac{T_s}{T_{HL}} = (0.701)^2\left(\dfrac{106}{30}\right)^2 \times 0.04$

$\therefore \quad T_s = 0.25\ T_{FL}$

Example 44.10. *A 220V, 10 H.P. shunt motor has field and armature resistances of 122W and 0.3W. respectively. Calculate the resistance to be inserted in the armature circuit to reduce the speed to 80% assuming motor η at full load to be 80%.*

(a) When torque is to remain constant.

(b) When torque is proportional to square of the speed. **[Nagpur University, Winter 1994]**

Solution. $\quad I_f = \dfrac{220}{112} = 1.8$ Amp.

Motor $\quad O/P = 10 \times 746 = 7460$ W

$\therefore \quad$ Motor $I/P = \dfrac{7460}{0.8} = 9300$ W

Line current $\quad I_L = \dfrac{9300}{220} = 42.2$ Amp.

$\therefore \quad I_a = 42.2 - 1.8 = 40.4$ A

$\therefore \quad E_{b_1} = 220 - 40.4 \times 0.3 = 208$ V

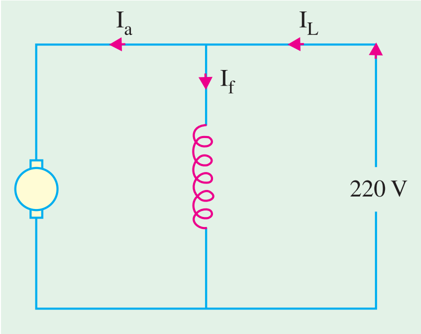


Fig. 44.11

Now $\quad \dfrac{N_2}{N_1} = \dfrac{E_{b_2}}{E_{b_1}} \quad \because \phi$ is constant

$0.8 = \dfrac{E_{b_2}}{E_{b_1}} \quad \therefore 0.8 = \dfrac{E_{b_2}}{208} \quad \Rightarrow \quad E_{b_2} = $ **166.4 V**

(a) $\because$ Torque remains constant and ϕ is constant

$\therefore$ I_a at reduced speed will also remain same

$\therefore \quad E_{b_2} = V - I_{a_2} R$ where R is total resistance

$$166.4 = 220 - 40.4 \times R \quad \therefore \quad R = \frac{220 - 166.4}{40.0} = 1.34\ \Omega$$

$\therefore$ Additional resistance in armature circuit = 1.34 – 0.3 = **1.04 Ω**

(b) $\quad \dfrac{T_2}{T_1} = \left(\dfrac{N_2}{N_1}\right)^2$

$$T \propto I_a \quad \text{also} \quad T \propto \phi I_a$$

$\because \quad T \propto I_a \quad (\because \phi$ is constant)

$$\frac{T_2}{T_1} = (0.8)^2 = 0.64$$

$\therefore \quad \dfrac{T_2}{T_1} = \dfrac{I_{a_2}}{I_{a_2}} \quad \therefore 0.64 = \dfrac{I_{a_2}}{40.4} \quad \Rightarrow \quad I_{a_2} = 25.3$ A

$$E_{b_2} = 220 - 25.3 \times R$$

$$166.4 = 220 - 25.5936 \times R$$

$\therefore$ R = **2.0943 Ω**

$\therefore$ Additional resistance = 2.0943 – 0.3 = **1.7943 Ω**

Example 44.11. *A 37.5 H.P., 220 V D.C. shunt motor with a full load speed of 535 r.p.m. is to be braked by plugging. Estimate the value of resistance which should be placed in series with it to limit the initial braking current to 200 amps. What would be the initial value of the electric braking torque and the value when the speed had fallen to half its full load value? Armature resistance of motor is 0.086 Ω and full load armature current is 140 amps.*

Solution. $E = V - I_a R_a$

Back e.m.f. of motor = $E = 220 - 140 \times 0.086 = 220 - 12 = 208$ Volts.

Total voltage during braking = $E + V$

= 220 + 208 = 428 V

$$R = \frac{V}{I}$$

Resistance required = $\frac{428}{200} = 2.14\ \Omega$

There is already 0.086 Ω present in aramature.

$\therefore$ Resistance to be added = 2.14 – 0.086 = 2.054 Ω

Torque $\propto \phi I$ or Torque $\propto I$ ($\because$ ϕ is constant for shunt motor)

$$\frac{\text{Initial braking torque}}{\text{Initial braking current}} = \frac{\text{F.L. toruqe}}{\text{F.L. current}}$$

$$\text{Power} = \text{Torque} \times \omega$$

$$\omega = \frac{2\pi N}{60} \text{ rad/sec.}$$

$$37.5 \times 746 = T \times \frac{2\pi \times 535}{60}$$

Full load torque = 499.33 Nw-m.

Initial braking torque = $499.33 \times \frac{200}{140} = 713.328$ Nw-m.

$\rightarrow$ At half – speed back e.m.f. falls to half its original value = 208/2 = 104 V

$$\text{Current} = \frac{220 + 104}{2.14} = 151 \text{ Amps.}$$

Electric braking torque at $\frac{1}{2}$ speed = $499.33 \times \frac{151}{140}$ = **538.56 Nw-m.**

Example 44.12. *A 500 V series motor having armature and field resistances of 0.2 and 0.3 Ω, runs at 500 r.p.m. when taking 70 Amps. Assuming unsaturated field find out its speed when field diverter of 0.684 Ω is used for following load whose torque*

(a) *remains constant*

(b) *varies as square of speed.*

Solution. When no diverter connected, E_{b_1} = 500 – 70 (0.2 + 0.3) = 465 V

(a) If I_{a_2} be the armature current when diverter is used, then current flowing through

$$\text{field} = I_{f2} = I_{a_2} \times \frac{0.684}{0.3 + 0.684} = 0.695\, I_{a_2}$$

$\because$ Load torque is constant

$$\therefore \quad I_{a_1} \phi_1 = I_{a_2} \phi_2 \qquad (\because \phi \ \alpha \ I_a)$$

$$\therefore \quad I_{a_1}\phi_1 = I_{a_2}(0.695)\,I_{a_2} \Rightarrow I_{a_2} = \frac{I_{a_1}}{\sqrt{0.695}} = \frac{70}{\sqrt{0.695}} = 84 \text{ A}$$

$$\therefore \text{ Field current} = I_{f_2} = 0.695\, I_{a_2} = 0.695 \times 84 = 58.4 \text{ A}$$

$$\text{Resistance of field with diverter} = \frac{0.3 \times 0.684}{0.3 + 0.684} = 0.208\ \Omega$$

Total field and armature resistance = $0.2 + 0.208 = 0.408\ \Omega$

$$E_{b_2} = 500 - 84\ (0.408) = 465.8 \text{ V}$$

$$\frac{N_1}{N_2} = \frac{E_{b_1}}{E_{b_2}} \times \frac{\phi_2}{\phi_1}$$

$$\frac{500}{N_2} = \frac{465}{465.8} \times \frac{58.4}{70} \Rightarrow N_2 = \mathbf{600\ r.p.m}$$

(*b*)
$$\frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right)^2 \quad \because \frac{T_1}{T_2} = \frac{I_{a_1}\phi_1}{I_{a_2}\phi_2} = \frac{I_{a_1} \cdot I_{a_1}}{I_{a_2} \times 0.695\, I_{a_2}}$$

$$\therefore \left(\frac{N_1}{N_2}\right)^2 = \frac{I_{a_1} \cdot I_{a_1}}{Ia_2^2 \times 0.695} \Rightarrow \frac{N_1}{N_2} = \frac{I_{a_1}}{I_{a_2}\sqrt{0.695}} = \frac{70}{I_{a_2}\sqrt{0.695}}$$

$$\frac{N_1}{N_2} = \frac{E_{b_1}}{E_{b_2}} \times \frac{\phi_2}{\phi_1}$$

$$\frac{70}{I_{a_2}\sqrt{0.695}} = \frac{465}{500 - I_{a_2}(0.2)\,0.208)} \times \frac{0.695\, I_{a_2}}{70}$$

$$I_{a_2}^2 + 7.42\, I_{a_2} - 9093 = 0$$

$$\Rightarrow \quad I_{a_2} = 91.7 \text{ A} \qquad \because \text{ negative value is absurd.}$$

$$\frac{N_1}{N_2} = \frac{70}{I_{a_2}\sqrt{0.695}} \Rightarrow \frac{500}{N_2} = \frac{70}{91.7\sqrt{0.695}}$$

$$\therefore \quad N_2 = \mathbf{546\ r.p.m.}$$

Example 44.13. *A 200 V series motor runs at 1000 r.p.m. and takes 20 Amps. Armature and field resistance is 0.4 W. Calculate the resistance to be inserted in series so as to reduce the speed to 800 r.p.m., assuming torque to vary as cube of the speed and unsaturated field.*

Solution.
$$\frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right)^3 = \left(\frac{1000}{800}\right)^3 = \frac{125}{64}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{I_{a_1}\phi_1}{I_{a_2}\phi_2} = \frac{20 \times 20}{I_{a_2} \times I_{a_2}} \qquad \because \phi \propto I_a \text{ for series motor.}$$

$$\frac{125}{64} = \frac{20^2}{I_{a_2}^2} \quad I_{a_2}\ I_{a_2} = 14.3 \text{ Amp}$$

$$E_{b_1} = 200 - 20 \times 0.4 = 192 \text{ V.}$$

$$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2} \times \frac{\phi_1}{\phi_2}$$

$$\frac{192}{E_{b_2}} = \frac{1000}{800} \times \frac{20}{14.3}$$

$$E_{b_2} = 110 \text{ V} ; \quad E_{b_2} = V - IR$$

$$110 = 200 - 14.3 \times R \; ; \; R = \frac{90}{14.3} = 6.3\ \Omega$$

Additional resistance required $= 6.3 - 0.4 =$ **5.9 Ω**

Example 44.14. *A 220V, 500 r.p.m. D.C. shunt motor with an armature resistance of 0.08Ω and full load armature current of 150 Amp. is to be braked by plugging. Estimate the value of resistance which is to be placed in series with the armature to limit initial braking current to 200 Amps. What would be the speed at which the electric braking torque is 75% of its initial value.*

Solution. Back e.m.f. of motor $= E_{b_1} = V - I_a R_a$

$$= 220 - 150 \times 0.08 = 208 \text{ V}$$

Voltage across armature when braking starts

$$= 220 + 208 = 428 \text{ V}$$

Initial braking current to be limited to 200 *A*.

$\therefore$ Resistance in armature circuit $= \dfrac{428}{200} = 2.14\ \Omega$

$\therefore$ External resistance required $= 2.14 - 0.08 = 2.06\ \Omega$

Since field Flux ϕ is constant therefore 75% torque will be produced when armature current is 75% of 200 Amp. *i.e.* 150Amp.

Let N_2 be the speed in r.p.m. at which 75% braking torque is produced. At this speed generated e.m.f. in armature

$$\because \quad \frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2} \; ; \; \frac{208}{E_{b_2}} = \frac{500}{N_2} \quad \therefore \quad E_{b_2} = \frac{208}{500} N_2$$

Voltage across armature when braking starts

$$150 \times 2.14 = \left(220 + \frac{208}{500} N_2\right) \text{Volts}$$

$$\therefore \quad N_2 = \textbf{243 r.p.m.}$$

Example 44.15. *A D.C. series motor operating at 250 V D.C. mains and draws 25 A and runs at 1200 r.p.m. R_a = 0.1 Ω and R_{se} = 0.3 Ω.*

A resistance of 25 Ω is placed in parallel with the armature of motor. Determine:

(i) The speed of motor with the shunted armature connection, if the magnetic circuit remains unsaturated and the load torque remains constant.

(ii) No load speed of motor. **[Nagpur University Winter 1995]**

Solution. $$\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \times \frac{\phi_1}{\phi_2}$$

Voltage across diverter $= 250 - 0.3\, I_2$

$$I_{div} = \frac{250 - 0.3 I_2}{25}$$

$$I_{a_2} = I_2 - \frac{250 - 0.3 I_2}{25}$$

$$= 1.012\ I_2 - 10$$

As T is constant $\therefore$ $\phi_1 \; I_{a_1} = \phi_2 \; I_{a_2}$

$$\therefore \quad I_{a_1}^2 = I_2 \,(I_{a_2})$$

$$(25)^2 = I_2 \;\; (1.012\ I_2 - 10\,)$$

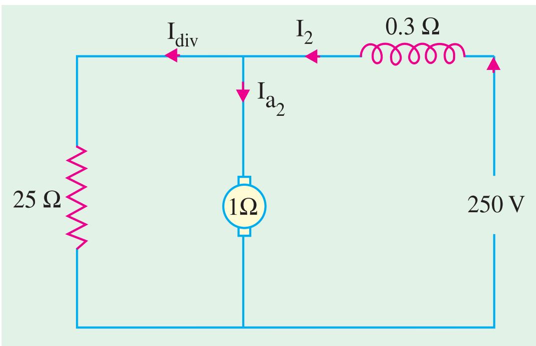


Fig. 44.12

$$1.012\, I_2^{\,2} - 10 \times I_2 - 625 = 0 \Rightarrow I_2 = 30.27\,A \text{ (neglecting negative value)}$$

$$(25)^2 = (30.27)\; I_{a_2} \Rightarrow I_{a_2} = 20.65 \text{ A}$$

$$E_{b_1} = V - I_{a_1}\,(R_a + R_{se}) = 250 - 25\,(0.4) = 240 \text{ Volts}$$

$$E_{b_2} = V - I_2\,(R_{se}) - I_{a_2}\,(R_a) = 250 - 30.27\,(0.3) - 0.1\,(20.65) = 238.85 \text{ V.}$$

$$\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N_2}{1200} = \frac{238.85}{240} \times \frac{2.5}{20.27} \quad \therefore \quad N_2 = \textbf{986 r.p.m.}$$

(ii) Series motor on no load.

Series motor can't be started on no load. When flux is zero, motor tries to run at infinite speed, which is not possible. So in the process, it tries to draw very high current from supply and fuse blows-out.

Example 44.16. *A 4 pole, 50Hz, slip ring Induction Motor has rotor resistance and stand still reactance referred to stator of 0.2 Ω and 1 Ω per phase respectively. At full load, it runs at 1440 r.p.m. Determine the value of resistance to be inserted in rotor in ohm/ph to operate at a speed of 1200 r.p.m., if :*

(i) *Load torque remains constant.* ***(ii)*** *Load torque varies as square of the speed.*

Neglect rtator resistance and leakage reactance.

Solution. ***(i)*** Load torque constant

$$\Rightarrow T \propto \frac{s}{R_2} \qquad N_s = 1500 \text{ rpm}$$

$$\therefore \quad T_1 \propto \frac{s_1}{R_2} \qquad T_2 \propto \frac{s_2}{(R_1 + r)}$$

$$s_1 = \frac{1500 - 1400}{1500} = 0.04 \; s_2 = \frac{1500 - 1240}{1500} = 0.2$$

As $\quad T_1 = T_2 \quad \therefore \quad \dfrac{s_1}{R_2} = \dfrac{s_2}{R_1 + r}$

$$\frac{0.04}{0.2} = \frac{0.2}{0.2 + r}$$

$$\therefore \quad r = 0.8\;\Omega$$

(ii) Load torque varies as square of the speed.

$$\frac{T_1}{T_2} = \left[\frac{N_1}{N_2}\right]^2 = \left[\frac{1440}{1200}\right]^2 = 1.44$$

$$\frac{T_1}{T_2} = 1.44 = \frac{\dfrac{R_2 s_1}{R_2^2 + (s_1\,X_2)^2}}{\dfrac{(R_2 + r)\,s_2}{(R_2 + r)^2 + (s_2\,X_2)^2}} = \frac{\dfrac{0.2 \times 0.04}{0.2^2 + (0.04 \times 1)^2}}{\dfrac{(0.02 + r)\,(0.2)}{(0.2 + r)^2 + (0.2 \times 1)^2}}$$

Substituting $\quad R_2 + r = R$

$$1.44 = \frac{0.1923}{\dfrac{0.2\,R}{R^2 + 0.04}}$$

$$\therefore \quad 0.1923\,R^2 - 0.288\,R + 0.007652 = 0$$

$$\Rightarrow \quad R = 1.47 \text{ and } 0.0272, \text{ But } R > 0.2 \;\therefore\; R = 1.47\;\Omega$$

$$\therefore \quad R = 0.2 + r\,;\; 1.47 = 0.2 + r \Rightarrow r = 1.27\;\Omega$$

Tutorial Problem No. 44.1

1. The characteristics of a series traction motor at 525 V are as follows :

current	:	50	70	80	90	A
speed	:	33.8	26.9	25.1	23.7	km/h
Gross torque	:	217	352	423	502	N-m

Determine the gross braking torque at a speed of 25.7 km/h when operating as self-excited series generator and loaded with an external resistance of 6 Ω. Resistance of motor = 0.5 Ω.
[382.4 N-m] ***(London Univ.)***

2. The characteristics of a series motor at 525 V are as follows :

current	:	75	125	175	225	A
speed	:	1200	950	840	745	r.p.m.

Calculate the current when operating as a generator at 1000 r.p.m. and loaded on a rheostat having a resistance of 3.25 Ω. The resistance of motor is 3.5 Ω. **[150 A]** ***(London Univ.)***

3. A train weighing 400 tonne travels a distance of 10 km down a gradient of 2%, getting its speed reduced from 40 to 20 km/h, the train resistance is = 50 N/t, allowance for rotational inertia = 10% and overall efficiency = 72%. Estimate (*i*) power and (*ii*) energy returned to the line.
[(*i*) 363 kW (*ii*) 121 kWh] ***(Elect. Power, Bombay Univ.)***

4. A 400-tonne train travels down a gradient of in 100 for 20 seconds during which period its speed is reduced from 80 km/h to 50 km/h by regenerative braking. Find the energy returned to the lines if the tractive resistance is 49 N/t and allowance for rotational inertia is 7.5%. Overall efficiency of motors is 75%. **[28.2 kWh]** ***(A.M.I.E.)***

5. A 400-tonne train travels down a gradient of 1 in 70 for 120 seconds during which period its speed is reduced from 80 km/h to 50 km/h by regenerative braking. Find the energy returned to the line if tractive resistance is 49 N/t and allowance for rotational inertia is 7.5%. Overall efficiency of motors is 75%. **[30.12 kWh]** ***(A.M.I.E.)***

6. A train weighing 500 tonne is going down a gradient of 20 in 1000. It is desired to maintain train speed at 40 km/h by regenerative braking. Calculate the power fed into the line. Tractive resistance is 40 N/t and allow rotational inertia of 10% and efficiency of conversion of 75%.
[650 kW] ***(Utilization of Elect. Power, A.M.I.E.)***

7. A 18.65 kW, 220-V D.C. shunt motor whith a full-load speed of 600 r.p.m. is to be braked by plugging. Estimate the value of the resistance which should be placed in series with it to limit the current to 130A. What would be the initial value of the electric braking torque and value when speed has fallen to half of its full-load vlaue ? Armature resistance of motor is 0.1 W. Full-load armature current is 95 A. **[3.211 Ω, 400.5 N-m, 302.57 N-m]** ***(Util of Elect. Power, A.M.I.E. Sec. B.)***

8. A 400-tonne train travels down a gradient of 1 in 70 for 120 seconds during which period its speed is reduced from 80 km/h to 50 km/h by regenerative braking. Find the energy returned to the lines if tractive resistance is 5 kg / tonne and allowance for rotational inertia is 7.5 %. Overall efficiency of motor is 75%. **[30.64%]**

9. What are the advantages of Electrical Drive over other Drives? What are the mainfeatures of Group Drive and an Individual Drive? ***(Nagpur University, Summer 2004)***

10. What are the essential requirements of starting of any motor? With the help of neat diagram explain 'open circuit transition' and 'closed circuit transition' in Auto transformer starting of Induction Motor. ***(Nagpur University, Summer 2004)***

11. What is the principle of speed control of D.C. motors for, below the base speed and above the base speed.Explain with neat N-T characteristics. ***(Nagpur University, Summer 2004)***

12. A 400 V, 25 h.p., 450 rpm, D.C. shunt motor is braked by plugging when running on full load. Determine the braking resistance necessary if the maximum braking current is not to exceed twice the full load current. Determine also the maximum braking torque and the braking torque when the motor is just reaching zero speed. The efficiency of the motor is 74.6% and the armature resistance is 0.2 Ω. ***(Nagpur University, Summer 2004)***

13. Mention the Advantage of PLC over conventional motor control.
(Nagpur University, Summer 2004)

14. Suggest the motors required for following Drives :-
(*i*) Rolling mills (*ii*) Marine drive (*iii*) Home appliances (*iv*) Pump
(*v*) Refrigeration and air-conditioning (*vi*) Lifts. *(Nagpur University, Summer 2004)*
15. Explain with neat block diagram the digital control of Electrical Drives. *(Nagpur University, Summer 2004)*
16. Explain Series parallel control of traction motor. *(Nagpur University, Summer 2004)*
17. Write short Notes on Speed reversal by contactor and relay. *(Nagpur University, Summer 2004)*
18. Write short Notes on Ratings of contactors. *(Nagpur University, Summer 2004)*
19. Write short Notes on Magnetic time-delay relay. *(Nagpur University, Summer 2004)*
20. Discuss the advantages and disadvantages of electric drive over other drives. *(J.N. University, Hyderabad, November 2003)*
21. Though a.c. is superior to d.c. for electric drives, sometimes d.c. is preferred. Give the reasons and mention some of the applications. *(J.N. University, Hyderabad, November 2003)*
22. A d.c. series motor drives a load, the torque of which varies as the square of the speed. The motor takes current of 30 amps, when the speed is 600 r.p.m. Determine the speed and current when the field winding is shunted by a diverter, the resistance of which is 1.5 times that of the field winding. The losses may be neglected. *(J.N. University, Hyderabad, November 2003)*
23. State the condition under which regenerative braking with d.c. services motor is possible and with the aid of diagrams of connection, explain the various methods of providing regeneration. *(J.N. University, Hyderabad, November 2003)*
24. Explain what you mean by ''Individual drive'' and ''Group drive''. Discuss their relative merits and demerits. *(J.N. University, Hyderabad, November 2003)*
25. A 500 V d.c. series motor runs at 500 r.p.m. and takes 60 amps. The resistances of the field and the armature are 0.3 and 0.2 Ohms, respectively. Calculate the value of the resistance to be shunted with the series field winding in order that the speed may be increased to 600 r.p.m., if the torque were to remain constant. Saturation may be neglected. *(J.N. University, Hyderabad, November 2003)*
26. A motor has the following duty cycle :
Load rising from 200 to 400 h.p. – 4 minutes
Uniform load 300 h.p. – 2 minutes
Regenerative braking h.p. Returned to supply from 50 to zero – 1 minute.
Remains idle for 1 minute.
Estimate the h.p. of the motor. *(J.N. University, Hyderabad, November 2003)*
27. What are various types of electric braking used? *(J.N. University, Hyderabad, November 2003)*
28. Explain how rheostatic braking is done in D.C. shunt motors and series motors. *(J.N. University, Hyderabad, November 2003)*
29. Describe how plegging, rheostics braking and regenerative braking are employed with D.C. series motor. *(J.N. University, Hyderabad, November 2003)*
30. Where is the use of Individual drive recommended and why? *(J.N. University, Hyderabad, November 2003)*
31. The speed of a 15 h.p. (Metric) 400 V d.c. shunt motor is to be reduced by 25% by the use of a controller. The field current is 2.5 amps and the armature resistance is 0.5 Ohm. Calculate the resistance of the controller, if the torque remains constant and the efficiency is 82%. *(J.N. University, Hyderabad, November 2003)*
32. Explain regenerative braking of electric motors. *(J.N. University, Hyderabad, November 2003)*
33. ''If a high degree of speed control is required, d.c. is perferable to a.c. for an electric drive''. Justify. *(J.N. University, Hyderabad, April 2003)*
34. A 200 V shunt motor has an armature resistance of 0.5 ohm. It takes a current of 16 amps on full load and runs at 600 r.p.m. If a resistance of 0.5 ohm is placed in the armature circuit, find the

ratio of the stalling torque to the full load torque. *(J.N. University, Hyderabad, April 2003)*

35. What are the requirements of good electric braking? *(J.N. University, Hyderabad, April 2003)*

36. Explain the method of rheostatic braking. *(J.N. University, Hyderabad, April 2003; Anna University, Chennai 2003)*

37. Mean horizontal Candlepower *(J.N. University, Hyderabad, April 2003)*

38. Mean hemispherical Candlepower *(J.N. University, Hyderabad, April 2003)*

39. Luminous flux. *(J.N. University, Hyderabad, April 2003)*

40. Define : (*i*) Luminous intensity (*ii*) Point source (*iii*) Lumen and (*iv*) Uniform point source. *(J.N. University, Hyderabad, April 2003)*

41. Prove that Luminous intensity of a point source is equal to the luminous flux per unit solid angle. *(J.N. University, Hyderabad, April 2003)*

42. Discuss the various factors that govern the choice of a motor for a given service. *(J.N. University, Hyderabad, April 2003)*

43. A 6 pole, 50 Hz slip ring induction motor with a rotor resistance per phase of 0.2 ohm and a stand still reactance of 1.0 ohm per phase runs at 960 r.p.m. at full load. Calculate the resistance to be inserted in the rotor circuit to reduce the speed to 800 r.p.m., if the torque remains unaltered. *(J.N. University, Hyderabad, April 2003)*

44. Compare the features of individual and group drives. *(J.N. University, Hyderabad, April 2003)*

45. What is an electric drive? Classify various types of electric drives and discuss their merits and demerits. *(J.N. University, Hyderabad, December 2002/January 2003)*

46. Suggest, with reasons the electric drive used for the following applications. (*i*) Rolling mills (*ii*) Textile mills (*iii*) Cement mills (*iv*) Paper mills (*v*) Coal mining (*vi*) Lift, Cranes, Lathes and pumps. *(J.N. University, Hyderabad, December 2002/January 2003)*

47. A 100 hp, 500 rpm d.c. shunt motor is driving a grinding mill through gears. The moment of inertia of the mill is 1265 kgm^2. If the current taken by the motor must not exceed twice full load current during starting, estimate the minimum time taken to run the mill upto full speed. *(J.N. University, Hyderabad, December 2002/January 2003)*

48. Explain the different methods of electric braking of a 3 phase induction motor. *(J.N. University, Hyderabad, December 2002/January 2003)*

49. A 50 hp, 400V, 750 rpm synchronous motor has a moment of inertia 20 kgm^2 and employs rheostatic braking for obtaining rapid stopping in case of emergency when the motor is running at full load, star connected braking resistor of 2 ohm per phase is switched on. Determine the time taken and the number of revolutions made before the motor is stopped. Assume as efficiency of 90% and a full load power factor of 0.95. *(J.N. University, Hyderabad, December 2002/January 2003)*

50. Explain regenerative braking of induction motor. *(J.N. University, Hyderabad, December 2002/January 2003)*

51. What is dynamic braking? *(Anna University, Chenni, Summer 2003)*

52. What is regenerative braking? *(Anna University, Chenni, Summer 2003)*

53. What are braking systems applicable to a DC shunt motor? *(Anna University, Chenni, Summer 2003)*

54. What for Series motor Regenerative Braking is not suited? *(Anna University, Chenni, Summer 2003)*

55. What are the important stages in controlling an electrical drive. *(Anna University, Chenni, Summer 2003)*

56. Explain rheostatic braking of D.C. motors. *(Anna University, Chennai 2003)*

OBJECTIVE TESTS – 44

1. A steel mill requires a motor having high starting torque, wide speed range and precise speed control. Which one of the following motors will you choose ?
(*a*) d.c. shunt motor
(*b*) synchronous motor
(*c*) d.c. series motor
(*d*) slip-ring induction motor.

2. Heavy-duty steel-works cranes which have wide load variations are equipped with .... motor.
(*a*) double squirrel-cage

(*b*) d.c. series
(*c*) slip-ring induction
(*d*) cumulative compound.

3. A reciprocating pump which is required to start under load will need ....motor.
(*a*) repulsion
(*b*) squirrel-cage induction
(*c*) synchronous
(*d*) double squirrel-cage induction.

4. Motors used in wood-working industry have .... enclosure.
(*a*) screen protected (*b*) drip proof
(*c*) TEFC (*d*) TE

5. Single-phase synchronous motors are used in teleprinters, clocks and all kinds of timing devices becasue of their
(*a*) low starting torque
(*b*) high power factor
(*c*) constant speed
(*d*) over-load capacity.

6. Which motor is generally used in rolling mills, paper and cement industries ?
(*a*) d.c. shunt motor
(*b*) double squirrel-cage motor
(*c*) slip-ring induction motor
(*d*) three-phase synchronous motor

7. Direct drive is used for power transmission only when
(*a*) negligible slip is required
(*b*) large amount of power is invloved
(*c*) speed of the driven machine equals the motor speed
(*d*) high-speed motor is to drive a low-speed machine.

8. Which type of enclosure will be most suitable for motors employed in atmospheres containing inflammable gases and vapours ?
(*a*) pipe-ventilated
(*b*) totally enclosed, fan-cool
(*c*) flame proof
(*d*) screen-protected.

9. While plugging d.c. motors, ... connections are reversed
(*a*) supply
(*b*) armature
(*c*) field
(*d*) both armature and field

10. During rheostatic braking of a d.c., motor,
(*a*) its field is disconnected from the supply
(*b*) its armature is reverse-connected
(*c*) it works as a d.c. generator
(*d*) direction of its field current is reversed.

11. Rheostatic braking may be applied to an induction motor provided
(*a*) separate d.c. source for field excitation is available
(*b*) it is a squirrel cage type
(*c*) it is slip-ring type
(*d*) variable external resistance is available

12. During regenerative braking of electric motors, they are
(*a*) disconnected from the supply
(*b*) reverse-connected to the supply
(*c*) made to run as generators
(*d*) made to stop.

13. Regenerative braking
(*a*) can be used for stopping a motor
(*b*) cannot be easily applied to d.c. series motors
(*c*) can be easily applied to d.c. shunt motors
(*d*) cannot be used wen motor load has over-hauling characteristics

14. Net energy saved during regenerative braking of an electric train
(*a*) increases with increase in specific resistance
(*b*) is high with high down gradient
(*c*) decreases with reduction in train speed due to braking
(*d*) is independent of the train weight.

15. The selection of an electric motor for any application depends on which of the following factors?
(*a*) Electrical characteristics
(*b*) Mechanical characteristics
(*c*) Size and reating of motors
(*d*) cost
(*e*) All of teh above

16. For a particular application the type of electric and control gear are determined by which of the following considerations?
(*a*) Starting torque
(*b*) Conditions of environment
(*c*) Limitation on starting current
(*d*) Speed control range and its nature
(*e*) all of the above

17. Which of the following motors is preferred for traction work?
(*a*) Universal motor
(*b*) D.C. series motor
(*c*) Synchronous motor
(*d*) three-phase induction motor

18. Which of the following motors always starts

on load?
(*a*) Conveyor motor (*b*) Floor mill motor
(*c*) Fan motor (*d*) All of the above

19. .... is preferred for automatic drives.
(*a*) Squirrel cage induction motor
(*b*) Synchronous motors
(*c*) Ward-Leonard controlled D.C. motors
(*d*) Any of the above

20. When the load is above .... a synchronous motor is found to be more economical.
(*a*) 2 kW (*b*) 20 kW
(*c*) 50 kW (*d*) 100 kW

21. The load cycle for a motor driving a power press will be ....
(*a*) variable load
(*b*) continuous
(*c*) continuous but periodical
(*d*) intermittent and variable load

22. Light duty cranes are used in which of the following?
(*a*) Power houses
(*b*) Pumping station
(*c*) Automobile workshops
(*d*) all of the above

23. While selecting an electric motor for a floor mill, which electrical characteristics will be of least significance?
(*a*) Running characteristics
(*b*) Starting charasteristics
(*c*) Efficiency
(*d*) Braking

24. Which of the following motors are preferred for overhead travelling cranes?
(*a*) Slow speed motors
(*b*) Continuous duty motors
(*c*) Short time rated motors
(*d*) None of the above

25. .... is preferred for synthetic fibre mills.
(*a*) Synchronous motor
(*b*) Reluctance motor
(*c*) Series motor
(*d*) Shunt motor

26. Ward-Leonard controlled D.C. drives are generally used for .... excavators.
(*a*) Light duty (*b*) Medium duty
(*c*) Heavy duty (*d*) All of the above

27. Which of the following motors is used for elevators?
(*a*) Induction motor
(*b*) Synchronous motor
(*c*) Capacitor start single phase motor
(*d*) Any of the above

28. Which part of a motor needs maximum attention for maintenance?
(*a*) Frame (*b*) Bearing
(*c*) Stator winding (*d*) Rotor winding

29. .... need frequent starting and stopping of electric motors.
(*a*) Paper mills
(*b*) Grinding mills
(*c*) Air-conditioners
(*d*) Lifts and hoists

30. Which feature, while selecting a motor for centrifugal pump, will be of least significance?
(*a*) Starting characteristics
(*b*) Operating speed
(*c*) Horse power
(*d*) Speed control

31. ...... motor is a constant speed motor.
(*a*) Synchronous motor
(*b*) Schrage motor
(*c*) Induction motor
(*d*) Universal motor

32. The starting torque is case of centrifugal pumps is generally
(*a*) less than running torque
(*b*) same as running torque
(*c*) slightly more than running torque
(*d*) double the running torque

33. Which of the following motors are best for the rolling mills?
(*a*) Single phase motors
(*b*) Squirrel cage induction motors
(*c*) Slip ring induction motors
(*d*) D.C. motors

34. .... is not a part of ball bearing?
(*a*) Inner race (*b*) Outer race
(*c*) Cage (*d*) Bush

35. The starting torque of a D.C. motor is independent of which of the following?
(*a*) Flux
(*b*) Armature current
(*c*) Flux and armature current
(*d*) Speed

36. Rotor of a motor is usually supported on .... bearings.
(*a*) ball or roller (*b*) needle
(*c*) bush (*d*) thrust

37. For which of the following applications D.C. motors are still preferred?
(*a*) High efficiency operation
(*b*) Reversibility

(c) Variable speed drive
(d) High starting torque

38. In a paper mill where constant speed is required
(a) synchronous motors are preferred
(b) A.C. motors are preferred
(c) individual drive is preferred
(d) group drive is preferred

39. A reluctance motor ......
(a) is provided with slip rings
(b) requires starting gear
(c) has high cost
(d) is compact

40. The size of an excavator is usually expressed in terms of
(a) 'crowd' motion (b) angle of swing
(c) cubic metres (d) travel in metres

41. For blowers which of the following motors in preferred?
(a) d.C. series motor
(b) D.C. shunt motor
(c) Squirrel cage induction motor
(d) Wound rotor induction motor

42. Belted slip ring induction motor is almost invariably used for
(a) water pumps
(b) jaw crushers
(c) centrifugal blowers
(d) none of the above

43. Which of the following is essentially needed while selecting a motor?
(a) Pulley (b) Starter
(c) Foundation pedal (d) Bearings

44. Reluctance motor is a ....
(a) variable torque motor
(b) low torque variable speed motor
(c) self starting type synchronous motor
(d) low noise, slow speed motor

45. .... method of starting a three phase induction motor needs six terminals.
(a) Star-delta
(b) Resistance starting
(c) Auto-transformer
(d) None of the above

46. In .... method of starting three phase induction motors the starting voltage is not reduced.
(a) auto-transformer
(b) star-delta
(c) slip ring
(d) any of the above

47. In jaw crushers a motor has to often start against ...... load.
(a) heavy (b) medium
(c) normal (d) low

48. For a motor-generator set which of the following motors will be preferred?
(a) Synchronous motor
(b) Slip ring induction motor
(c) Pole changing induction motor
(d) Squirrel cage induction motor

49. Which of the following motors is usually preferred for kiln drives?
(a) Cascade controlled A.C. motor
(b) slip ring induction motor
(c) three phase shunt would commutator motor
(d) Any of the above

50. Heat control switches are used in ....
(a) transformers
(b) cooling ranges
(c) three phase induction motors
(d) single phase

51. .... has relatively wider range of speed control
(a) Synchronous motor
(b) Ship ring induction motor
(c) Squirrel cage induction motor
(d) D.C. shunt motor

52. In squirrel cage induction motors which of the following methods of starting cannot be used?
(a) Resistance in rotor circuit
(b) Resistance in stator circuit
(c) Auto-transformer starting
(d) Star-delta starting

53. In which of the following applications the load on motor changes in cyclic order?
(a) Electric shovels
(b) Cranes
(c) Rolling mills
(d) All of the above

54. Flame proof motors are used in
(a) paper mills
(b) steel mills
(c) moist atmospheres
(d) explosive atmospheres

55. Which of the following machines has heavy fluctuation of load?
(a) Printing machine
(b) Punching machine
(c) Planer
(d) Lathe

56. For derries and winches which of the following drives can be used?
(a) Pole changing squirrel cage motors
(b) D.C. motors with Ward-leonard control

(*c*) A.C. slip ring motors with variable resistance
(*d*) Any of the above

57. Battery operated scooter for braking uses
(*a*) plugging
(*b*) mechanical braking
(*c*) regenerative braking
(*d*) rheostatic braking

58. .... has least range of speed control.
(*a*) Slip ring induction motor
(*b*) Synchronous motor
(*c*) D.C. shunt motor
(*d*) Schrage motor

59. .... has the least value of starting torque to full load torque ratio.
(*a*) D.C. shunt motor
(*b*) D.C. series motor
(*c*) Squirrel cage induction motor
(*d*) Slip ring induction motor

60. In case of .... speed control by injecting e.m.f. in the rotor circuit is possible.
(*a*) d.c. shunt motor
(*b*) schrage motor
(*c*) synchronous motor
(*d*) slip ring induction motor

61. A pony motor is used for the starting which of the following motors?
(*a*) Squirel cage induction motor
(*b*) Schrage motor
(*c*) Synchronous motor
(*d*) None of the above

62. In .... the speed can be varied by changing the position of brushes.
(*a*) slip ring motor
(*b*) schrage motor
(*c*) induction motor
(*d*) repulsion motor

63. In which of the following applications variable speed operation is preferred?
(*a*) Exhaust fan
(*b*) Ceiling fan
(*c*) Refrigerator
(*d*) Water pump

64. Heavy duty cranes are used in
(*a*) ore handling plants
(*b*) steel plants
(*c*) heavy engineering workshops
(*d*) all of the above

65. the travelling speed of cranes varies from
(*a*) 20 to 30 m/s
(*b*) 10 to 15 m/s
(*c*) 5 to 10 m/s
(*d*) 1 to 2.5 m/s

66. Besides a constant speed a synchronous rotor possesses which of the following advantages?
(*a*) Lower cost
(*b*) Batter efficiency
(*c*) High power factor
(*d*) All of the above

67. By the use of which of the following D.C. can be obtained from A.C.?
(*a*) Silicon diodes
(*b*) Mercury are rectifier
(*c*) Motor generator set
(*d*) any of the above

68. Which of the following motors is preferred when quick speed reversal is the main consideration?
(*a*) Squirrel cage induction motor
(*b*) Wound rotor induction motor
(*c*) Synchronous motor
(*d*) D.C. motor

69. Which of the following motors is preferred when smooth and precise speed control over a wide range is desired?
(*a*) D.C. motor
(*b*) Squirrel cage induction motor
(*c*) Wound rotor induction motor
(*d*) Synchronous motor

70. For crane travel which of the following motors is normally used?
(*a*) Synchronous motor
(*b*) D.C. differentially compound motor
(*c*) Ward-Leonard controlled D.C. shunt motor
(*d*) A.C. slip ring motor

71. The capacity of a crane is expressed in terms of
(*a*) type of drive
(*b*) span
(*c*) tonnes
(*d*) any of the above

72. the characteristics of drive for crane hoisting and lowering are which of the following?
(*a*) Precise control
(*b*) Smooth movement
(*c*) Fast speed control
(*d*) All of the above

73. Which of the following motors is preferred for boom hoist of a travelling crane?
(*a*) Single phase motor
(*b*) Synchronous motor
(*c*) A.C. slip ring motor

(*d*) Ward-Leonard controlled D.C. shunt motor

74. A wound rotor induction motor is preferred, as compared to squirrel cage induction motor, when major consideration is
(*a*) slop speed operation
(*b*) high starting torque
(*c*) low windage losses
(*d*) all of the above

75. Which of the following motors has series characteristics?
(*a*) Shadel pole motor
(*b*) Repulsion motor
(*c*) Capacitor start motor
(*d*) None of the above

76. Which of the following happens when star-delta starter is used?
(*a*) Starting voltage is reduced
(*b*) Starting current is reduced
(*c*) Both (*a*) and (*b*)
(*d*) None of the above

77. For a D.C. shunt motor which of the following is incorrect?
(*a*) Unsuitable for heavy duty starting
(*b*) Torque varies as armature current
(*c*) Torque-armature current is a straight line
(*d*) Torque is zero for zero armature current

78. For which of the following applications motor has to start with high acceleration?
(*a*) Oil expeller
(*b*) Floor mill
(*c*) Lifts and hoists
(*d*) centrifugal pump

79. Which of the following types of motor enclosure is safest?
(*a*) totally enclosed
(*b*) Totally enclosed fan cooled
(*c*) Open type
(*d*) Semi closed

80. While selecting motor for an air conditioner which of the following characteristics is of great importance?
(*a*) Type of bearings
(*b*) Type of enclosure
(*c*) Noise
(*d*) Arrangement for power transmission
(*e*) None of the above

81. The diameter of the rotor shaft for an electric motor depends on which of the following?
(*a*) r.p.m. only
(*b*) Horse power only
(*c*) Horse power and r.p.m.
(*d*) Horse power, r.p.m. and power factor

82. Which of the following alternatives will be cheaper?
(*a*) A 100 H.P. A.C. three phase motor
(*b*) Four motors of 25 H.P. each
(*c*) Five motors of 20 H.P. each
(*d*) Ten motors of 10 H.P. each

83. The cost of an induction motor will increase as
(*a*) horsepower rating increases but r.p.m. decreases
(*b*) horsepower rating decreases but r.p.m. increases
(*c*) horsepower rating and operating speed increases
(*d*) horsepower rating and operating speed decreases

84. in series motor which of the following methods can be used for changing the flux per pole?
(*a*) Tapped field control
(*b*) Diverter field control
(*c*) Series-parallel control
(*d*) Any of the above

ANSWERS

1. (*c*)	**2.** (*b*)	**3.** (*d*)	**4.** (*c*)	**5.** (*c*)	**6.** (*d*)	**7.** (*c*)	**8.** (*c*)	**9.** (*b*)	**10.** (*c*)
11. (*a*)	**12.** (*c*)	**13.** (*c*)	**14.** (*b*)	**15.** (*e*)	**16.** (*e*)	**17.** (*b*)	**18.** (*d*)	**19.** (*c*)	**20.** (*d*)
21. (*d*)	**22.** (*d*)	**23.** (*d*)	**24.** (*c*)	**25.** (*b*)	**26.** (*c*)	**27.** (*a*)	**28.** (*b*)	**29.** (*d*)	**30.** (*d*)
31. (*a*)	**32.** (*a*)	**33.** (*d*)	**34.** (*d*)	**35.** (*d*)	**36.** (*a*)	**37.** (*c*)	**38.** (*c*)	**39.** (*d*)	**40.** (*c*)
41. (*c*)	**42.** (*b*)	**43.** (*b*)	**44.** (*c*)	**45.** (*a*)	**46.** (*c*)	**47.** (*a*)	**48.** (*a*)	**49.** (*d*)	**50.** (*b*)
51. (*d*)	**52.** (*a*)	**53.** (*d*)	**54.** (*d*)	**55.** (*b*)	**56.** (*d*)	**57.** (*b*)	**58.** (*b*)	**59.** (*c*)	**60.** (*d*)
61. (*c*)	**62.** (*b*)	**63.** (*b*)	**64.** (*d*)	**65.** (*d*)	**66.** (*c*)	**67.** (*d*)	**68.** (*c*)	**69.** (*a*)	**70.** (*d*)
71. (*c*)	**72.** (*d*)	**73.** (*c*)	**74.** (*b*)	**75.** (*b*)	**76.** (*c*)	**77.** (*a*)	**78.** (*c*)	**79.** (*b*)	**80.** (*c*)
81. (*c*)	**82.** (*a*)	**83.** (*a*)	**84.** (*d*)						

GO To FIRST

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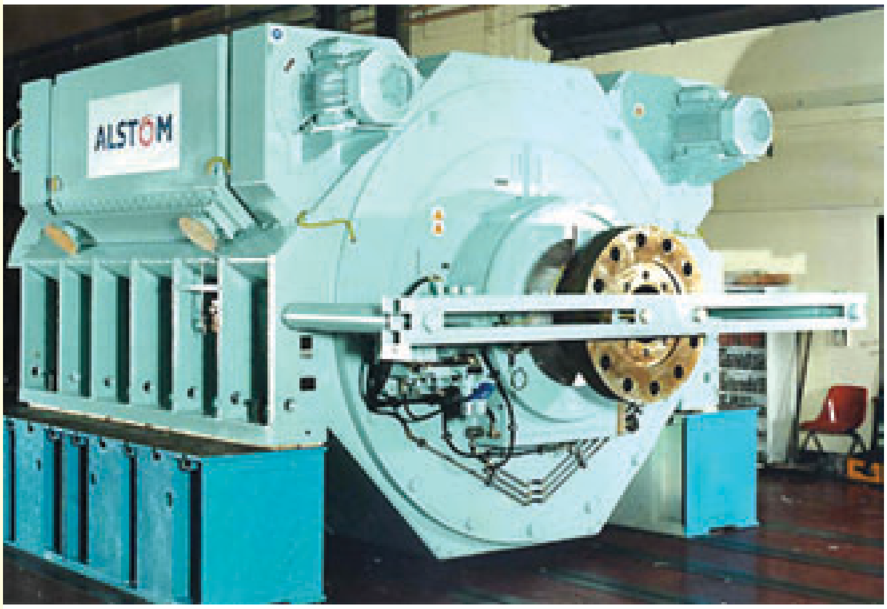
CHAPTER 45

Learning Objectives

- Size and Rating
- Estimation of Motor Rating
- Different Types of Industrial Loads
- Heating of Motor or Temperature Rise
- Equation for Heating of Motor
- Heating Time Constant
- Equation for Cooling of Motor or Temperature Fall
- Cooling Time Constant
- Heating and Cooling Curves
- Load Equalization
- Use of Flywheels
- Flywheel Calculations
- Load Removed (Flywheel Accelerating)
- Choice of Flywheel

RATING AND SERVICE CAPACITY



Generator converts mechanical energy into electrical energy using electromagnetic induction

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45.1. Size and Rating

The factors which govern the size and rating of motor for any particular service are its maximum temperature rise under given load conditions and the maximum torque required. It is found that a motor which is satisfactory from the point of view of maximum temperature rise usually satisfies the requirement of maximum torque as well. For class-A insulation, maximum permissible temperature rise is 40ºC whereas for class – B insulation, it is 50ºC. This temperature rise depends on whether the motor has to run continuously, intermittently or on variable load.

Different ratings for electrical motors are as under:

1. Continuous Rating. It is based on the maximum load which a motor can deliver for an indefinite period without its temperature exceeding the specified limits and also possesing the ability to take 25% overload for a period of time not exceeding two hours under the same conditions.

For example, if a motor is rated continuous 10 KW, it means that it is capable of giving an output of 10 KW continuously for an indefinite period of time and 12.5 KW for a period of two hours without its temperature exceeding the specified limits.

2. Continuous Maximum Rating. It is the load capacity as given above but without overload capacity. Hence, these motors are a little bit inferior to the continuous-rated motors.

3. Intermittent Rating. It is based on the output which a motor can deliver for a specified period, say one hour or ½ hour or ¼ hour without exceeding the temperature rise.

This rating indicates the maximum load of the motor for the specified time followed by a no-load period during which the machine cools down to its original temperature.

45.2. Estimation of Motor Rating

Since primary limitation for the operation of an electric motor is its temperature rise, hence motor rating is calculated on the basis of its average temperature rise. The average temperature rise depends on the average heating which itself is proportional to the square of the current and the time for which the load persists.

For example, if a motor carries a load L_1 for time t_1 and load L_2 for time t_2 and so on, then

$$\text{Average heating} \propto L_1^2 t_1 + L_2^2 t_2 + \ldots\ldots + L_n^2 t_n$$

In fact, heating is proportional to square of the current but since load can be expressed in terms of the current drawn, the proportionality can be taken for load instead of the current.

$$\therefore \text{ size of the motor } = \sqrt{\frac{L_1^2 t_1 + L_2^2 t_2 + \ldots\ldots + L_n^2 t_n}{t_1 + t_2 + \ldots\ldots t_n}}$$

Generally, load on a motor is expressed by its load cycle. Usually, there are periods of no-load in the cycle. When motor runs on no-load, heat generated is small although heat dissipation continues at the same rate as long as the machine is running. Hence, there is a difference in the heating of a motor running at no-load and when at rest. It is commonly followed practice in America to consider the period at rest as one – third while calculating the size of motor. It results in giving a higher motor rating which is advantageous and safe.

Example 45.1 *An electric motor operates at full-load of 100 KW for 10 minutes, at ¾ full load for the next 10 minutes and at ½ load for next 20 minutes, no-load for the next 20 minutes and this cycle repeats continuously. Find the continuous rating of the suitable motor.*

Solution.

$$\text{Size of the motor required} = \sqrt{\frac{100^2 \times 10 + 75^2 \times 10 + 50^2 \times 20 + 0 \times 20}{10 + 10 + 20 + 20}}$$

$$= 61 \text{ KW}$$

According to American practice, we will consider the period of rest as (20/3) minutes. In that case, the motors size is

$$= \sqrt{\frac{100^2 \times 10 + 75^2 \times 10 + 50^2 \times 20 + 0 \times 20}{10 + 10 + 20 + (20/3)}}$$

$= 66$ **KW**

Example 45.2. *An electric motor has to be selected for a load which rises uniformly from zero to 200 KW in 10 minutes after which it remains constant at 200 KW for the next 10 minutes, followed by a no-load period of 15 minutes before the cycle repeats itself. Estimate a suitable size of continuously rated motor.*

Solution.

$$\text{Motor size} = \sqrt{\frac{(200/2)^2 \times 10 + (200)^2 \times 10 + 0 \times 15}{10 + 10 + (15 \times 1/3)}}$$

$= 140$ **KW**

According to American practice, no-load has been taken as one third.

Example 45.3. *A certain motor has to perform the following duty cycle:*

100 KW for 10 minutes	*No-load for 5 minutes*
50 KW for 8 minutes	*No-load for 4 minutes*

The duty cycle is repeated indefinitely. Draw the curve for the load cycle. Assuming that the heating is proportional to the square of the load, determine suitable size of a continuously-rated motor. **[Utilisation of Electric Power A.M.I.E.]**

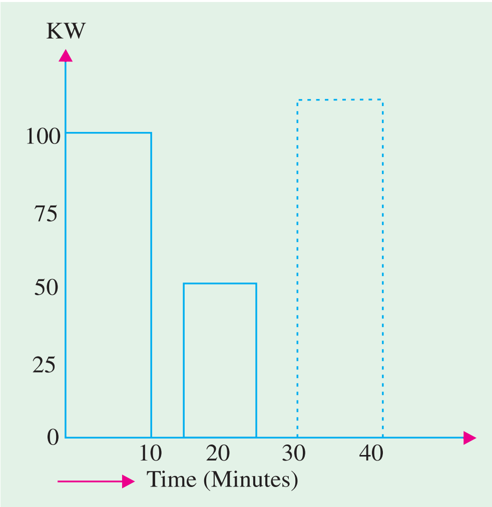

Fig. 45.1

Solution.

As explained above, heating is prorortional to the square of the current and hence, to the square of the load.

∴ size of the continuously-rated motor

$$\sqrt{\frac{100^2 \times 10 + 50^2 \times 8}{10 + 5 + 8 + 4}}$$

$= 66.67$ **kW**

Hence, the motor of **70 KW** would be adequate. The curve of the load cycle is shown in Fig. 45.1

The ultimate usefulness of the above factors is to select a motor of as small a size as possible compatible with temperature rise and to ensure that the motor has ample overload torque to cater for maximum-load conditions. Obviously, over-motoring of any industrial drive will result in a waste of electrical energy, a low power factor and unnecessarily high capital cost for the motor and control gear.

45.3. Different Types of Industrial Loads

The three different types of industrial loads under which electric motors are required to work are as under:

(i) continuous load ***(ii)*** intermittent load and ***(iii)*** variable or fluctuating load.

The size of the motor depends on two factors. Firstly, on the temperature rise which, in turn, will depend on whether the motor is to operate on continuous, intermittent or variable load. Secondly, it will depend on the maximum torque to be developed by the motor. Keeping in mind the load torque requirements, the rating of the motor will be decided by the load conditions as described below.

(*i*) Continuous Load. In such cases, the calculation of motor size is simpler because the loads like pumps and fans require a constant power input to keep them operating. However, it is essential to calculate the KW rating of the motor correctly. If the KW rating of the motor is less than what is required, the motor will overheat and consequently burn out. If, on the other hand, KW rating is more than what is needed by the load, the motor will remain cool but will operate at lower efficiency and power.

(*ii*) Intermittent Loads. Such loads can be of the following two types:

(*a*) In this type of load, motor is loaded for a short time and then shut of for a sufficient by long time, allowing the motor to cool down to room temperature as shown in Fig. 45.2. In such cases, a motor with a short time rating is used as in a kitchen mixie.

Torque motors are designed to provide maximum torque at locked rotor or near stalled conditions. Their applications are in servo and positioning systems, tension reels, automatic door openers, and filament winding equipment.

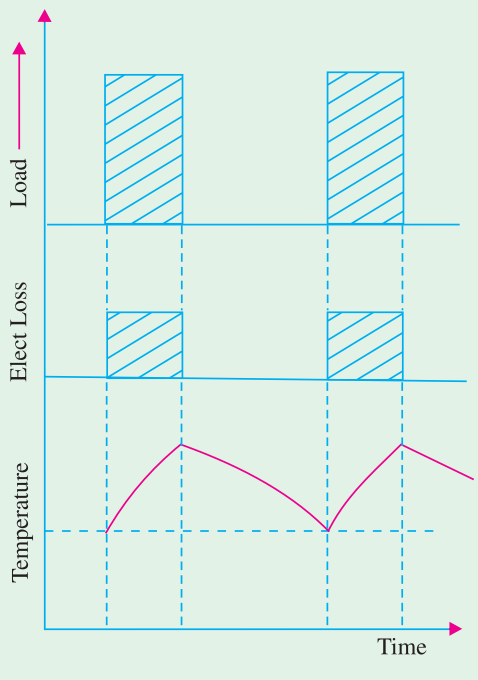

Fig. 45.2

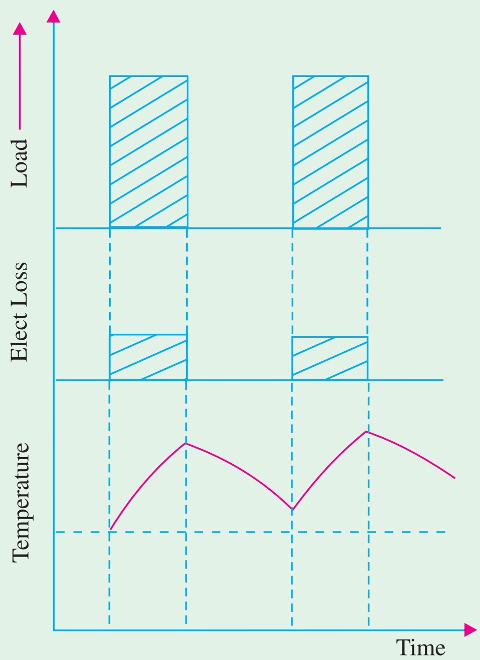

Fig. 45.3

(*b*) In this type of load, motor is loaded for a short time and then it is shut off for a short time. The shut off time is so short that the motor cannot cool down to the room temperature as shown in Fig.45.3. In such cases, a suitable continuous or short-time rated motor is chosen which, when operating on a given load cycle, will not exceed the specified temperature limit.

(*iii*) Variable Loads. In the case of such loads, the most accurate method of selecting a suitable motor is to draw the heating and cooling curves as per the load fluctuations for a number of motors. The smallest size motor which does not exceed the permitted temperature rise when operating on the particular load cycle should be chosen for the purpose.

However, a simpler but sufficiently accurate method of selection of a suitable rating of a motor is to assume that heating is proportional to the square of the current and hence the square of the load. The suitable continuous rating of the motor would equal the r.m.s. value of the load current.

Example 45.4. *A motor has to perform the following duty cycle*

100 H.P.	*For*	*10 min*
No Load	”	*5 min*
60 H.P.	”	*8 min*
No Load	”	*4 min*

which is repeated infinitely. Determine the suitable size of continuously rated motor.

Solution.

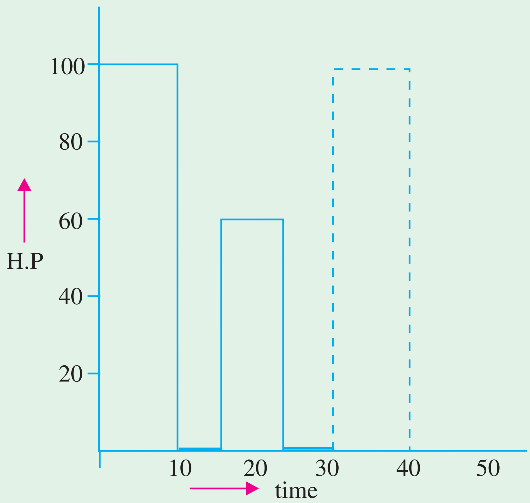

Fig. 45.4

$$\text{R.M.S. H.P.} = \sqrt{\left\{\frac{1}{\text{Time for one cycle}}\right\}\int \text{HP}^2 dt}$$

$$\text{R.M.S. H.P.} = \left[\frac{\sum \text{HP}^2 \times \text{time}}{\text{Time for one cycle}}\right]^{\frac{1}{2}}$$

$$= \sqrt{\frac{100^2 \times 10 + 50^2 \times 8}{10 + 5 + 8 + 4}} = 69.07 \text{ H.P.}$$

$\simeq$ **75 H.P. motor can be used.**

Example 45.5. *A motor working in a coal mine has to exert power starting from zero and rising uniformly to 100 H.P. in 5 min after which it works at a constant rate of 50 H.P. for 10 min. Then, a no load period of 3 min. The cycle is repeated indefinitely, estimate suitable size of motor.*
[Nagpur University Summer 2000]

Solution.

(*a*) For time period : 0 - 5 min

$$\Rightarrow \quad y = mx + c$$

$$\text{Slope} = \frac{(100-0)}{5}$$

$$m = 20 \text{ HP/min}$$

$$\therefore \quad y = 20\,x + 0$$

$$\mathbf{y = 20\,x}$$

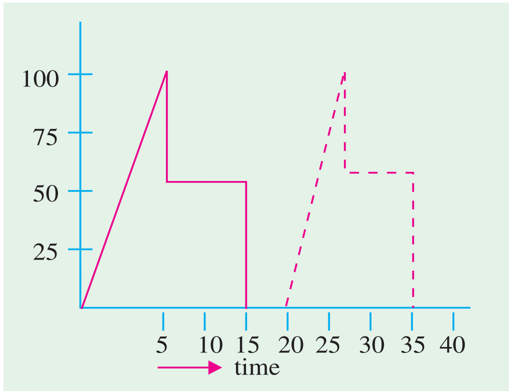

Fig. 45.5

(*b*) For total time period : 0 - 18 min

R.M.S. H P^2

$$= \left\{ \left[\int_0^5 y^2\,dx \right] + 50^2 \times 10 + 0^2 \times 3 \right\} / 18$$

$$\Rightarrow \text{H.P.}^2 \times 18 = \left[\int_0^5 (20\,x)^2\,dx \right] + 25000 = \left[\frac{400\,x^3}{3} \right]_0^5 + 25000$$

$$\therefore \text{H.P.}^2 \times 18 = \frac{400 \times 125}{3} + 25000$$

$$\Rightarrow \text{H.P.} = \sqrt{\frac{41666.67}{18}} = 48.11 \text{H.P.} \simeq 50 \text{ H.P. motor can used}$$

or Same problem can be solved by Simpson's 1/3rd Rule of Integration

$$\text{H. P.} = \sqrt{\frac{\frac{1}{3} \times 100^2 \times 5 + 50^2 \times 10}{18}}$$

H. P. = 48.11 H.P. ; $\therefore$ **H. P. $\simeq$ 50 H.P.**

Example 45.6. *A motor has following duty cycle*

Load rising from 200 to 400 H.P.	-	*4 min.*
Uniform load 300 H. P.	-	*2 min.*
Regenerative braking - H.P. returned to supply from 50 to zero	-	*1 min.*
Remaining idle for	-	*1 min.*

Estimate suitable H. P. rating of the motor. motor can be used.
[Nagpur University Winter 1994]

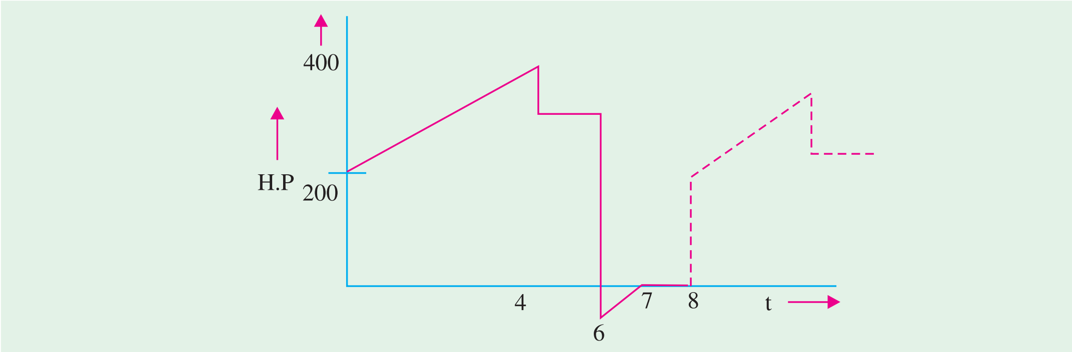

Fig. 45.6

Solution.

$$\text{H. P.} = \sqrt{\frac{\frac{1}{3}(H_1^2 + H_1H_2 + H_2^2)\, t_1 + H_3^2\, t_2 + \frac{1}{3}H_4^2\, t_3}{8}}$$

$$= \sqrt{\frac{\frac{1}{3}(200^2 + 200 \times 400 + 400^2) \times 4 + 300^2 \times 2 + \frac{1}{3}50^2 \times 1}{8}}$$

$$= \sqrt{\frac{1662500}{24}} = \mathbf{263\ H.\ P.}$$

Note. During regenerative braking, even though H.P. is returned to line, machine will be carrying current. So far heating is concerned, it is immaterial whether machine is taking current from or giving current to line.

This problem can be solved by another method as follows:-

(a) For time period : 0 - 4 min

$$\rightarrow \int_0^4 (50x + 200)^2\, dx = \int_0^4 (250x^2 + 20000x + 40000)\, dx$$

$$= 2500\left[\frac{x^3}{3}\right]_0^4 + 20000\left[\frac{x^2}{2}\right]_0^4 + 40000x$$

$$= 2500\left[\frac{4^3}{3}\right] + 20000\left[\frac{4^2}{2}\right] + 40000 \times 4 = \mathbf{373333.3\ H.P.}$$

(b) For time period : 4 - 6 min

$$\rightarrow (300)^2 \times 2 = \mathbf{180000\ H.P.}$$

(c) For time period : 6 - 7 min

$$\rightarrow \int_0^1 (50x)^2\, dx = 2500\left[\frac{x^3}{3}\right]_0^1 = \frac{2500}{3} = 833.33\,\text{H.P}$$

$$\therefore \text{R.M.S. H.P.} = \sqrt{\frac{373333.33 + 180000 + 833.33}{8}} = 263.1$$

$$\simeq \mathbf{300\ \ H.P.\ motor\ will\ be\ suitable}$$

Example 45.7. *The load cycle of a motor for 15 min. in driving some equipment is as follows :*

0 - 5 min	-	*30 H. P.*
5 - 9 min	-	*No Load*
9 - 12 min	-	*45 H. P.*
12 - 15 min	-	*No Load*

The load cycle is repeated indefinitely. Suggest a suitable size of continuously rated motor.

Solution.

$$\text{R.M.S.} \quad \text{H.P} = \left[\frac{30^2 \times 5 + 45^2 \times 3}{15}\right]^{\frac{1}{2}}$$

$$= 26.55 \ \text{H. P.}$$

$\therefore$ R.M.S. H.P $\simeq$ **30 H. P. motor will be suitable.**

Example 45.8. *A motor driving a colliery winder has the following acceleration period*

load cycle 0 - 15 sec.	:	*Load rising uniformly from 0 - 1000 H.P.*
Full speed period	:	*15 - 85 sec. Load const. at 600 H.P.*
Decceleration period	:	*85 to 95 sec. regenerative braking the H. P. returned uniformly from 200 to 0 H. P.*
95 - 120 sec.	:	*Motor stationary.*

Estimate the size of continuously rated motor.

Solution.

$$\text{R.M.S. H. P.} = \left[\frac{\frac{1}{3}(1000)^2 \times 15 + 600^2 \times 70 + \frac{1}{3}(200)^2 \times 10}{120}\right]^{\frac{1}{2}}$$

$$= 502 \ \text{H. P.}$$

$\cong$ **505 H. P. motor can be used.**

45.4. Heating of Motor or Temperature Rise

The rise in temperature of a motor results from the heat generated by the losses and an expression for this temperature rise is obtained by equating the rate at which heat is being generated by these losses to the rate at which heat is being absorbed by the motor for raising the temperature of motor and in dissipation from the surfaces exposed to cooling media.

So long as the temperature of machine rises, the generated heat will be stored in body and the rest will be dissipated to cooling medium depending upon the temperature difference. This is called as unstable or transient situation.

If the temperature of body rises, it has to store heat. The amount of heat *i.e.* stored depends upon the heat capacity of the body. If the temperature of the machine remains constant *i.e.* it doesn't rise, then no further storage of heat takes place and all the heat *i.e.* generated must be dissipated. So rate of heat generation in motor equals rate of heat dissipation from the cooling surface. This is called a stable situation.

45.5. Equation for Heating of Motor

Let,

$W \rightarrow$ Heat generated in motor due to powerloss in watts.

$G \rightarrow$ Weight of motor (kg)

S $\rightarrow$ Average specific heat in (Watt - Sec.) to raise the temperature of unit weight through 1°C.

$G \times S$ $\rightarrow$ Heat required to raise the temperature of motor through 1°C (Watt - Sec.)

θ $\rightarrow$ Temperature rise above cooling medium in °C.

θ_f $\rightarrow$ Final temperature rise in °C.

A $\rightarrow$ Cooling surface area of motor.

λ $\rightarrow$ Rate of heat dissipation from the cooling surface.
[(Watts/Unit area/°C rise in temperature.) above cooling medium]

$A\lambda$ $\rightarrow$ Rate of heat dissipation in Watts /°C rise in temperature for a motor.

Assumptions

1. Loss '*W*' remains constant during temperature rise.
2. Heat dissipation is proportional to the temperature difference between motor and cooling medium.
3. Temperature of cooling medium remains constant.

{Rate of heat generation in motor}
= {Rate of heat absorption by the motor} + {Rate of heat dissipation from cooling surface}

$$\rightarrow \quad W = GS\frac{d\theta}{dt} + A\lambda\theta$$

$$\text{or} \quad W - A\lambda\theta = GS\frac{d\theta}{dt}$$

$$\rightarrow \quad \frac{W}{A\lambda} - \theta = \frac{GS}{A\lambda}\frac{d\theta}{dt}$$

$$\rightarrow \quad \frac{d\theta}{\left(\frac{W}{A\lambda} - \theta\right)} = \frac{dt}{\frac{GS}{A\lambda}}$$

By integrating,

$$\log_e\left(\frac{W}{A\lambda} - \theta\right) = -\frac{A\lambda}{GS}t + C \qquad \text{..... (2)}$$

At $t = 0$, $\theta = \theta_1$ [Initial temperature rise *i.e.* difference between the temperature of cooling medium and temperature of motor, during starting]

If starting from cold position, $\theta_1 = 0$

Substituting the values of t and θ in above equation.

$$C = \log_e\left(\frac{W}{A\lambda} - \theta_1\right)$$

$$\therefore \quad (2) \Rightarrow \log_e\left[\frac{\left(\frac{W}{A\lambda} - \theta\right)}{\left(\frac{W}{A\lambda} - \theta_1\right)}\right] = -\frac{A\lambda}{GS}t$$

$$\text{by taking antilog,} \quad \frac{\left(\frac{W}{A\lambda} - \theta\right)}{\left(\frac{W}{A\lambda} - \theta_1\right)} = e^{-\frac{A\lambda}{GS}t}$$

$$\therefore \quad \theta = \frac{W}{A\lambda} - \left(\frac{W}{A\lambda} - \theta_1\right) e^{-\frac{A\lambda}{GS}t} \qquad \text{..... (3)}$$

When, the final temperature rise of θ_f is reached, all the heat generated is dissipated from the cooling surface so that,

equation (1) becomes $W = A\lambda\ \theta_f$ or $\theta_f = \frac{W}{A\lambda}$

And $\frac{GS}{A\lambda}$ = Heating time constant

$$\therefore \quad \frac{A\lambda}{GS} = \frac{1}{T}$$

Then equation (3) becomes;

$$\theta = \theta_f - (\theta_f - \theta_1)\, e^{\frac{-t}{T}}$$

If starting from cold, then $\theta_1 = 0$

$$\therefore \quad \theta = \theta_f\,(1 - e^{\frac{-t}{T}})$$

45.6. Heating Time Constant

Heating time constant of motor is defined as the time required to heat up the motor upto 0.633 times its final temperature rise.

$\theta = (1 - e^{-t/T})$

At $t = T$, $\theta = 0.633\ \theta_f$

After time $t = T$ θ reaches to 63.3 % of θ_f

$t = 2T$ θ reaches to 86.5 % of θ_f

$t = 3T$ θ reaches to 95 % of θ_f

$t = 4T$ θ reaches to 98.2 % of θ_f

$t = 5T$ θ reaches to 99.3 % of θ_f

T = Heating time constant.
= 90 min for motors upto 20 H.P.
= 300 min for larger motors.

45.7. Equation for Cooling of Motor or Temperature Fall

If rate of heat generation is less than rate of heat dissipation, cooling will take place.

$\therefore$ {Rate of heat generation in motor}+{Rate of heat absorption by motor}={Rate of heat dissipation from cooling surface}

$$W + GS\frac{d\theta}{dt} = A\lambda'\theta$$ where λ' = Rate of heat dissipation during cooling surface

$$W - A\lambda'\theta = -GS\frac{d\theta}{dt} \qquad \Rightarrow \frac{W}{A\lambda'} - \theta = -\frac{GS}{A\lambda'}\frac{d\theta}{dt}$$

$$\theta - \frac{W}{A\lambda'} = \frac{GS}{A\lambda'}\frac{d\theta}{dt} \qquad \Rightarrow \frac{d\theta}{\theta - \frac{W}{A\lambda'}} = \frac{dt}{\frac{GS}{A\lambda'}}$$

$$\therefore \quad \int \frac{d\theta}{\theta - \frac{W}{A\lambda'}} = \int \frac{dt}{\frac{GS}{A\lambda'}}$$

$$\log_e\left(\theta - \frac{W}{A\lambda'}\right) = -\frac{A\lambda'}{GS}t + C$$

At $t = 0$ Let $\theta = \theta_0$ Difference of temperature between cooling medium and motor (Temperature rise at which cooling starts.)

$$\therefore \quad C = \log_e\left(\theta_0 - \frac{W}{A\lambda'}\right)$$ Put this value of C in the abve equation.

$$\therefore \quad \log_e \frac{\theta - \frac{W}{A\lambda'}}{\theta_0 - \frac{W}{A\lambda'}} = -\frac{A\lambda'}{GS}t$$

If θ_f' is final temperature drop (above that of cooling medium), then at this temperature whatever heat is generated will be dissipated.

$$\therefore \quad W = A\lambda'\theta_f' \quad \Rightarrow \quad \theta_f' = \frac{W}{A\lambda'}$$

$$\therefore \quad \log_e\left\{\frac{\theta - \theta_f'}{\theta_0 - \theta_f}\right\} = -\frac{t}{T'}$$ Where T' is cooling time constant $= \frac{GS}{A\lambda'}$

$$\therefore \quad \frac{\theta - \theta_f'}{\theta_0 - \theta_f'} = e^{-t/T} \quad \Rightarrow \quad (\theta - \theta_f') = (\theta_0 - \theta_f')e^{\frac{-t}{T'}}$$

$$\boxed{\theta = \theta_f' + (\theta_0 - \theta_f')e^{\frac{-t}{T'}}}$$

If motor is disconnected from supply, there will be no losses taking place and so final temperature reached will be ambient temperature. Hence $\theta_f' = 0$ ($\because W = 0$)

$$\boxed{\therefore \quad \theta = \theta_0 . e^{\frac{-t}{T'}}}$$

If $t = T'$, then $\theta = \theta_0 . e^{-1} \Rightarrow \theta = \frac{\theta_0}{e} = 0.368\,\theta_0$; $\therefore \theta = 0.368\,\theta_0$

45.8. Cooling Time Constant

Cooling time constant is defined as the time required to cool machine down to 0.368 times the initial temperature rise above ambient temperature.

By putting different values of T' in $\theta = \theta_0 . e^{-t/T'}$

∴ After time t	=	T'	θ has fallen to	36.8% of θ_0
t	=	$2T'$	θ has fallen to	13.5% of θ_0
t	=	$3T'$	θ has fallen to	5% of θ_0

t	=	$4T'$	θ has fallen to	1.8% of θ_0
t	=	$5T'$	θ has fallen to	0.7% of θ_0

45.9. Heating and Cooling Curves

(*a*) Motor continuously worked on Full Load.

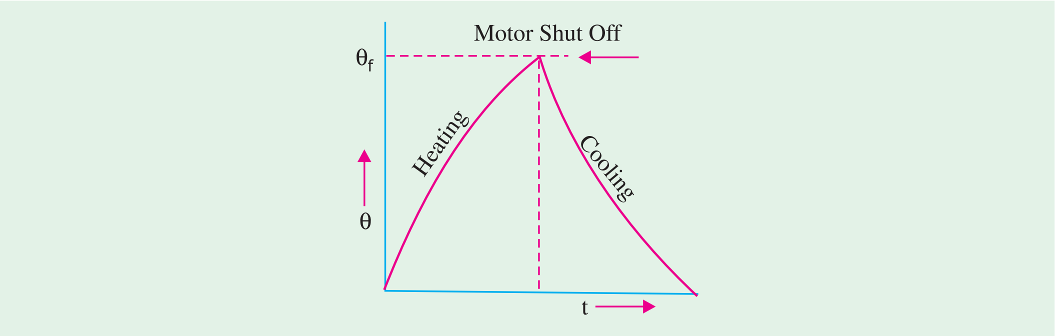

Fig. 45.7

Maximum permissible temperature rise.

Motor reaches final temperature rise and then cooling is carried out to ambient temperature.

(*b*) Motor Run for short time

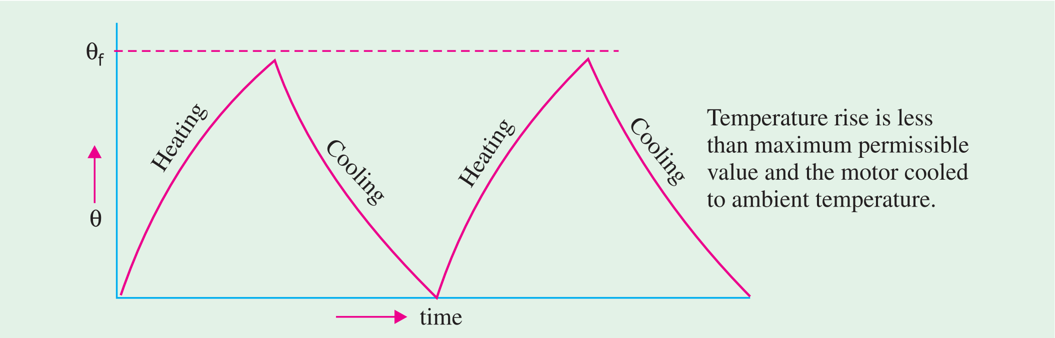

Fig. 45.8

(*c*) Cooling period not sufficient to cool down the motor to its ambient temperature.

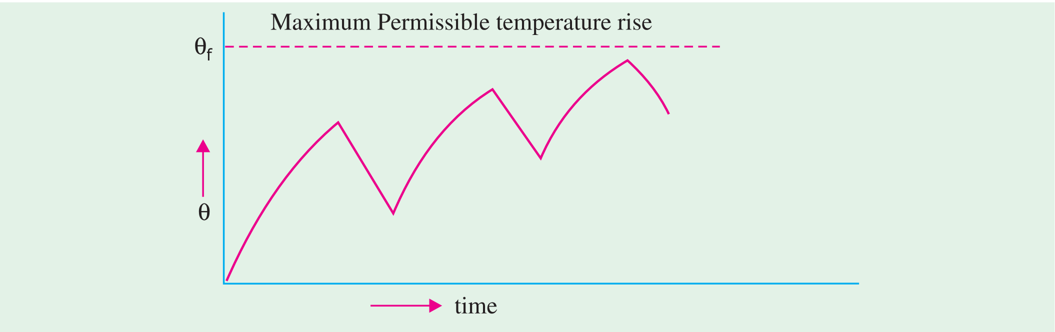

Fig. 45.9

* For intermittent loads, a motor of smaller rating can be used without exceeding maximum permissible temperature rise.

Example 45.9. *A 40 KW motor when run continuously on full load, attains a temperature of 35°C, above the surrounding air. Its heating time constant is 90 min. What would be the 1/2 hour rating of the motor for this temperature rise? Assume that the machine cools down completely between each load period and that the losses are proportional to square of the load,*

Solution.

Let 'P' KW be the ½ hour rating of the motor

θ_f – Final temperature rise at P K W

θ_f' – Final temperature rise at 40 KW

$\therefore$ Losses at P KW $\propto P^2$

Losses at 40 K W $\propto 40^2$

$$\Rightarrow \quad \frac{\theta_f}{\theta_f'} = \frac{\text{Losses at } P\text{KW}}{\text{Losses at 40 KW}} = \left(\frac{P}{40}\right)^2 \quad \therefore \quad \theta_f = \left(\frac{P}{40}\right)^2 \theta_f'$$

As the machine cools down completely, for 'P' KW the equation will be

$$\theta = \theta_f\left(1 - e^{\frac{1}{T}}\right) \quad \text{where} \quad \theta_f = \left(\frac{P}{40}\right)^2 \times 35$$

$$\Rightarrow \quad 35 = \left(\frac{P}{40}\right)^2 \times 35\left(1 - e^{-\frac{0.5}{1.5}}\right); \qquad \therefore \; P = \mathbf{75.13\ KW}$$

Example 45.10. *Determine the one - hour rating of a 15 H.P. motor having heting time constant of 2 hours. The motor attains the temperature rise of 40°C on continuous run at full load. Assume that the losses are proportional to square of the load and the motor is allowed to cool down to the ambient temperature before being loaded again.* **[Nagpur University Summer 2001]**

Solution.

Let 'P' H. P be one - hour rating of the motor

$$\text{Losses at this load} = \text{Original losses} \times \left(\frac{P}{15}\right)^2.$$

Let θ_f be the final temperature rise at P H.P. and θ_f' at 15 H P

$$\therefore \quad \frac{\theta_f}{\theta_f'} = \frac{\text{Losses at P H.P.}}{\text{Original Losses}} = \left(\frac{P}{15}\right)^2$$

$$\theta_f = \theta_f'\left(\frac{P}{15}\right)^2 = 40\left(\frac{P}{15}\right)^2$$

$$\theta = \theta_f\left(1 - e^{\frac{-t}{T}}\right)$$

$$40 = 40\left[\frac{P}{15}\right]^2\left(1 - e^{\frac{-1}{2}}\right)$$

P = 23.96 H.P.

$P \simeq$ **24 H.P.**

Example 45.11. *The heating and cooling time constants of a motor are 1 hour and 2 hours respectively. Final temperature rise attained is 100°C. This motor runs at full load for 30 miniutes and then kept idle for 12 min. and the cycle is repeated indefinitely. Determine the temperature rise of motor after one cycle.* **[Nagpur University Winter 1997]**

Solution.

$$\theta = \theta_f \left(1 - e^{\frac{-t}{T}}\right)$$

$$= 100\left(1 - e^{\frac{-30}{60}}\right) = 39.34°$$

$$\theta = \theta_0\, e^{-\frac{t}{T'}} = 39.34 e^{\frac{-12}{120}} = \mathbf{35.6°C}$$

= Temperature rise of motor after 1 cycle.

Example 45.12. *Calculate the maximum overload that can be carried by a 20 KW output motor, if the temperature rise is not to exceed 50°C after one hour on overload. The temperature rise on full load, after 1 hour is 30°C and after 2 hours is 40°C. Assume losses proportional to square of load.*

Solution.

$$\theta = \theta_f \left(1 - e^{\frac{-t}{T}}\right)$$

$$30 = \theta_f \left(1 - e^{\frac{-1}{T}}\right) \quad \text{and} \quad 40 = \theta_f \left(1 - e^{-\frac{2}{T}}\right)$$

$$\frac{1 - e^{\frac{-2}{T}}}{1 - e^{\frac{-1}{T}}} = \frac{40}{30} \qquad \text{Put } x = e^{\frac{-1}{T}}$$

$$\therefore \frac{1 - x^2}{1 - x} = \frac{4}{3} \qquad \therefore \frac{(1-x)\,(1+x)}{(1-x)} = \frac{4}{3}$$

$$\Rightarrow \quad 1 + x = \frac{4}{3}$$

$$\therefore \; x = \frac{1}{3} = e^{-1/T} \quad \Rightarrow \quad T = 0.91 \text{ hrs.}$$

To find $\theta_f \Rightarrow$ $30 = \theta_f \left(1 - e^{\frac{-1}{T}}\right) \Rightarrow 30 = \theta_f \left(1 - \frac{1}{3}\right) \Rightarrow \theta_f = 45°C$

After 1 hr. $\quad 50 = \theta_f \left(1 - \frac{1}{e^T}\right)$

$$50 = \theta_f \left(1 - \frac{1}{3}\right) \qquad \therefore \; \theta_f = 75°C$$

Let the maximum overload capacity of 20 KW motor is *P* KW

$$\therefore \frac{\theta_f}{\theta'_f} = \frac{\text{Losses at PK W}}{\text{Original Losses}} = \left(\frac{P}{20}\right)^2$$

$$\therefore \quad \frac{75}{45}=\left(\frac{P}{20}\right)^2 \qquad \therefore \quad P = \mathbf{25.8\ kW}$$

Example 45.13. *In a transformer the temperature rise is 25°C after 1 hour and 37.5°C after 2 hours, starting from cold conditions. Calculate its final steady temperature rise and the heating time constant. If the transformer temperature falls from the final steady value to 40°C in 1.5 hours when disconnected, calculate its cooling time constant. Ambient temperature is 30°C.*

Solution. $\theta = \theta_f \left(1 - e^{\frac{-1}{T}}\right)$

$$25 = \theta_f \left(1 - e^{\frac{-1}{T}}\right) \qquad \text{and} \qquad 37.5 = \theta_f \left(1 - e^{\frac{-2}{T}}\right)$$

$$\therefore \quad \frac{37.5}{25} = \frac{1 - e^{\frac{-2}{T}}}{1 - e^{\frac{-1}{T}}}, \qquad \text{Put } x = e^{-\frac{1}{T}}$$

$$1.5 = \frac{1 - x^2}{1 - x}, \qquad 1.5 = \frac{(1-x)(1+x)}{(1-x)}$$

$$1.5 = 1 + x \qquad \therefore \quad x = 0.5$$

$$\therefore \quad e^{-\frac{1}{T}} = 0.5 \qquad \therefore \quad T = 1.44 \text{ hrs}$$

$$25 = \theta_f \left(1 - e^{\frac{-1}{T}}\right) \quad \Rightarrow \quad 25 = \theta_f \left(1 - e^{\frac{-1}{1.44}}\right) \quad \Rightarrow \quad \theta_f = \mathbf{50°C}$$

Cooling : Temperature rise after 1.5 hours above ambient temperature = 40 – 30 = 10°C.

$\because$ The transformer is disconnected $\theta = \theta_0 \, e^{\frac{-t}{T'}}$

$$10 = 50\, e^{\frac{-1.5}{T'}} \qquad \therefore \quad T' = \mathbf{0.932\ hrs}$$

Example 45.14. *The initial temperature of machine is 45°C. Calculate the temperature of machine after 1.2 hours, if its final steady temperature rise is 85°C and the heating time constant is 2.4 hours. Ambient temperature is 25°C*

Solution.

$$\theta = \theta_f - (\theta_f - \theta_1)\, e^{-t/T} \qquad \theta = 85 - (85 - 20)\; e^{-12/24}$$

θ = 45.54°C - Temperature rise above cooling medium

$\therefore$ Temperature of machine after 1.2 hours is = 45 . 54 + 25 = **70 .54°C**

Example 45.15. *The following rises were observed in a temperature rise test on a D.C. machine at full loads.*

After 1 hour — 15°C
After 2 hours — 25°C

Find out *(i)* *Final steady temperature rise and time constant.*
(ii) *The steady temperature rise after 1 hour at 50% overload, from cold. Assume that the final temperature rise on 50% overload is 90°C.*

[Nagpur University Summer 1998]

Solution. $\theta = \theta_f\,(1 - e^{\frac{-t}{T}})$, as motor is starting from cold.

$$15 = \theta_f\,(1 - e^{\frac{-1}{T}}) \quad \text{and} \quad 25 = \theta_f\,(1 - e^{-\frac{2}{T}})$$

$$\therefore \quad \frac{25}{15} = \frac{\theta_f\left(1 - e^{\frac{-2}{T}}\right)}{\theta_f\left(1 - e^{\frac{-1}{T}}\right)}$$

$$\therefore \quad e^{\frac{-1}{T}} = \frac{25}{15} - 1 \quad , \quad e^{\frac{-1}{T}} = \frac{2}{3}$$

$\therefore$ T = 2.466 hours, $\quad 15 = \theta_f\left(1 - e^{\frac{-1}{T}}\right)$

by putting value of T,

$$15 = \theta_f\left(1 - e^{\frac{-1}{2.466}}\right)$$

$\Rightarrow$ $\quad \theta_f$ = **45°C**

(ii) On 50% overload θ_f = 90°C

$\therefore$ Final temperature rise after 1 hour at 50% overload is $\theta = \theta_f\,(1 - e^{\frac{-t}{T}})$

$$\theta = 90\left(1 - e^{\frac{-1}{2.466}}\right) = \mathbf{30^\circ C}$$

45.10. Load Equalization

If the load fluctuates between wide limits in space of few seconds, then large peak demands of current will be taken from supply and produce heavy voltage drops in the system. Large size of conductor is also required for this.

Process of smoothing out these fluctuating loads is commonly referred to as load equalization and involves storage of energy during light load periods which can be given out during the peak load period, so that demand from supply is approximately constant. Tariff is also affected as it is based on M.D. (Maximum Demand)

For example, in steel rolling mill, when the billet is in between the rolls it is a peak load period and when it comes out it is a light load period, when the motor has to supply only the friction and internal losses, as shown in figure 45.10.

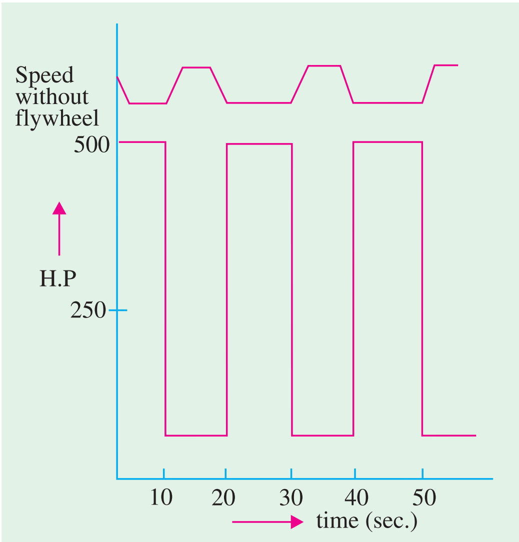

Fig. 45.10

45.11. Use Of Flywheels

The method of Load Eqalization most commonly employed is by means of a flywheel. During peak load period, the flywheel deccelarates and gives up its stored kinetic energy, thus reducing the load demanded from the supply. During light load periods, energy is taken from supply to accelerate flywheel, and replenish its stored energy ready for the next peak. Flywheel is mounted on the motor shaft near the motor. The motor must have drooping speed characteristics, that is, there should be a drop in speed as the load comes to enable flywheel to give up its stored energy. When the Ward - Leonard system is used with a flywheel, then it is called as Ward - Leonard Ilgner control.

45.12. Flywheel Calculations

The behaviour of flywheel may be determined as follows.

Fly wheel Decelerating :- (or Load increasing)

Let T_L $\rightarrow$ Load torque assumed constant during the time for which load is applied in kg-m

T_f $\rightarrow$ Torque supplied by flywheel in kg-m

T_o $\rightarrow$ Torque required on no load to overcome friction internal losses etc., in kg-m

T_m $\rightarrow$ Torque supplied by the motor at any instant, in kg-m

ω_o $\rightarrow$ No Load speed of motor in rad/sec.

ω $\rightarrow$ Speed of motor at any instant in rad/sec.

s $\rightarrow$ motor slip speed (ω_o - ω) in rad/sec.

I $\rightarrow$ Moment of inertia of flywheel in kg-m^2

g $\rightarrow$ Acceleration due to gravity in m/sec^2

t $\rightarrow$ time in sec.

When the flywheel deacelerates, it gives up its stored energy.

$\rightarrow$ $T_m = T_L - T_f$ or $T_L = T_m + T_f$ ..... **(1)**

Energy stored by flywheel when running at speed 'ω' is 1/2 $I\omega^2/g$.

If speed is reduced from ω_0 to ω.

The energy given up by flywheel is

$$=\frac{1}{2}\frac{I}{g}\left(\omega_0^2-\omega^2\right)$$

$$=\frac{1}{2}\frac{I}{g}\left(\omega_0+\omega\right)\left(\omega_0-\omega\right) \quad \text{.... (2)}$$

$\left(\frac{\omega_0+\omega}{2}\right)$ = mean speed. Assuming speed drop of not more than 10%., this may be assumed equal to ω.

$\therefore \left(\frac{\omega_0+\omega}{2}\right) \simeq \omega$ Also $(\omega_0-\omega)=s$

$\therefore$ From equation (2), Energy given up $=\frac{I}{g}\omega.s$

Power given up $=\frac{I}{g}\omega.\frac{ds}{dt}$

but Torque $= \frac{\text{Power}}{\omega}$

∴ Torque supplied by flywheel.

$$T_f = \frac{I}{g}\frac{ds}{dt}$$

∴ From equation (1) , $T_m = T_L - \frac{I}{g}\frac{ds}{dt}$

For values of slip speed upto 10% of No - load speed, slip is proportional to Torque

or $\quad s = K\,T_m$

$$\therefore \quad T_m = T_L - \frac{I}{g}K\frac{dT_m}{dt}$$

This equation is similar to the equation for heating of the motor W – Aλθ = G.S. $\frac{d\theta}{dt}$

i.e. $\quad (T_L - T_m) = \frac{I}{g}K\frac{dT_m}{dt} \qquad \Rightarrow \; g\frac{dt}{IK} = \frac{dT_m}{(T_L - T_m)}$

By integrating both sides.

$$-ln(T_L - T_m) = \frac{tg}{IK} + C_1 \qquad \text{.... (3)}$$

At t = 0, when load starts increasing from no load *i.e.* $T_m = T_o$

Hence, at $t = 0 \quad T_m = T_o$

∴ $C_1 = -\ln(T_L - T_o)$

By substituting the value of C_1 above, in equation (3) $-\text{in}\;(T_L - T_m) = \frac{tg}{IK} - \text{In}\;(T_L - T_0)$

$$\therefore \; ln\left(\frac{T_L - T_m}{T_L - T_0}\right) = -\frac{tg}{\text{IK}} \qquad \Rightarrow \left(\frac{T_L - T_m}{T_L - T_0}\right) = e^{\frac{-tg}{Ik}}$$

$$\Rightarrow \; (T_L - T_m) = (T_L - T_0)\,e\frac{-tg}{Ik} \qquad \therefore \; T_m = T_L - (T_L - T_0)\;e^{\frac{-tg}{Ik}}$$

If the Load torque falls to zero between each rolling period, then $T_m = T_L - \left(1 - e^{\frac{-tg}{IK}}\right) \; (\therefore \; T_0 = 0)$

45.13. Load Removed (Flywheel Accelerating)

Slip speed is decreasing and therefore $\frac{ds}{dt}$ is negative

$$T_m = T_0 + T_f = T_0 - \frac{I}{g}\frac{ds}{dt} \quad \Rightarrow \quad T_0 - T_m = \frac{I}{g}K\frac{dT_m}{dt}$$

$$\frac{g\,dt}{I\,\text{K}} = \frac{dT_m}{T_0 - T_m}$$

After integrating both sides,

$-\text{In}\;(T_0 - T_m) = \frac{tg}{IK} + C$ At t = 0, $T_m = T_m{}'$ motor torque at the instant, when load is removed

∴ $C = -\ln(T_o - T_m{}')$ Putting this value of C in the above equation

$$-\;\ln\;(T_0 - T_m) = \frac{tg}{IK} - \text{In}\;(T_0 - T_m{}')$$

$$\therefore \quad ln\left(\frac{T_0 - T_m}{T_0 - T_m{'}}\right) = \frac{-tg}{IK} \qquad\qquad \therefore \quad T_0 - T_m = (T_0 - T_m{'})\, e^{\frac{-tg}{IK}}$$

$$\therefore \quad T_m = T_0 + (T_m{'} - T_0)\, e^{\frac{-tg}{IK}}$$

Where $T_m{'}$ = the motor torque, at the instant the load is removed.

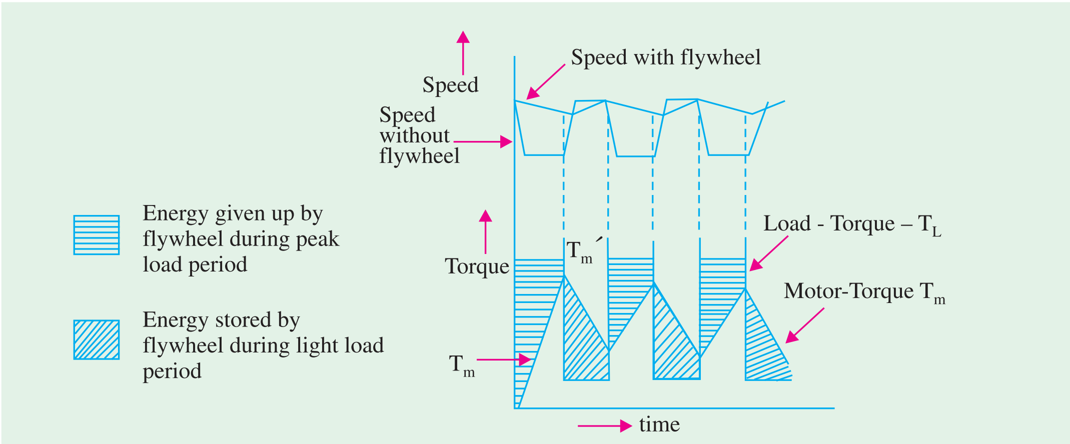

Fig. 45.11. Rolling mill drive with Flywheel

45.14. Choice of Flywheel

There are two choices left for selecting a flywheel to give up its maximum stored energy:

1. Large drop in speed and small flywheel (But with this the quality of production will suffer, since a speed drop of 10 to 15% for maximum load is usually employed).
2. Small drop in speed and large flywheel. (This is expensive and creates additional friction losses. Also design of shaft and bearing of motor is to be modified.) So compromise is made between the two and a proper flywheel is chosen.

The above figure shows the flywheel of a motor as a separate part

Example 45.16. *The following data refers to a 500 H.P. rolling mill, induction motor equipped with a flywheel.*

No load speed	$\rightarrow$	*40 r.p.m.*
Slip at full load (torque)	$\rightarrow$	*12%*
Load torque during actual rolling	$\rightarrow$	*41500 kg - m*
Duration of each rolling period	$\rightarrow$	*10 sec.*

Determine inertia of flywheel required in the above case to limit motor torque to twice its full load value. Neglect no - load losses and assume that the rolling mill torque falls to zero between each rolling period. Assume motor slip proportional to full load torque.

[Nagpur University Summer 1996]

Solution. $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 40}{60} = 4.189 \text{ rad/sec}$

given, $T_0 = 0$ $\qquad T_m = T_L - (T_L - T_0)\, e^{\frac{-tg}{IK}}$

$t = 10$ sec.

$T_L = 41500$ kg - m

$T_m = 2 \times T_{\text{Full Load}}$

Now $T_{Full\,Load} = \dfrac{500 \times 735.5}{0.88 \times 4.189}$ N-m. $\quad \because$ s=12%

$(1-s) = 0.88$

$= 99765$ N-m

$= 10169.7$ kg-m

$\therefore \quad T_m = 2 \times 10169.7$

$= 20339.5$ kg

$s = \omega_0 - \omega$

$= \dfrac{2\pi}{60}(N_0 - N)$

$= \dfrac{2\pi}{60}(40 - 0.88(40))$

$= \dfrac{2\pi}{60}(4.8) = 0.503$ rad/ sec.

$s = K\ T_{FL}$

$0.503 = K\,(10169.7) \quad \Rightarrow \quad K = 4.91 \times 10^{-5}$

$$T_m = T_L - (T_L - T_0)\ e^{\frac{-tg}{IK}}$$

$\because T_0 = 0 \qquad \therefore T_m = T_L\left(1 - e^{\frac{-tg}{IK}}\right)$

$$20339.5 = 41500\left(1 - \frac{-10 \times 981}{e^{1 \times 4.91 \times 10^{-5}}}\right)$$

$\therefore$ I = **2.9663 × 10^6 kg - m^2**

Example 45.17. *A 6 pole, 50 Hz Induction Motor has a flywheel of 1200 kg-m² as moment of inertia. Load torque is 100 kg - m. for 10 sec. No load period is long enough for the flywheel, to regain its full speed. Motor has a slip of 6% at a torque of 50 kg-m. Calculate*

(i) Maximum torque exerted by motor.

(ii) Speed at the end of deaceleration period. **[Nagpur University Winter 1996]**

Solution. *(i)* $T_m = T_L - (T_L - T_0)\, e^{\frac{-tg}{IK}}$

Assume $T_0 = 0, \quad \rightarrow \quad T_m = T_L\,(1 - e^{\frac{-tg}{IK}})$

$T_L = 100$ kg – m, $\quad t = 10$ sec, $\quad g = 9.81$ m/sec^2, $\quad I = 1200$ kg – m^2 $\quad s = \omega_0 - \omega$

$$s = KT \quad \therefore \quad \text{slip speed} = s = \frac{2\pi}{60}(N_0 - N)$$

$$N_S = \frac{120\,f}{p} = \frac{120\times 50}{6} = 1000 \text{ rpm} = N_0$$

N = 0.94 × 1000 = 940 rpm.

$$\therefore \quad S = \frac{2\pi}{60}(1000 - 940) = 2\pi \text{ rad/sec} \quad \text{or} \quad s = \frac{2\pi N}{60} = \frac{2\pi\times 1000\times 0.06}{60}$$

$$s = 2\pi = 6.283 \text{ rad/sec}$$

$$\therefore\ K = \frac{S}{T} = \frac{2\pi}{50} = 0.04\pi = 0.125$$

$$\therefore \quad T_m = T_L\,(1 - e^{\frac{-tg}{IK}}) = \left(1 - e^{\frac{-10\times 9.81}{1200\times 0.04\pi}}\right)$$

T_m = **47.83 kg - m**

(ii) Slip speed = 0.04 π × 47.8 rad / sec

s = KT_m

s = 0.125 (47.83)

s = 5.98 rad/sec

$$s = 5.98\frac{60}{2\pi}\text{ rpm} = 57.5 \text{ rpm} = \text{slip speed}$$

∴ Actual speed = 1000 – 57.5 = **942.5 rpm**

Example 45.18. *An Induction Motor equipped with a flywheel is driving a rolling mill which requires a Load Torque of 1900 N - m for 10 sec. followed by 250 N - m for 30 sec. This cycle being repeated indefinitely. The synchronous speed of motor is 750 r.p.m and it has a slip of 10% when delivering 1400 N-m Torque. The total Moment of Inertia of the flywheel and other rotating parts is 2100 kg-m². Draw the curves showing the torque exerted by the motor and the speed for five complete cycles, assuming that initial torque is zero.* **[Nagpur University Summer 1998]**

Solution.

T_L = 1900 N- m for 10 sec.

T_L = 250 N- m for 30 sec.

T_0 = 0 (assumed)

Slip = 10% at 1400 N – m torque

Slip = 750 × 0.1 = 75 r.p.m.

Ns = 750 r.p.m.

s = 10%

I = 2100 Kg- m², T_m = 1400 N – m

$$= \frac{75\times 2\pi}{60} = 7.85 \text{ rad/sec.}$$

$$\text{s} = \text{K T}_m\,;\ K = \frac{S}{T_m} = \frac{7.85}{1400} = 0.0056$$

(i) **During 1st cycle :**

(a) Flywheel de-acelerating :

$$T_m = T_L - (T_L - T_0)\,e^{\frac{-t}{IK}} \qquad \text{[When torque is taken in N - m.]}$$

→ After 10 sec

$$T_m = 1900 - (1900 - 0)\,e^{-0.085\times 10} \qquad \because \frac{1}{IK} = 0.085$$

T_m = 1088 N- m
Slip = 0.0056 × 1088 = 6.08 rad/sec
Slip = 58 r.p.m.
Speed = 750 – 58 = **692 r.p.m.**

(*b*) Flywheel acacelerating (Off Load Period)

Flywheel

$$T_m = T_0 + (T_m{}' - T_0)\,e^{-\frac{t}{IK}}$$

T_0 = No load torque = 280 N- m
$T_m{}'$ = 1088 N-m (T_m at the beginning of the period *i.e.* the motor torque at the instant when load is removed)
After 30 sec.,

T_m = 280 + (1088 – 280) $e^{-0.085\times 30}$

T_m = 343 N-m

∴ Slip at this T_m = 0.0056 × 343 = 1.92 rad/ sec = 18.34 r.p.m

∴ Speed = (750 – 18.34) r.p.m. = **731.6 r.p.m.**

(*ii*) During 2nd cycle :

(*a*) Flywheel deacelerating T_0 = 343 N-m.

T_m = 1900 – (1900 – 343) $e^{-0.085\times 30}$
= 1235 N-m.

∴ Slip at this T_m = 0.0056 × 1235 = 6.92 rad/ sec
= 66 r.p.m.

→ speed = 750 – 66 = **684 r.p.m.**

(*b*) Off Load Period:

$$T_m = 280 + (1235 - 280)\,e^{-0.085\times 30}$$

= 354.6 N-m.

∴ Slip at this T_m = 0.0056 × 354.6 = 1.99 rad/sec
= 19 r.p.m.

speed = 750 – 19 = 731.0 r.p.m.

(*iii*) During 3rd Cycle :

(*a*) On Load period : T_m can be found as above.

T_m = 1263 N – m
Speed = **683.6 r.p.m.**

(*b*) Off Load period

T_m = 354.6 N – m
speed = **731.0 r.p.m.**

Initial condition at the beginning of the 3rd peak load are thus practically the same as that at the beginning of 2nd. Therefore Motor Torque in this and all succeeding load cycles will follow a similar curve to that in second period.

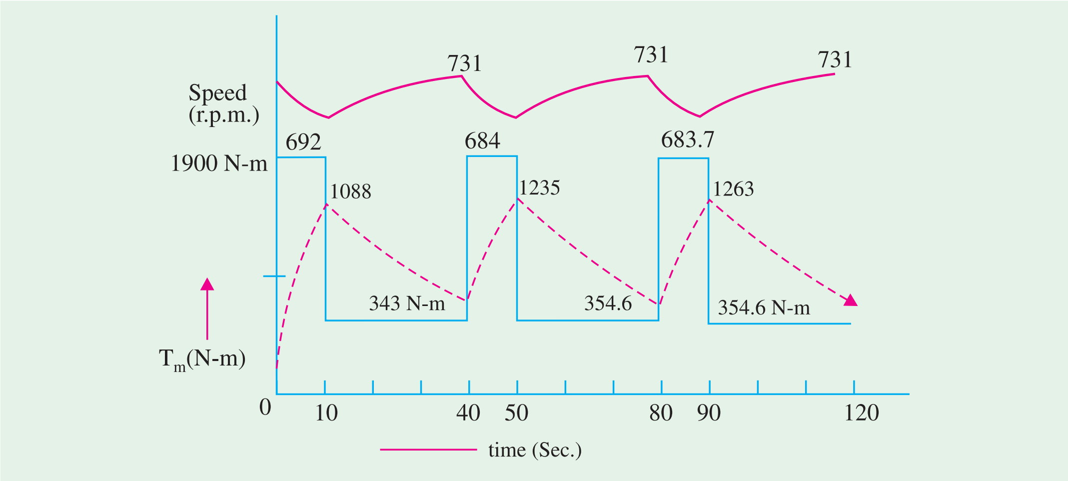

Fig. 45.12

Example 45.19. *A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no-load period, the flywheel regains its original speed. The motor torque is required to be limited to 85 kg-m. Determine moment of inertia of flywheel. The no - load speed of motor is 500 r.p.m. and it has a slip of 10% on full load.*

Solution.

$$T_m = T_L - (T_L - T_0)\, e^{\frac{-tg}{IK}}$$

$$T_m = T_L\left(1 - e^{\frac{-tg}{IK}}\right), \qquad \because\ T_0 = 0\ \text{kg} - \text{m}$$

T_m = 85 kg-m, T_L = 150 kg-m, T_0 = 0 kg-m, t = 15 sec., I = ?, g = 9.81 m/sec^2

s = K $T_{F.L.}$, where s = $\omega_0 - \omega$

$$\frac{2\pi(500)\times 0.1}{60} = K\times 85 \Rightarrow K = 0.0617$$

$$\therefore\ 85 = 150\left(1 - e^{\frac{-15\times 9.81}{I\times 0.627}}\right); \qquad \therefore\ I = \mathbf{2884\ kg\text{-}m^2}$$

Example 45.20. *A 3 – ϕ, 50 KW, 6 pole, 960 r.p.m. induction motor has a constant load torque of 300 N-m and at wide intervals additional torque of 1500 N-m for 10 sec. Calculate*

(a) *The moment of inertia of the flywheel used for load equalization, if the motor torque is not to exceed twice the rated torque.*

(b) *Time taken after removal of additional load, before the motor torque becomes 700 N-m.*

Solution.

(a) $$P = T\times\omega \qquad \therefore\ T = P/\omega$$

$$\therefore\ T_{F.L.} = \frac{50\times 10^3}{\frac{2\pi\times 960}{60}} = 497.36\ \text{N-m.}$$

$$\therefore\ T_m = 2\times T_{FL} = 2\times 497.36 = 994.72\ \text{N} - \text{m}$$

$$T_L = 1500 + 300 = 1800 \text{ N-m}$$

$$N_S = \frac{120\ f}{P} = 1000 \text{ r.p.m.}$$

$\therefore$ F.L. slip =1000 – 960 = 40 rpm = 40 r.p.m. = 4%

$\therefore$ $s = K\ T_{F.L.}$

$$\frac{2\pi(40)}{60} = K \times 497.36 \qquad \therefore \quad K = 8.42 \times 10^{-3}$$

$$T_m = T_L - (T_L - T_0)\, e^{\frac{-t}{1K}} \quad \text{as torque is in N - m}$$

$$994.72 = 1800 - (1800 - 300) \times e^{\frac{-10}{I \times 8.42 \times 10^{-3}}}$$

$\therefore$ I = **1909 kg– m²**

(*b*)
$$T_m = T_0 + (T_m{}' - T_0)\, e^{\frac{-t}{IK}}$$

$$700 = 300 + (994.72 - 300)\, e^{\frac{-t}{1909 \times 8.42 \times 10^{-3}}}$$

$\therefore$ t = **8.87 sec.**

Example 45.21. *A 3-phase, 8 pole, 50 c.p.s. Induction Motor equipped with a flywheel supplies a constant load torque of 100 N-m and at wide intervals an additional load torque of 300 N-m for 6 sec. The motor runs at 735 r.p.m., at 100 N-m torque. Find moment of inertia of the flywheel, if the motor torque is not to exceed 250 N-m.*

Solution.

$$T_0 = 100\ N - m. \qquad \therefore \quad T_L = 100 + 300 = 400 \text{ N - m}$$

$$N_S = \frac{120\ f}{P} = \frac{120\ f}{P} = 750 \text{ rpm.}$$

Slip at 100 N-m torque = 750 – 735 = 15 r.p.m.

$s = K\ T_m$

$$\frac{2\pi}{60}(15) = K(100) \quad \therefore \quad K = 0.0157 \quad ; \qquad T_m = T_L - (T_L - T_0)\, e^{\frac{-t}{IK}}$$

$$250 = 400 - (400 - 100)\, e^{\frac{-6}{I(0.0157)}}$$

$\therefore$ I = **552 kg - m²**

Example 45.22. *A 6 pole, 50 Hz, 3 - ϕ wound rotor Induction Motor has a flywheel coupled to its shaft. The total moment of inertia is 1000 kg-m². Load torque is 1000 N-m for 10 sec. followed by a no load period which is long enough for the motor to reach its no – load speed. Motor has a slip of 5% at a torque of 500 N-m. Find*

(a) Maximum torque developed by motor

(b) Speed at the end of deaceleration period. **[Nagpur University Winter 1996]**

Solution.

(*a*)
$$T_m = T_L\left(1 - e^{\frac{-t}{IK}}\right)$$

$$s = KT_m \qquad \text{But } N_S = \frac{120\, f}{P} = 1000\,\text{r.p.m.}$$

$$\frac{2\pi}{60}(1000 \times 0.05) = K\,(500)\;;\quad K = 6.2 \times 10^{-3}$$

$$T_m = 1000\left(1 - e^{\frac{-10}{1000 \times 6.2 \times 10^{-3}}}\right);\quad T_m = \mathbf{796.39\ N-m}$$

(*b*) $s = K\,T_{F.L.}$

$$\frac{2\pi}{60}(100 - N) = 6.2 \times 10^{-3} \times 790.39$$

$\therefore$ N = **952.2 r.p.m.**

Example 45.23. *A motor fitted with a flywheel supplies a load torque of 1000 N-m. for 2 sec. During no load period, the flywheel regains its original speed. The motor torque is to be limited to 500 N-m. Find moment of inertia of the flywheel. No load speed of the motor is 500 r.p.m. and its full load slip is 10%.*

Solution.

$s = K\,T_{F.L.}$

$$\frac{2\pi}{60}(500 \times 0.1) = K\,500\,, \qquad K = 0.0104\;; \qquad T_m = T_L\left(1 - e^{\frac{-t}{IK}}\right)$$

$$500 = 1000\left\{1 - e^{\frac{-2}{1(0.0104}}\right\};\quad I = \mathbf{277.44\ kg\text{-}\ m^2}$$

Tutorial Problem No. 45.1

1. A motor driving a colliery winding equipment has to deliver a load rising uniformly from zero to a maximum of 1500 KW in 20 sec. during the accelerating period, 750 KW for 40 sec. during the full speed period and during the deceleration period of 10 sec., when regenerative braking is taking place from an initial value of 250 KW to zero and then a no load period of 20 sec. Estimate remittable KW rating of the motor. **[648 KW]**

2. A constant speed drive has the following duty cycle:

Load rising from 0 to 400 KW	–	5 minutes
Uniform load of 400 KW	–	5 minutes
Regenerative power of 400 KW returned to supply	–	4 minutes
Remains idle for	–	2 minutes

Estimate power rating of motor. **[380 H. P.]**

[*Nagpur University Winter 96*]

3. Determine the rated current of a transformer for the following duty cycle:
- 500 A for 3 minutes
- Sharp increase to 1000 A and constant at this value for 1 minute
- Gradually decreasing to 200 A. for 2 minutes
- Constant at this value for 2 minutes
- Gradually increasing to 500 A during 2 minutes repeated indefinitely. **[540 A]**

4. An induction motor has to perform the following duty cycle:

75 KW for 10 miniutes, No load for 5 minutes
45 KW for 8 minutes, No load for 4 minutes

which is repeated indefinitely.
Determine suitable capacity of a continuously rated motor. **[70 H. P.]**

5. A 25 H.P. motor has heating time constant of 90 min. and when run continuously on full load attains a temperature of 45°C. above the surrounding air. What would be the half hour rating of the motor for this temperature rise, assuming that it cools down completely between each load period and that the losses are proportional to square of the load. **[47 H.P.]**

6. At full load of 10 H.P., temperature rise of a motor is 25° C. after 1 hr. and 40°C after 2 hrs. Find (*a*) Heating time constant of motor, (*b*) Final temperature rise on full load.
[T = 1. 96 hrs, θ_f = 62.5°C]

7. A totally enclosed motor has a temperature rise of 20°C after half an hour and 35°C after one hour on full load. Determine temperature rise after 2 hours on full load. **[54.68°C]**

8. A 25 H.P., 3-ϕ, 10 pole, 50 c.p.s. induction motor provided with a flywheel has to supply a load torque of 800 N-m for 10 sec, followed by a no load period, during which the flywheel regains its full speed. Full load slip of motor is 4% and torque-speed curve may be assumed linear over the working range. Find moment of inertia of flywheel, if the motor torque is not to exceed twice the full load torque. Assume efficiency = 90%. **[718 kg-m^2]**

9. A motor fitted with a flywheel has to supply a load torque of 200 kg-m for 10 sec, followed by a no load period. During the no load period, the motor regains its speed. It is desired to limit the motor torque to 100 kg-m. What should be the moment of inertia of flywheel. No load speed of motor is 500 r.p.m. and has a slip of 10% at a torque of 100 kg-m. . **[I = 2703 kg-m^2]**

10. A 50 Hz., 3-ϕ, 10 pole, 25H.P., induction motor has a constant load torque of 20 kg-m and at wide intervals additional torque of 100 kg-m for 10 sec. Full load slip of the motor is 4% and its efficiency is 88%. Find -
(*a*) Moment of inertial of flywheel , if motor torque not to exceed twice full load torque.
(*b*) Time taken after removal of additional load, before motor torque is 45 kg- m.
[I = 1926 kg-m^2, t = 9.99 sec.]

11. Define the following terms regarding the ratings of motor :-
(*i*) Continuous rating (*ii*) short time rating (*iii*) Intermittent rating.
(Nagpur University, Summer 2004)

12. With the help of heating and cooling curves define and explain the terms :
(*i*) Heating time constant (*ii*) Cooling time constant. ***(Nagpur University, Summer 2004)***

13. What do you mean by 'load-equilisation' it is possible to apply this scheme for reversible drive? Why? ***(Nagpur University, Summer 2004)***

14. A motor is equipped with the flywheel has to supply a load torque of 600 N-m for 10 seconds, followed by no load period long enought for flywheel to regain its full speed. It is desired to limit the motor torque of 450 N-m. What should be moment of inertia of the flywheel? the no load speed of the motor is 600 rpm and has 8% slip at a torque of 450 N-m. The speed-torque characteristics of the motor can be assumed to be a straight line in the region of interest.
(Nagpur University, Summer 2004)

15. A motor has the following load cycle :
Accelerating period 0-15 sec Load rising uniformly from 0 to 1000 h.p.
Full speed period 15-85 sec Load constant at 600 h.p.
Decelerating period 85-100 sec h.p. returned to line falls uniformly 200 to zero
Decking period 100-120 sec Motor stationary. Estimate the size the motor.
(J.N. University, Hyderabad, November 2003)

16. A motor driving a load has to deliver a load rising uniformly from zero to maximum of 2000 h.p. in 20 sec during the acceleration period, 1000 h.p. for 40 sec during the full speed period and during the deceleration period of 10 sec when regenerating braking taking place the h.p. returned to the supply falls from 330 to zero. The interval for decking before the next load cycle starts is 20 sec. Estimate the h.p. Rating of the motor. ***(J.N. University, Hyderabad, November 2003)***

17. Draw and explain the output vs. time characteristics of any three types of loads.
(J.N. University, Hyderabad, November 2003)

18. Discuss series and parallel operation of series and shunt motors with unequal wheeldiameters. Comment on the load sharing in each case. *(J.N. University, Hyderabad, November 2003)*

19. Discuss the various factors that govern the size and the rating of a motor for a particular service. *(J.N. University, Hyderabad, April 2003)*

20. A motor has to deliver a load rising uniformly from zero to a maximum of 1500 Kw in 20 sec during the acceleration period, 1,000 Kw for 50 sec during the full load period and during the deceleration period of 10 sec when regenerative braking takes place the Kw returned to the supply falls from an initial value of 500 to zero uniformly. The interval for decking before the next load cycle starts is 20 sec. Estimate the rating of the motor. *(J.N. University, Hyderabad, April 2003)*

21. Derive an expression for the temperature rise of an equipment in terms of the heating time constant. *(J.N. University, Hyderabad, April 2003)*

22. At full load of 10 h.p., the temperature rise of a motor is 25 degree C after one hour, and 40 degree C after 2 hours. Find the final temperature rise on full load. Assume that the iron losses are 80% of full load copper losses. *(J.N. University, Hyderabad, April 2003)*

23. Explain what you mean by Lord Equalization and how it is accomplished. *(J.N. University, Hyderabad, April 2003)*

24. A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no load period the flywheel regains its originalspeed. The motor torque is required to be limited to 85 kg-m. Determine the moments of inertia of the flywheel. The no load speed of the motor is 500 r.p.m. and it has a slip of 10% on full load. *(J.N. University, Hyderabad, April 2003)*

25. Discuss the various losses that occur in magnetic conductors which cause the temperature rise in any electrical apparatus and suggest how they can be reduced. *(J.N. University, Hyderabad, April 2003)*

26. The outside of a 12 h.p. (metric) motor is equivalent to a cylinder of 65 cms diameter and 1 meter length. The motor weighs 400 Kg and has a specific heat of 700 Joules per kg per degree C. The outer surface is capable of heat dissipation of 12 W per meter square per degree C. Find the final temperature rise and thermal constant of the motor when operating at full load with an effciency of 90%. *(J.N. University, Hyderabad, April 2003)*

27. "A flywheel is not used with a synchronous motor for load equalization". Discuss. *(J.N. University, Hyderabad, April 2003)*

28. A 25 h.p. 3-phase 10 pole, 50 Hz induction motor fitted with flywheel has to supply a load torque of 750 Nw-m for 12 sec followed by a no load period during which the flywheel regains its original speed. Full load slip of the motor is 4% and the torque-speed curve is linear. Find the moment of inertia of the flywheel if the motor torque is not to exceed 2 times the full load torque. *(J.N. University, Hyderabad, April 2003)*

29. Explain what do you mean by Load Equalization and how it is accomplished. *(J.N. University, Hyderabad, April 2003)*

30. A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no load period the flywheel regains its original speed. The motor torque is required to be limited to 85 kg-m. Determine the moments of inertia of the flywheel. The no load speed of the motor is 500 r.p.m. and it has aslip of 10% on full load. *(J.N. University, Hyderabad, April 2003)*

31. A 100 hp motor has atemperature rise of 50^{o}C when running continuously on full load. It has a time constant of 90 minutes. Determine 1/2 hr rating of the motor for same temperature rise. Assume that the losses are proportional to the square of the load and motor cools completely between each load period. *(J.N. University, Hyderabad, December 2002/January 2003)*

32. Explain 'load equalisation'. How this can be achieved in industrial drives. *(J.N. University, Hyderabad, December 2002/January 2003)*

33. Obtain the expression for temperature rise of a electrical machine. State the assumptions made if any. *(J.N. University, Hyderabad, December 2002/January 2003)*

34. A 75 kW, 500 rpm dc shunt motor is used to drive machinery for which the stored energy per kW is 5400 Joules. Estimate the time taken to start the motor, if the load torque is equal to full load torque during the starting period and the current is limited to 1 1/2 times the full load current. *(J.N. University, Hyderabad, December 2002/January 2003)*

OBJECTIVE TESTS – 45

1. Heat dissipation is assumed proportional to
   (*a*) Temperature difference
   (*b*) Temperature difference between motor and cooling medium
   (*c*) Temperature of cooling medium
2. Temperature of cooling medium is assumed
   (*a*) constant (*b*) variable
3. When the motor reaches final temperature rise its temperature remains
   (*a*) constant
   (*b*) falls
   (*c*) rises.
4. For intermittent load, a motor of smaller rating can be used
   (*a*) true (*b*) false
5. If motor is disconnected from supply, final temperature reached will be the ambient temperature
   (*a*) true (*b*) false
6. Final temperature rise is theoretically attained only after
   (*a*) fixed time
   (*b*) variable time
   (*c*) infinite time
7. Motor is derated when taken at altitude
   (*a*) Yes (*b*) No
8. The rolling mill load
   (*a*) is constant
   (*b*) fluctuates widely within long intervals of time
   (*c*) fluctuates widely within short intervals of time
   (*d*) varies
9. Size of motor is decided by
   (*a*) load
   (*b*) current
   (*c*) heat produced in motor
   (*d*) torque
10. Tariff is affected by sudden load drawn by motor
    (*a*) true (*b*) false
11. Flywheel helps in smoothing only
    (*a*) speed fluctuations
    (*b*) current fluctuations
    (*c*) both of the above
12. To use flywheel, motor should have
    (*a*) constant speed characteristics
    (*b*) drooping speed characteristics
    (*c*) variable speed characteristics
13. During light load period
    (*a*) flywheel absorbs energy
    (*b*) flywheel gives up energy
    (*c*) flywheel does nothing
14. During peak load periods
    (*a*) flywheel absorbs energy
    (*b*) flywheel gives up energy
    (*c*) flywheel does nothing
15. Large size of flywheel
    (*a*) can be used practically
    (*b*) can't be used practically

ANSWERS

1. (*b*) **2.** (*a*) **3.** (*a*) **4.** (*a*) **5.** (*a*) **6.** (*c*) **7.** (*a*) **8.** (*c*) **9.** (*c*) **10.** (*a*)
11. (*c*) **12.** (*b*) **13.** (*a*) **14.** (*b*) **15.** (*b*)

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CONTENTS

CHAPTER 46

Learning Objectives

- Classes of Electronic AC Drives
- Variable Frequency Speed Control of a SCIM
- Variable Voltage Speed Control of a SCIM
- Chopper Speed Control of a WRIM
- Electronic Speed Control of Synchronous Motors
- Speed Control by Current-fed D.C. Link
- Synchronous Motor and Cycloconverter

ELECTRONIC CONTROL OF A.C. MOTORS

Efficient control of motors becomes critical where high precision, accuracy, flexibility, reliability and faster response are of paramount importance. Electronic and digital controls are employed in such conditions

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46.1. Classes of Electronic A.C. Drives

AC motors, particularly, the squirrel-cage and wound-rotor induction motors as well as synchronous motors lend themselves well to electronic control of their speed and torque. Such a control is usually exercised by varying voltage and frequency. Majority of the electronic a.c. drives can be grouped under the following broad classes :

1. **static frequency changers** like cyclo-converters which convert incoming high line frequency directly into the desired low load frequency. Cyclo-converters are used both for synchronous and squirrel-cage induction motors.
2. **variable-voltage controllers** which control the speed and torque by varying the a.c. voltage with the help of SCRs and gate turn-off thyristors (GTOs).
3. **rectifier-inverter systems** with natural commutation.
4. **rectifier-inverter systems** with self-commutation.

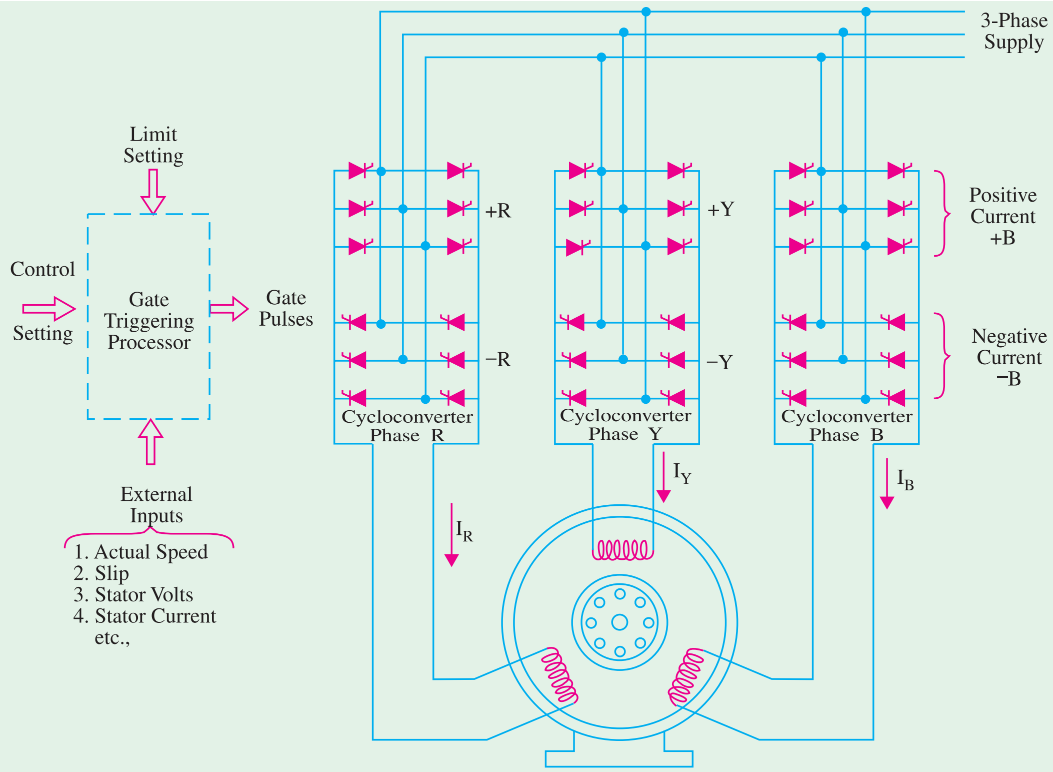


Fig. 46.1

46.2. Variable-frequency Speed Control of a SCIM

Fig. 46.1 shows a 3-phase SCIM connected to the outputs of three 3-phase cycloconverters. As seen, each cyclo-converter consists of two 3-phase thyristor bridges, each fed by the same 3-phase, 50-Hz line. The +*R* bridge generates the positive half-cycle for *R*-phase whereas –*R* generates the negative half. The frequency of the cycloconverter output can be reduced to any value (even upto zero) by controlling the application of firing pulses to the thyristor gates. This low frequency permits excellent speed control. For example, the speed of a 4-pole induction motor can be varied from zero to 1200 rpm on a 50-Hz line by varying the output frequency of the cycloconverter from zero to 40 Hz. The stator voltage is adjusted in proportion to the frequency in order to maintain a constant flux in the motor.

This arrangement provides excellent torque/speed characteristics in all 4-quadrants including regenerative braking. However, such cycloconverter-fed motors run about 10°C hotter than normal and hence require adequate cooling. A small part of the reactive power required by SCIM is provided by the cycloconverter, the rest being supplied by the 3-phase line. Consequently, power factor is poor which makes cycloconverter drives feasible only on small and medium power induction motors.

46.3. Variable Voltage Speed Control of a SCIM

In this method, the speed of a SCIM is varied by varying the stator voltage with the help of three sets of SCRs connected back-to back (Fig. 46.2). The stator voltage is reduced by delaying the firing (or triggering) of the thyristors. If we delay the firing pulses by 100°, the voltage obtained is about 50% of the rated voltage which decreases the motor speed considerably.

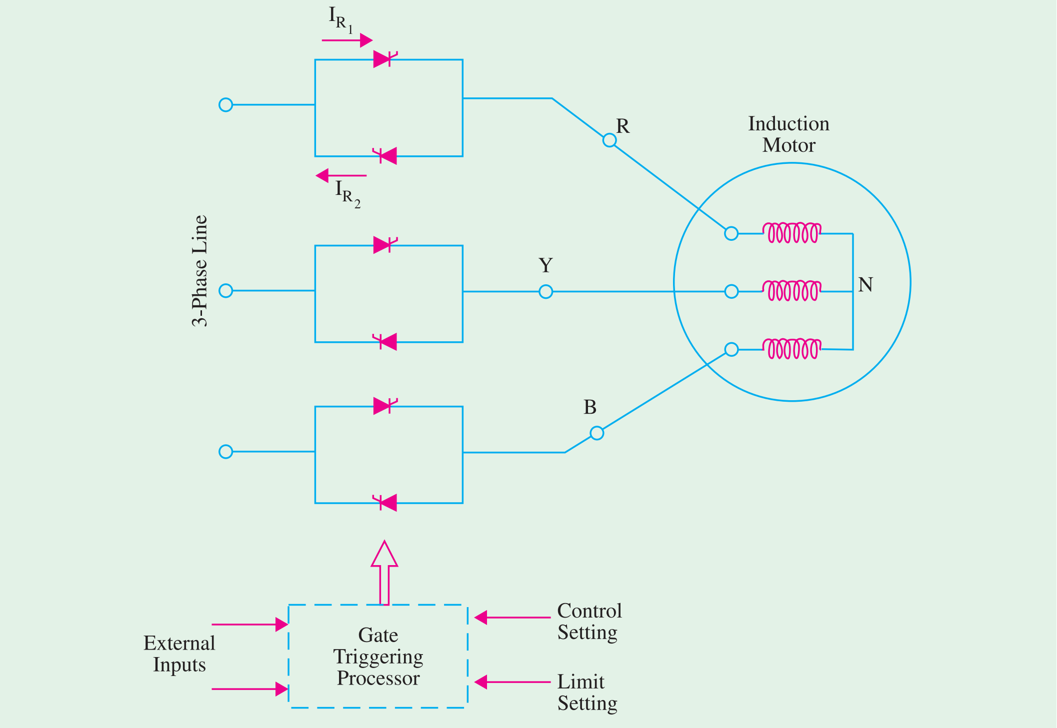
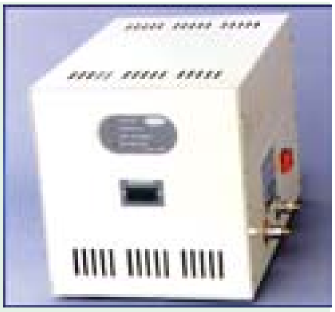


Fig. 46.2

Unfortunately, I^2R losses are considerable due to distortion in voltage. Moreover, p.f. is also low due to large lag between the current and voltage. Hence, this electronic speed control method is feasible for motors rated below 15 kW but is quite suitable for small hoists which get enough time to cool off because of intermittent working. Of course, p.f. can be improved by using special thyristors called gate turn-off thyristors (GTOs) which force the current to flow almost in phase with the voltage (or even lead it).



A commonly used electronic power inverter

46.4. Speed Control of a SCIM with Rectifier-Inverter System

A rectifier-inverter system with a d.c. link is used to control the speed of a SCIM. The inverter used is a self-commuted type (different from a naturally commutated type) which converts d.c. power into a.c. power at a frequency determined by the frequency of the pulses applied to the thyristor gates. The rectifier is connected to the 3-phase supply line whereas the inverter is connected to the stator of the SCIM.

Two types of links are used :

1. constant-current d.c. link —for speed control of *individual* motors.

2. constant-voltage d.c. link —for speed control of several motors.

As shown in Fig. 46.3, the constant-current link supplies constant current to the inverter which then feeds it sequentially (through proper switching sequence) to the three phases of the motor. Similarly, the constant-voltage dc link (Fig. 46.4) provides a constant voltage to the inverter which is switched from one phase of the motor to the next in a proper sequence.

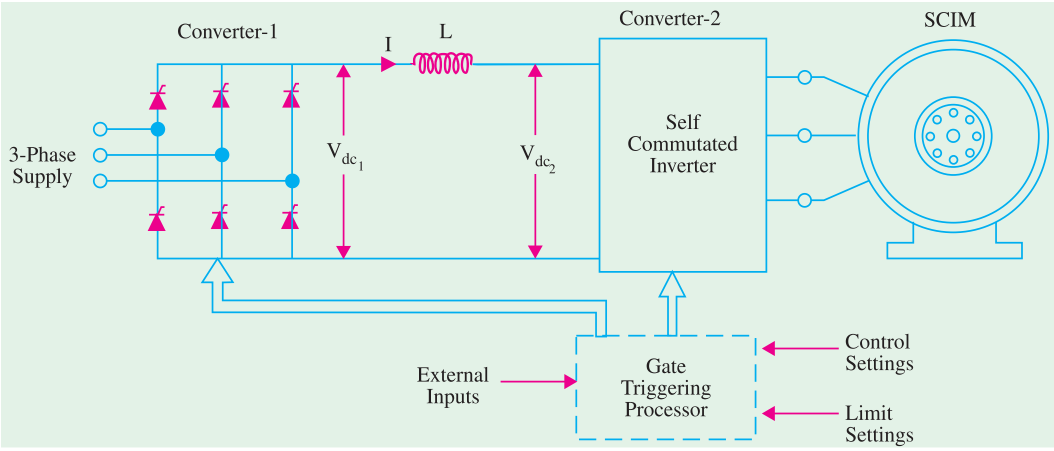


Fig. 46.3

The arrangement of Fig. 46.3 gives speed control with high efficiency in all 4 quadrants in addition to the facility of regenerative braking. Heavy inertia loads can be quickly accelerated because motor develops full break-down torque right from the start. The output frequency of the inverter varies over a range of 20 : 1 with a top frequency of about 1 kHz. The a.c. voltage supplied by the inverter is changed in proportion to the frequency so as to maintain the stator flux constant.

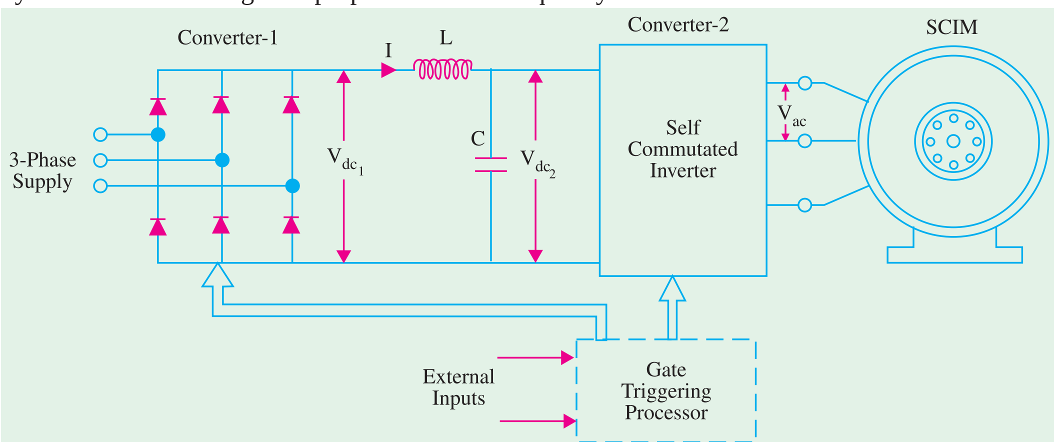


Fig. 46.4

Consequently, d.c. link voltage V_{dc1} has to be reduced as the motor speeds up. This is accomplished by increasing the firing angle of the thyristors in converter 1. Unfortunately, this leads to increase in the reactive power drawn from the 3-phase line which results in poor power factor. To improve the p.f., use of capacitors is necessary. The direction of rotation can be changed by altering the phase sequence of the pulses that trigger the gates of converter 2.

The voltage-fed frequency converter of Fig. 46.4 consists of a rectifier and a self-commutated inverter connected by a d.c. link and is often used for group drives in textile mills. The 3-phase bridge rectifier produces d.c. voltage V_{dc1} which is smoothened up by the LC filter before being applied to the inverter. The inverter successively switches its output ac voltage V_{ac} to the three phases of the motor. This voltage is varied in proportion to the frequency so as to maintain constant flux in the motor. Since, V_{ac} depends on V_{dc2} which itself depends on V_{dc1}, it is V_{dc1} which is changed as frequency varies. In this system, motor speed can be controlled from zero to maximum while developing full breakdown torque.

A simple electronic control system

46.5. Chopper Speed Control of a WRIM

As discussed in Art. 30.18 (*d*), the speed of a WRIM can be controlled by inserting three variable resistors in the rotor circuit. The all-electronic control of speed can be achieved by connecting a 3-phase bridge rectifier across the rotor terminals and then feed the rotor output to a single fixed resistor or R_0 via a chopper (Fig. 46.5).

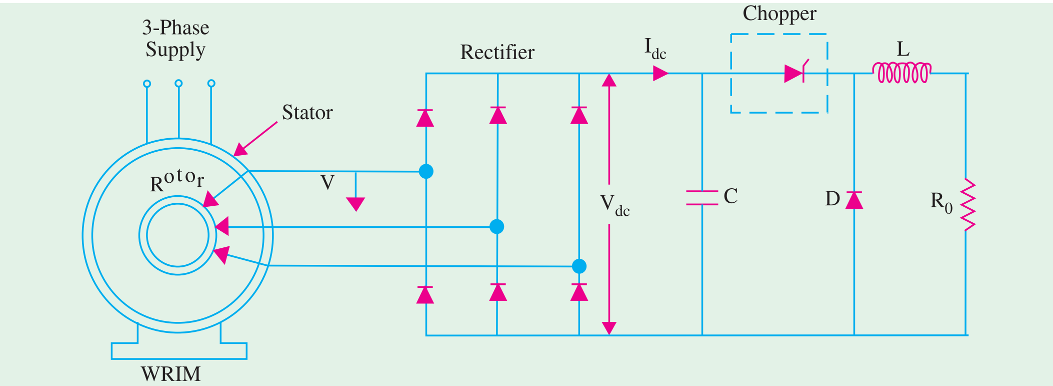


Fig. 46.5

The capacitor C supplies the high current pulses drawn by the chopper. The inductor L and free-wheeling diode D serve the same purpose as discused in Art. 28.14. By varying the chopper on-time TON, the apparent resistance R_a across the bridge rectifier can be made either high or low. The value of apparent resistance is given by $R_a = R_0/f^2\, T_{ON}^2$ where f is the OFF/ON switching frequency of the chopper. The resulting torque/speed characteristic is similar to the one obtained with a 3-phase rheostat.

Example 46.1. *The wound-rotor induction motor of Fig.43.5 is rated at 30-kW, 975 rpm, 440-V, 50 Hz. The open-circuit line voltage is 400 V and the load resistance is 0.5 Ω. If chopper frequency is 200 Hz, calculate* T_{ON} *so that the motor develops a gross torque of 200 N-m at 750 rpm. Also, calculate the magnitude of the current pulses drawn from the capacitor.*

Solution. Obviously, N_S = 1000 rpm. Hence, slip at 750 rpm is = (1000 – 750)/1000 = 0.25. The rotor line voltage at 750 rpm is = $sE_2 = 0.25 \times 400 = 100$ V.

The d.c. voltage of 3-phase bridge rectifier is $V_{dc} = 1.35\ V = 1.35 \times 100 = 135$ V.

Now, $T_g = P^2/2\pi N_s; P^2 = T_g \times 2\pi N_s = 200 \times 2\pi \times (1000/60) = 20{,}950$ W

Part of P^2 dissipated as heat $= sP^2 = 0.25 \times 20{,}950 = 5{,}238$ W

The power is actually dissipated in R_0 and is, obviously, equal to the rectifier output $V_{dc}.I_{dc}$.

$$\therefore \qquad V_{dc}\,.\,I_{dc} = 5238 \quad \text{or} \quad I_{dc} = 5238/135 = 38.8 \text{ A}$$

The apparent resistance at the input to the chopper is

$$R_a = V_{dc}/I_{dc} = 135/38.8 = 3.5\ \Omega$$

Now, $$R_a = R_0/f^2 T_{ON}^2 \text{ or } T_{ON} = \sqrt{R_0/f^2R_a} = \sqrt{0.5/200^2 \times 3.5} = \mathbf{1.9\ ms}$$

Current in R_0 can be found from the relation

$$I_0^2 R_0 = 5238 \quad \text{or} \quad I_0 = \sqrt{5238/0.5} = \mathbf{102\ A}$$

As seen, capacitor delivers current pulses of magnitude I_0^2 A and a pulse width of 1.9 ms at the rate of 200 pulses/second. However, the rectifier continuously charges C with a current of 38.8 A.

46.6. Electronic Speed Control of Synchronous Motors

The speed of such motors may be controlled efficiently by using *(i)* current-fet delink and *(ii)* cycloconverter as discussed below :

46.7. Speed Control by Current-fed DC Link

As shown in Fig. 46.6, the typical circuit consists of three converters two of which are connected between the three-phase source and the synchronous motor whereas the third converter (bridge rectifier) supplies dc field excitation for the rotor. Converter-1 (*C*-1) acts as a controlled rectifier and feeds d.c. power to converter-2 (*C*-2). The converter-2 behaves as a naturally commutated inverter whose a.c. voltage and frequency are established by the motor. The function of the smoothing inductor L is to maintain a ripple-free current in the d.c. link between the two converters. Converter-1 acts as a current source and controls I.

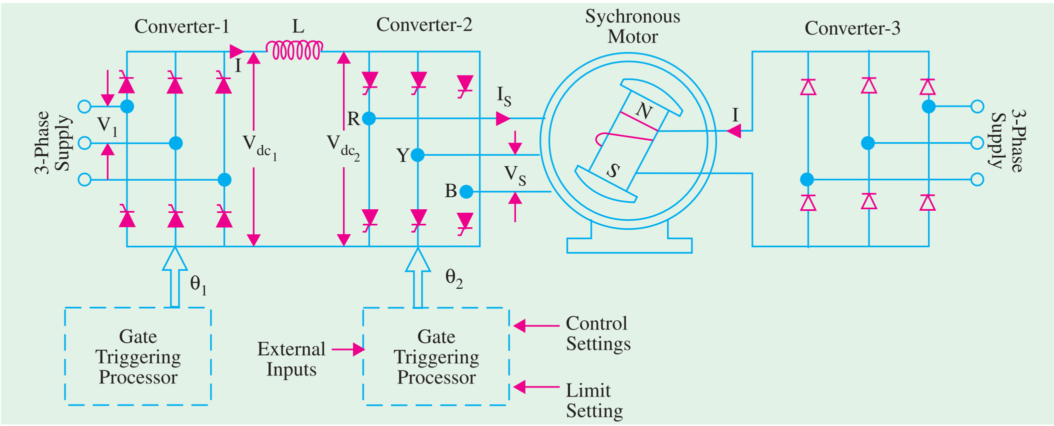


Fig. 46.6

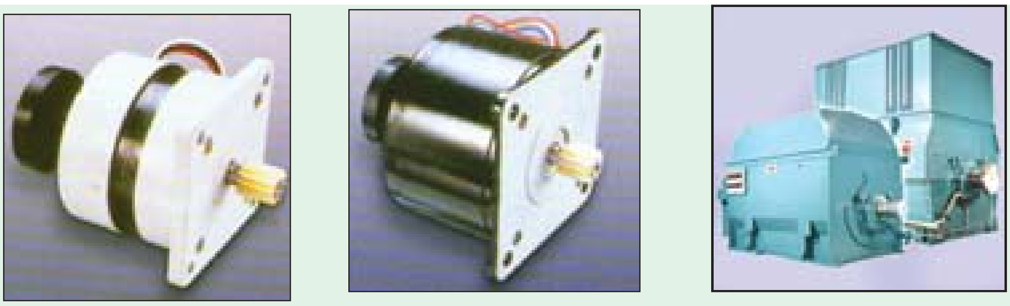
The converter-2 is naturally commutated by voltage V_s induced across motor terminals by its revolving magnetic flux. The revolving flux which depends on the stator currents and the d.c. field exciting current I_{f1} is usually kept constant. Consequently, V_s is proportional to motor speed.

As regards various controls, information picked off from various points is processed in the gate-triggering processors which then send out appropriate gate firing pulses to converters 1 and 2. The processors receive information about the desired rotor speed, its actual speed, instantaneous rotor position, field current, stator voltage and current etc. The processors check whether these inputs represent normal or abnormal conditions and send appropriate gate firing pulses either to correct the situation or meet a specific demand.

Gate triggering of C-1 is done at line frequency (50 Hz) whereas that of C-2 is done at motor frequency. In fact, gate pulses of C-2 are controlled by rotor position which is sensed by position transducers mounted at the end of the shaft. The motor speed can be increased by increasing either d.c. link current I or exciting current I_f.

Now, $$V_{dc2} = 1.35\ V_s \cos\alpha_1 \quad \text{and} \quad V_{dc1} = 1.35\ V_s \cos\alpha_1$$

where V_{dc2} = d.c. voltage generated by *C*-2, V_{dc1} = d.c. voltage supplied by *C*-1



Special features of A C Synchronous motors: 1. Bi-directional, 2. Instantaneous Start, Stop and Reverse, 3. Identical Starting and Running Currents, 4. Residual Torque always present, 5. No damage due to stalling, 6. Low speed of 60 rpm. Applications of AC Synchronous Motors are found in: 1. Actuators, 2. Remote control of switches 3. Winding machines, 4. Machine tool applications, 6. Valve controls, 6. Printing machines, 7. Automatic welding machines, 8. Medical equipment, 9. Conveyor systems, 10. Paper feeders

α_2 = firing angle of *C*-2 ; α_1 = firing angle of *C*-1

The firing angle α_1 is automatically controlled and supplies *I* which is sufficient to develop the required torque. This method of speed control is applied to motors ranging from 1 kW to several MW. Permanent-magnet synchronous motors used in textile industry and brushless synchronous motors for nuclear reactor pumps are controlled by this method.

46.8. Synchronous Motor and Cycloconverter

As shown in Fig. 46.7, the arrangement consists of three cycloconverters connected to the three

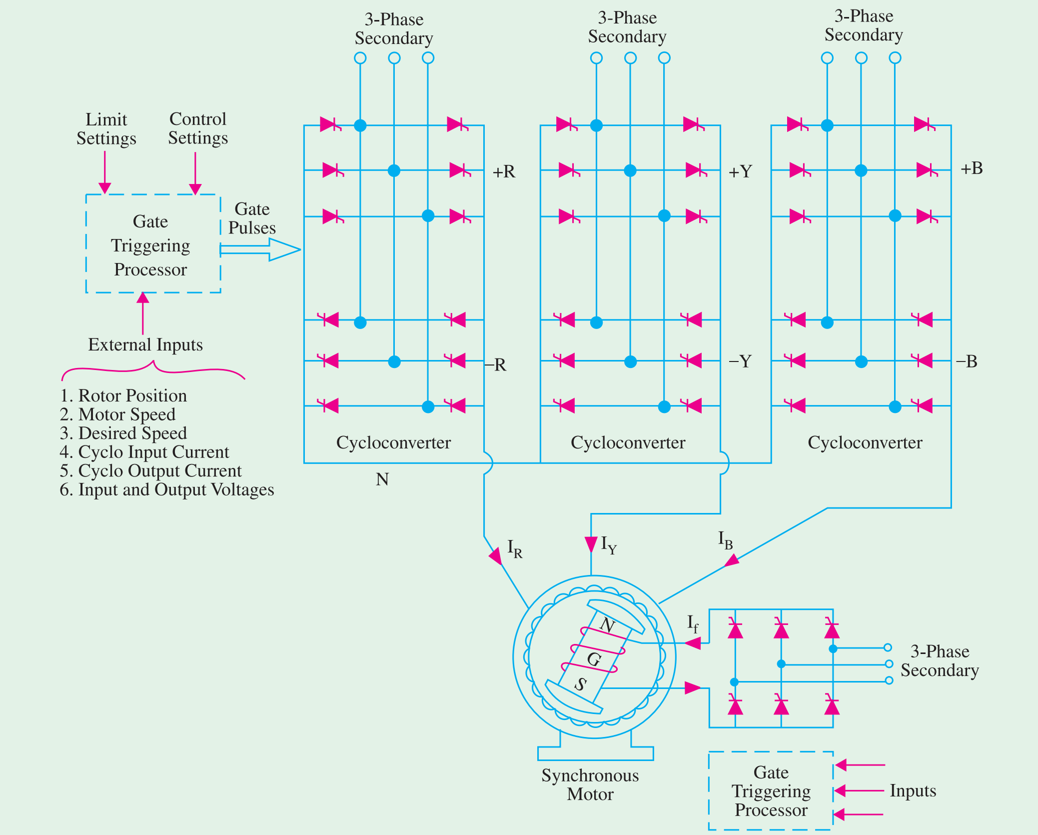


Fig. 46.7

one phases of the synchronous motor and one controlled rectifier for supplying field exciting current I_f to the rotor. Each cycloconverter is composed of two three-phase bridges and supplies a single-phase output. As is well known, a cycloconverter can directly convert a.c. power at higher frequency to one at a lower frequency. With a line frequency of 50 Hz, the cycloconverter output frequency can be varied from zero to 10 Hz.

The cycloconverters and the controlled rectifier function as current sources. The air-gap flux is kept constant by controlling the magnitude of the stator currents and exciting current I_f. By proper timing of gate pulses, motor can be made to operate at unity power factor.

The speed of cycloconverter-driven large slow-speed synchronous motors can be continuously varied from zero to 15 rpm. Such low speeds permit direct drive of the ball mill without using a gear reducer. Such high-power low-speed systems are also being introduced as propeller drives on board the ships.

46.9. Digital Control of Electric Motors

Advantages of Digital Control

1. High precision and accuracy
2. Better speed regulation
3. Faster response
4. Flexibility
5. Better time response
6. Improved performance
7. Economical
8. Easy software control
9. Reliability
10. The greatest advantage of the digital control is that by changing the program, desired control technique can be implemented without any change in the hardware.

The speed information can be fed into microcomputer using a D.C. Tacho (Speed encoder) and A/D converter (Speed *I/P* module). The motor current data is usually fed into the computer through a fast A/D converter. A synchronizing circuit interface (Line synchronizing circuit) is required so that the micro-computer can synchronize the generation of the firing pulse data with the supply line frequency. The gate pulse generator is shown as receiving a firing signal from microcomputer.

A set of instruction (Program) is stored in memory and those are executed by computer for proper functioning of a drive. A typical program flow - chart for this drive system is shown in figure (46.9). The sequence of instructions allows the computer to process data for speed regulation, current regulation and reversal operation.

46.10. Application of Digital Control

The above operations can be clearly understood by considering one of the applications of Digital Control system, such as Digital Control System for Speed Control of D.C. drives using a Micro-computer :

Various components and their operations shown in Fig. 46.8 are discussed below :

(i) Thyristor Converter

PC based control systems can be built where a phase-controlled rectifier supplies a D.C. motor. The main control to be handled is to turn on & off SCRs. Thyristor power converter in this case is a dual converter – one for forward and other for reverse direction.

(ii) Gate Pulse Generator and Amplifier

PC is used for firing angle control of dual converter. It can be programmed using suitable software to perform the function of firing range selection, firing pulse generation, etc. The firing pulses so obtained are amplified, if needed to turn *ON* the SCR reliably. Changeover signal decides whether to

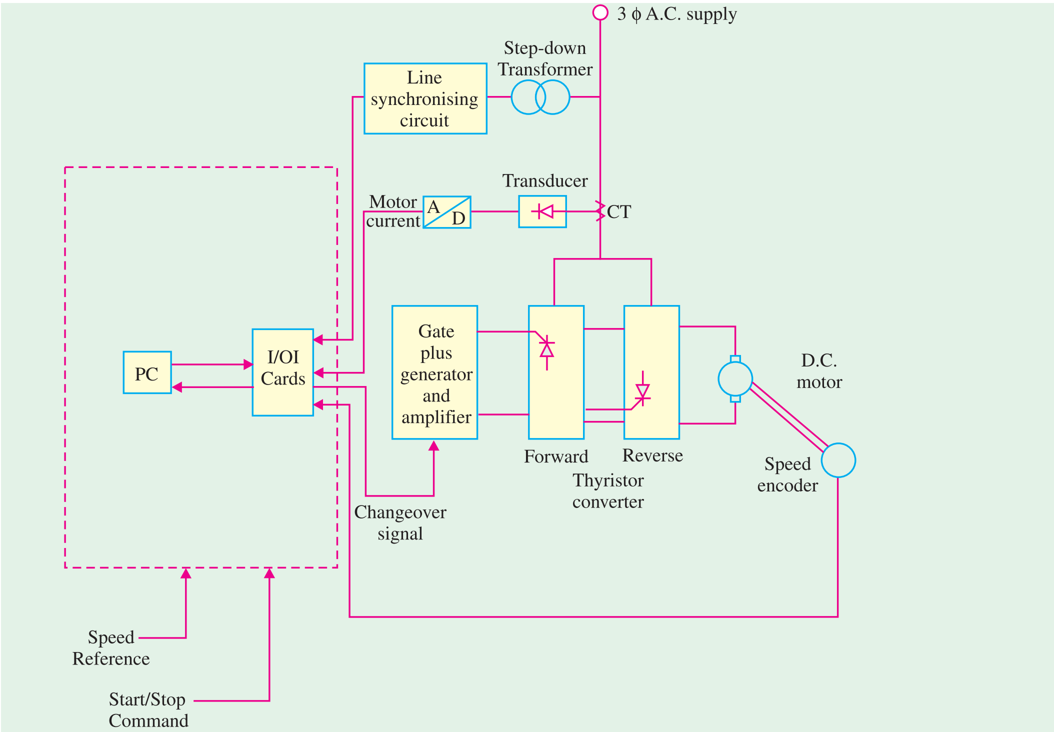


Fig. 46.8

switch *ON* forward or reverse group of SCRs. The gate pulse generator is shown as receiving a firing signal from *PC*.

(iii) Speed Encoder and Input Module

The speed information can be fed to PC through speed input module. The speed measurement is done digitally by means of speed/shaft encoder. It consists of a disc with definite number of holes drilled on it. This disc is fixed on to the shaft. Using a light source and a phototransistor; a series of pulses is obtained, as the shaft rotates. This pulse train is processed and shaped. These optically coded pulses are counted to get actual speed of motor.

(iv) A/D Converter and Transducer

The motor current drawn from supply is stepped down with the help of current transformer. It is converted to D.C. voltage output with the help of current transducer. As PC can't process analog signals, this analog current signal is fed to A/D converter to obtain digital signal which is fed to PC.

(v) Line Synchronizing Circuit

This is required so that PC can synchronize the generation of firing pulse data, with supply line frequency.

(vi) I/O Cards

Input/ Output cards are required to interface PC with the outside world.

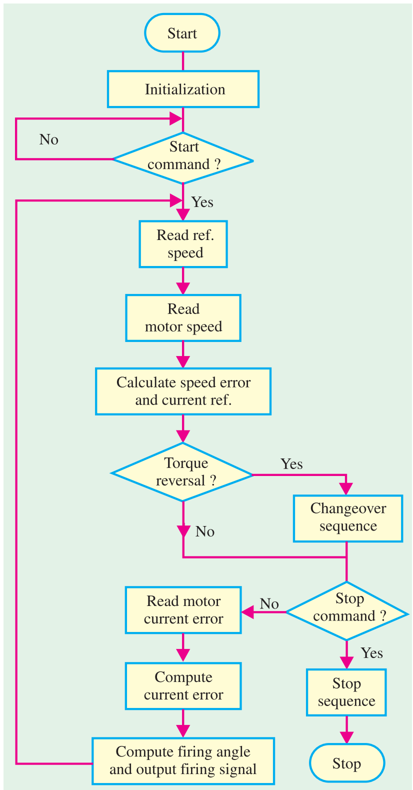


Fig. 46.9. Program flow chart for digital control of motor.

OBJECTIVE TESTS – 46

1. The function of a cycloconverter is to convert
   (*a*) ac power into d.c. power
   (*b*) direct current into alternating current
   (*c*) high ac frequency directly to low ac frequency
   (*d*) a sine wave into a rectangular wave.
2. Major disadvantage of using three sets of SCRs for variable-voltage speed control of a SCIM is the
   (*a*) considerable I^2R loss
   (*b*) poor power factor
   (*c*) long delay of thyristor firing pulses
   (*d*) necessity of using a processor.
3. In the current-fed frequency converter arrangement for controlling the speed of an individual SCIM, the direction of rotation can be reversed by
   (*a*) changing the output frequency of the inverter
   (*b*) altering the phase sequence of pulses that trigger converter-2
   (*c*) interchanging any two line leads
   (*d*) reversing the d.c. link current.
4. In the chopper speed control method for a WRIM, the motor speed inversely depends on
   (*a*) fixed resistor across the rectifier
   (*b*) chopper switching frequency
   (*c*) chopper ON time TON
   (*d*) both (*b*) and (*c*).
5. In the synchronous motor drive using current-fed dc link
   (*a*) converter-2 functions as a self-commutated inverter
   (*b*) converter-1 works as an uncontrolled rectifier
   (*c*) converter-3 is a controlled rectifier
   (*d*) gate triggering of converter-2 is done at motor frequency.
6. In the three cycloconverter drive of a synchronous motor
   (*a*) each cycloconverter produces a 3-phase output
   (*b*) all cycloconverters act as voltage sources
   (*c*) a 3-phase controlled rectifier provides field exciting current.
   (*d*) air-gap flux is kept constant by controlling stator currents only.

ANSWERS

1. (*c*) **2.** (*a*) **3.** (*b*) **4.** (*d*) **5.** (*d*) **6.** (*c*)

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CHAPTER **48**

ELECTRIC WELDING

Learning Objectives

- Definition of Welding
- Welding Processes
- Four Positions of Arc Welding
- Electrodes for Metal Arc Welding
- Advantages of Coated Electrodes
- Arc Welding Machines
- V-I Characteristics of Arc Welding D.C. Machines
- D.C. Welding Machines with Motor Generator Set
- AC Rectified Welding Unit
- AC Welding Machines
- Carbon Arc Welding
- Submerged Arc Welding
- Gas Shield Arc Welding
- TIG Welding
- MIG Welding
- MAG Welding
- Resistance Welding
- Spot Welding
- Seam Welding
- Projection Welding
- Butt Welding
- Flash Butt Welding
- Upset Welding
- Stud Welding
- Electrogas Welding
- Electron Beam Welding

Electricity is used to generate heat necessary to melt the metal to form the necessary joints

CONTENTS

48.1. Definition of Welding

It is the process of joining two pieces of metal or non-metal at faces rendered plastic or liquid by the application of heat or pressure or both. Filler material may be used to effect the union.

48.2. Welding Processes

All welding processes fall into two distinct categories :

1. Fusion Welding—It involves melting of the parent metal. Examples are:

(*i*) Carbon arc welding, metal arc welding, electron beam welding, electroslag welding and electrogas welding which utilize electric energy and

(*ii*) Gas welding and thermit welding which utilize chemical energy for the melting purpose.

2. Non-fusion Welding—It does not involve melting of the parent metal. Examples are:

(*i*) Forge welding and gas non-fusion welding which use chemical energy.

(*ii*) Explosive welding, friction welding and ultrasonic welding etc., which use mechanical energy.

(*iii*) Resistance welding which uses electrical energy.

Proper selection of the welding process depends on the (*a*) kind of metals to be joined (*b*) cost involved (*c*) nature of products to be fabricated and (*d*) production techniques adopted.
The principal welding processes have been tabulated in Fig. 48.1

48.3. Use of Electricity in Welding

Electricity is used in welding for generating heat at the point of welding in order to melt the material which will subsequently fuse and form the actual weld joint. There are many ways of producing this localised heat but the two most common methods are as follows :

1. resistance welding—here current is passed through the inherent resistance of the joint to be welded thereby generating the heat as per the equation $I^2 Rt/J$ kilocalories.

2. arc welding—here electricity is conducted in the form of an arc which is established between the two metallic surfaces

48.4. Formation and Characteristics of Electric Arc

An electric arc is formed whenever electric current is passed between two metallic electrodes which are separated by a short distance from each other. The arc is started by momentarily touching the positive electrode (anode) to the negative metal (or plate) and then withdrawing it to about 3 to 6 mm from the plate. When electrode first touches the plate, a large short-circuit current flows and as it is later withdrawn from the plate, current continues to flow in the form of a spark across the air gap so formed. Due to this spark (or discharge), the air in the gap becomes ionized *i.e.* is split into negative electrons and positive ions. Consequently, air becomes conducting and current is able to flow across the gap in the form of an arc.

As shown in Fig. 48.2, the arc consists of ***lighter*** electrons which flow from cathode to anode and ***heavier*** positive ions which flow from anode to cathode. Intense heat is generated when high-velocity electrons strike the anode. Heat generated at the cathode is much less because of the low velocity of the impinging ions. It is found that nearly **two-third** of the heat is developed at the anode which burns into the form of a crater where temperature rises to a value of 3500-4000°C. The remaining one-third of the heat is developed near the cathode. The above statement is true in all d.c. systems of welding where positive side of the circuit is the hottest side. As a result, an electrode connected to the positive end of the d.c. supply circuit will burn 50% faster than if connected to the negative end. This fact can be used for obtaining desired penetration of the base metal during welding.

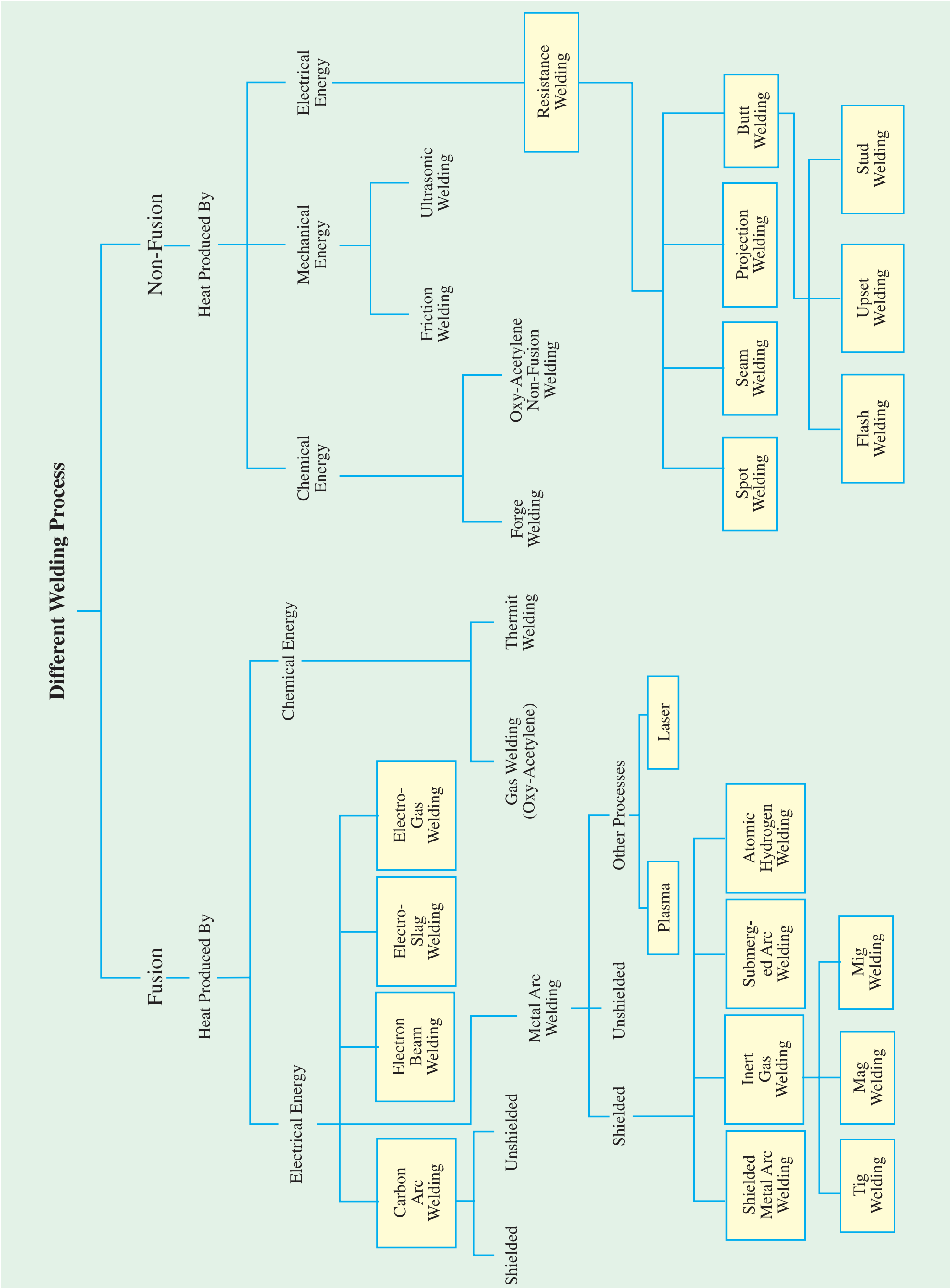

Fig. 48.1

If positive supply end is connected to the base metal (which is normally grounded), penetration will be greater due to more heat and, at the same time, the electrode will burn away slowly [Fig. 48.3 (*a*)] since it is connected to the negative end of the supply. If supply connections are reversed, the penetration of heat zone in the base metal will be comparatively shallow and, at the same time, electrode will burn fast [Fig. 48.3 (*b*)]. AC supply produces a penetration depth that is nearly halfway between that achieved by the d.c. positive ground and negative ground as shown in Fig. 48.3 (*c*).

It may be noted that with a.c. supply, heat is developed equally at the anode and cathode due to rapid reversal of their polarity. The arc utilized for arc welding is a low-voltage high-current discharge. The voltage required for striking the arc is higher than needed for maintaining it. Moreover, amperage increases as voltage decreases after the arc has been established. Fig 48.4 shows V/I characteristics of an electric arc for increasing air-gap lengths. The voltage required to strike a d.c. arc is about 50-55 V and that for a.c. arc is 80-90 V. The voltage drop across the arc is nearly 15-20 V. It is difficult to maintain the arc with a voltage less than 14 V or more than 40 V.

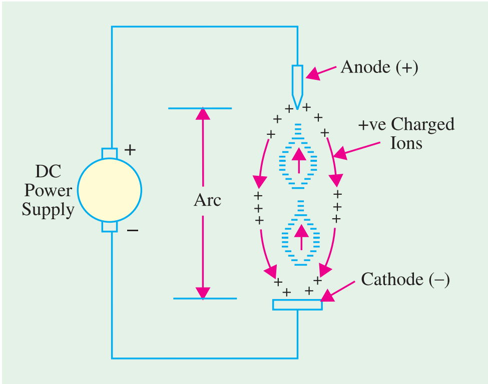

Fig. 48.2

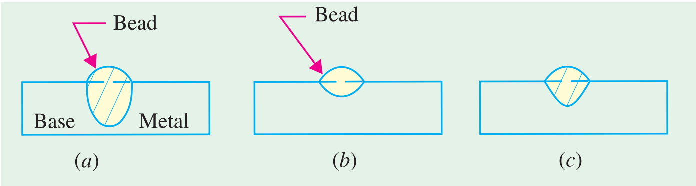

Fig. 48.3

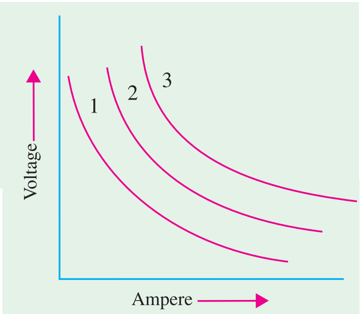

Fig. 48.4

48.5. Effect of Arc Length

In metal arc welding, a fairly short arc length is necessary for getting good welds. Short arc length permits the heat to be concentrated on the workpiece, is more stable because effect of magnetic blow is reduced and the vapours from the arc surround the electrode metal and the molten pool thereby preventing air from destroying the weld metal. When arc length is long

1. large amount of heat is lost into the surrounding area thus preventing good penetration and fusion;
2. arc flame is very unstable since effect of magnetic blow is increased. Hence, arc flame will have a tendency to blow out;
3. air is able to reach the molten globule of metal as it passes from the electrode to the weld and weld pool. It leads to the contamination of the weld due to absorption of oxygen and nitrogen;
4. weld deposits have low strength, poor ductility, high porosity, poor fusion and excessive spatter.

The length of arc required for welding will depend on the kind of electrode used, its coating, its diameter, position of welding and the amount of current used. Usually, shorter arc length are necessary for vertical, horizontal and overhead welding than for flat welding.

An electric arc is produced when electricity is passed between two electrodes

48.6. Arc Blow

An arc column can be considered as a flexible current-carrying conductor which can be easily deflected by the magnetic field set up in its neighbourhood by the positive and negative leads from the d.c. welding set. The two leads carry currents in the opposite directions and hence, set up a repulsive magnetic force which pulls the arc away from the weld point particularly when welding corners where field concentration is maximum. The deflection of the arc is called **arc blow**. This condition is encountered only with d.c. welding sets and is especially noticeable when welding with bare electrodes. It is experienced most when using currents above 200 A or below 40 A.

Due to arc blow, heat penetration in the required area is low which leads to incomplete fusion and bead porosity apart from excessive weld spatter.

Arc blow can be avoided by using a.c. rather than d.c. welding machines because reversing currents in the welding leads produce magnetic fields which cancel each other out thereby eliminating the arc blow. However, with d.c. welding machines, arc blow effects can be minimized by ***(i)*** welding away from the earth ground connection, ***(ii)*** changing the position of the earth connection on the work, ***(iii)*** wrapping the welding electrode cable a few turns around the work, ***(iv)*** reducing the welding current or electrode size, ***(v)*** reducing the rate of travel of the electrode and ***(vi)*** shortening the arc column length etc.

48.7. Polarity in DC Welding

Arc welding with the electrode connected to the positive end of the d.c. supply is called reverse polarity.* Obviously, the workpiece is connected to the negative end.

A better name for d.c. reverse polarity (DCRP) *is* ***electrode-positive*** as shown in Fig. 48.5 (*a*). As stated earlier in Art. 48.4, two-third of the arc heat is developed at the anode. Hence, in DCRP welding, electrode is the hottest whereas workpiece is comparatively cooler. Consequently, electrode burns much faster but weld bead is relatively shallow and wide. That is why thick and heavily-coated electrodes are used in DCRP welding because they require more heat for melting.

Arc welding with the electrode connected to the negative end of the d.c. supply is called **straight polarity**.** Obviously, the workpiece is connected to the positive end as shown in Fig. 48.5 (*b*). A better name for d.c. straight polarity (DCSP) is ***electrode-negative***.

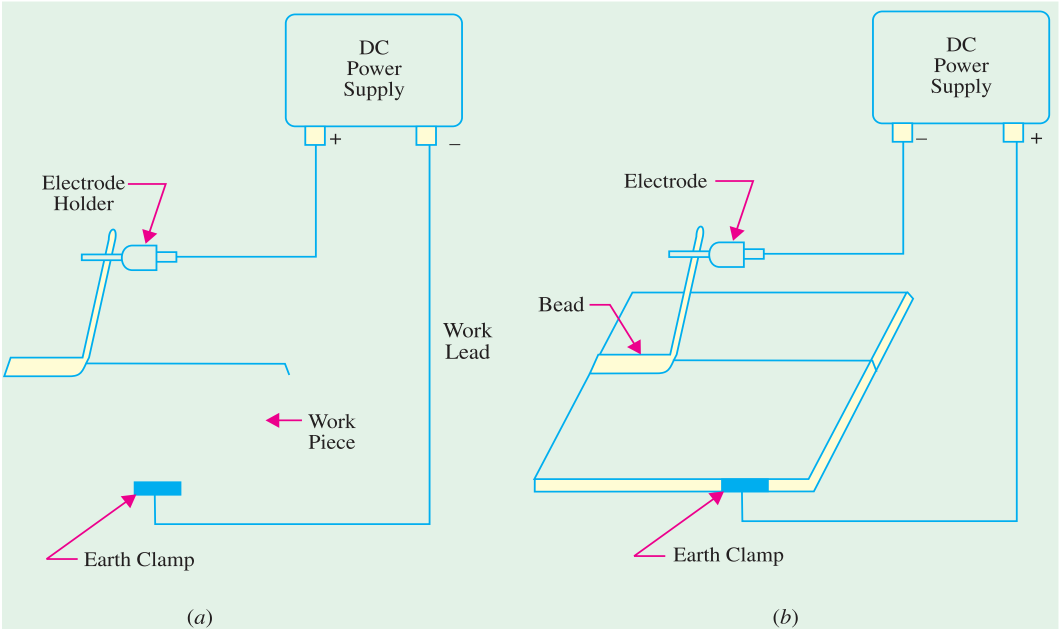

Fig. 48.5

* In British literature, it is called straight polarity.

** In British literature, it is called reverse polarity.

In DCSP welding, workpiece is the hottest, hence base metal penetration is narrow and deep. Moreover, bare and medium-coated electrodes can be used in this welding as they require less amount of heat for melting.

It is seen from the above discussion that polarity necessary for the welding operation is determined by the type of electrode used.

It is also worth noting that in a.c. welding, there is no choice of polarity because the circuit becomes alternately positive, first on one side and then on the other. In fact, it is a combination of D CSP and D CRP.

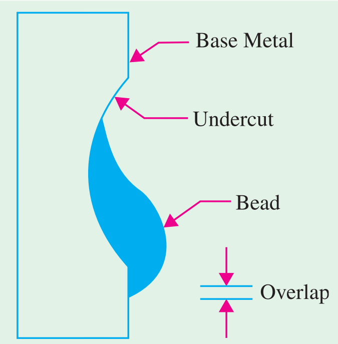

Fig. 48.6

48.8. Four Positions of Arc Welding

There are four basic positions in which manual arc welding is done.

1. Flat position. It is shown in Fig. 48.7 (*a*). Of all the positions, flat position is the easiest, most economical and the most used for all shielded arc welding. It provides the strongest weld joints. Weld beads are exceedingly smooth and free of slag spots. This position is most adaptable for welding of both ferrous and non-ferrous metals particularly for cast iron.

2. Horizontal Position. It is the second most popular position and is shown in Fig. 48.7 (*b*). It also requires a short arc length because it helps in preventing the molten puddle of the metal from sagging. However, major errors that occur while welding in horizontal position are under-cutting and over-lapping of the weld zone (Fig. 48.6).

3. Vertical Position. It is shown in Fig. 48.7 (*c*). In this case, the welder can deposit the bead either in the uphill or downhill direction. Downhill welding is preferred for thin metals because it is faster than the uphill welding. Uphill welding is suited for thick metals because it produces stronger welds.

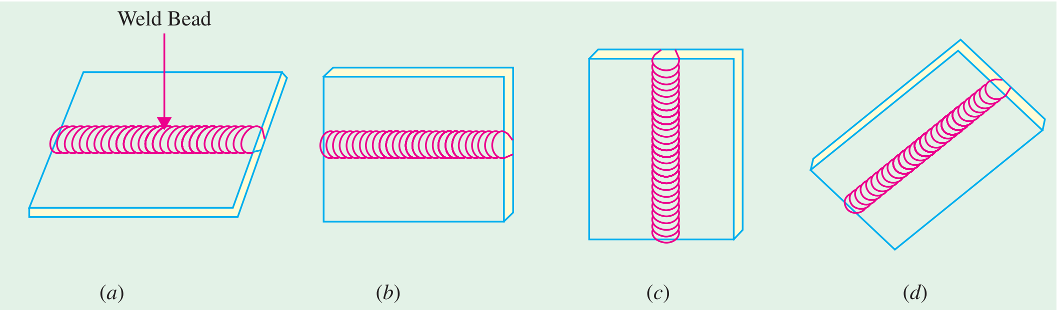

Fig. 48.7.

4. Overhead Position. It is shown in Fig. 48.7 (*d*). Here, the welder has to be very cautious otherwise he may get burnt by drops of falling metal. This position is thought to be the most hazardous but not the most difficult one.

48.9. Electrodes for Metal Arc Welding

An electrode is a filler metal in the form of a wire or rod which is either bare or coated uniformly with flux. As per IS : 814-1970, the contact end of the electrode is left bare and clean to a length of 20-30 mm. for inserting it into electrode holder (Fig. 48.8).

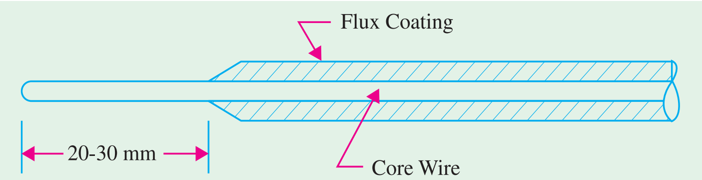

Fig. 48.8

Metal arc welding was originally done with bare electrodes which consisted of a piece of wire or rod of the same metal as the base metal. However, due to atmospheric contamination, they produced brittle and poor quality welds.

Hence, bare wire is no longer used except for automatic welding in which case arrangement is made to protect the weld area from the atmosphere by either powdered flux or an inert gas. Since 1929, coated electrodes are being extensively used for shielded arc welding. They consist of a metal core wire surrounded by a thick flux coating applied by extrusion, winding or other processes. Depending on the thickness of the flux coating, coated electrodes may be classified into ***(i)*** lightly-dusted (or dipped) electrodes and ***(ii)*** semi-coated (or heavy-coated) electrodes. Materials commonly used for coating are ***(i)*** titanium oxide ***(ii)*** ferromanganese ***(iii)*** silica flour ***(iv)*** asbestos clay ***(v)*** calcium carbonate and ***(vi)*** cellulose with sodium silicate often used to hold ingredients together.

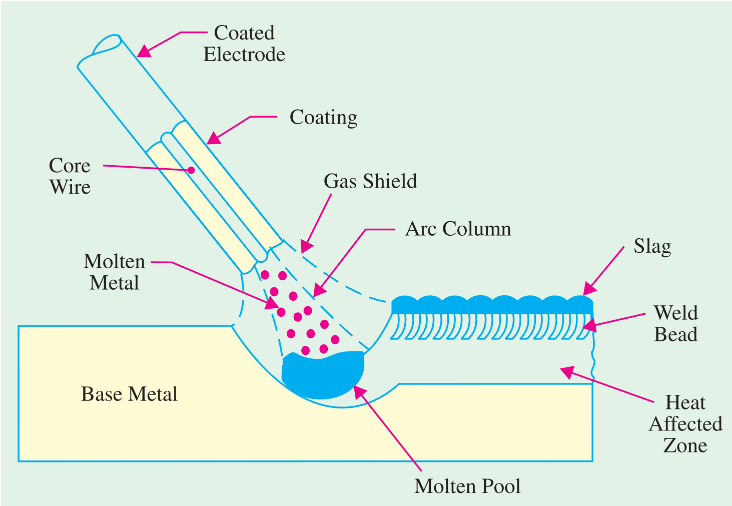

Fig. 48.9

Electrode coating contributes a lot towards improving the quality of the weld. Part of the coating burns in the intense heat of the arc and provides a gaseous shield around the arc which prevents oxygen, nitrogen and other impurities in the atmosphere from combining with the molten metal to cause a poor quality brittle and weak weld. Another portion of the coating flux melts and mixes with the impurities in the molten pool causing them to float to the top of the weld where they cool in the form of slag (Fig. 48.9). This slag improves the bead quality by protecting it from the contaminating effects of the atmosphere and causing it to cool down more uniformly. It also helps in controlling the basic shape of the weld bead.

The type of electrode used depends on the type of metal to be welded, the welding position, the type of electric supply whether a.c. or d.c. and the polarity of the welding machine.

48.10. Advantages of Coated Electrodes

The principal advantages of using electrode coating are as under :

1. It stablizes the arc because it contains ionizing agents such as compounds of sodium and potassium.

2. It fluxes away impurities present on the surface being welded.

3. It forms slag over the weld which ***(i)*** protects it from atmospheric contamination ***(ii)*** makes it cool uniformly thereby reducing the changes of brittleness and ***(iii)*** provides a smoother surface by reducing 'ripples' caused by the welding operation.

4. It adds certain materials to the weld metal to compensate for the loss of any volatile alloying elements or constituents lost by oxidization.

5. It speeds up the welding operation by increasing the rate of melting.

6. It prevents the sputtering of metal during welding.

7. it makes it possible for the electrode to be used on a.c. supply. In a.c. welding, arc tends to cool and interrupt at zero-current positions. But the shielding gases produced by the flux keep the arc space ionized thus enabling the coated electrodes to be used on a.c. supply.

It is worth noting that efficiency of all coated (or covered) electrodes is impaired by dampness. Hence, they must always be stored in a dry space. If dampness is suspected, the electrodes should be dried in a warm cabinet for a few hours.

48.11. Types of Joints and Types of Applicable Welds

Bureau of Indian Standards (B.I.S.) has recommended the following types of joints and the welds applicable to each one of them (Fig. 48.10).

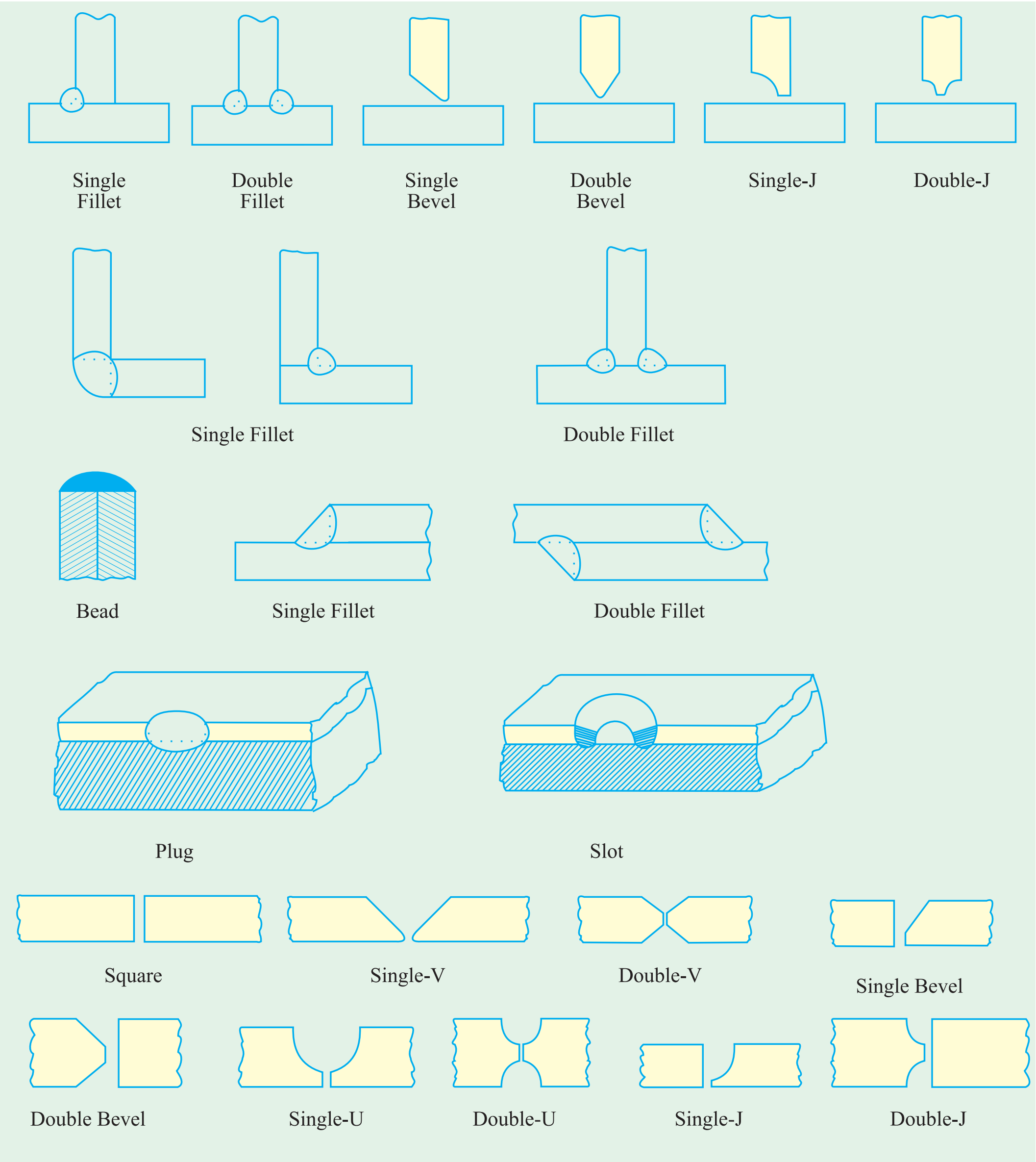

Fig. 48.10

1. Tee joint — with six types of welds.
2. Corner joint — with two types of welds.
3. Edge joint — with one type of weld.
4. Lap joint — with four types of welds.
5. Butt joint — with nine types of welds.

48.12. Arc Welding Machines

Welding is never done directly from the supply mains. Instead, special welding machines are used which provided currents of various characteristics. Use of such machines is essential for the following reasons :

1. To convert a.c. supply into d.c. supply when d.c. welding is desired.
2. To reduce the high supply voltage to a safer and suitable voltage for welding purposes.
3. To provide high current necessary for arc welding without drawing a corresponding high current from the supply mains.
4. To provide suitable voltage/current relationships necessary for arc welding at minimum cost.

There are two general types of arc welding machines :

(*a*) d.c. welding machines

(*i*) motor-generator set

(*ii*) a.c. transformers with rectifiers

(*b*) a.c. welding machines

48.13. V-I Characteristics of Arc Welding DC Machines

It is found that during welding operation, large fluctuations in current and arc voltage result from the mechanism of metal transfer and other factors. The welding machine must compensate for such changes in arc voltage in order to maintain an even arc column. There are three major voltage/current characteristics used in modern d.c. welding machines which help in controlling these current fluctuations :

1. drooping arc voltage (DAV).
2. constant arc voltage (CAV).
3. rising arc voltage (RAV).

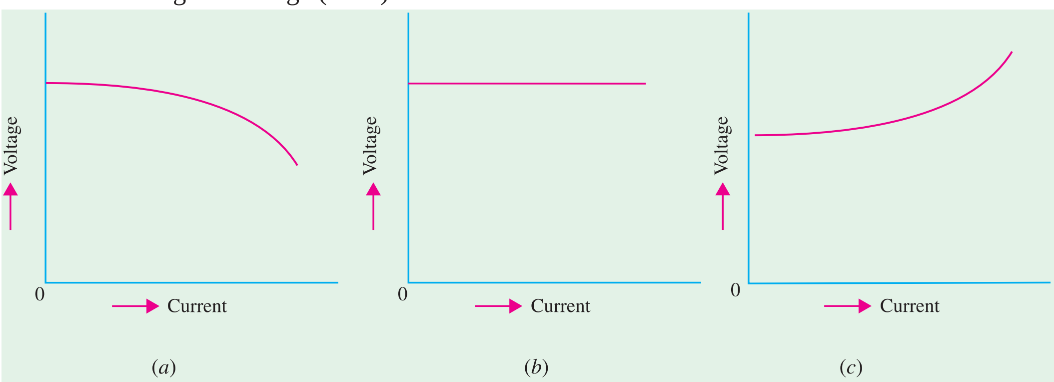

Fig. 48.11

The machines with DAV characteristics have high open-circuit voltage which drops to a minimum when arc column is started. The value of current rises rapidly as shown in Fig. 48.11 (*a*). This type of characteristic is preferred for manual shield metal arc welding.

The CAV characteristic shown in Fig. 48.11 (*b*) is suitable for semi-automatic or automatic welding processes because voltage remains constant irrespective of the amount of current drawn.

Because of its rising voltage characteristic, RAV has an advantage over CAV because it maintains a constant arc gap even if short circuit occurs due to metal transfer by the arc. Moreover, it is well-adopted to fully automatic process.

DC welding machines can be controlled by a simple rheostat in the exciter circuit or by a combination of exciter regulator and series of field taps. Some arc welding are equipped with remote-controlled current units enabling the operator to vary voltage-amperage requirement without leaving the machines.

DC and AC welding machines

48.14. DC Welding Machines with Motor Generator Set

Such a welding plant is a self-contained single-operator motor-generator set consisting of a reverse series winding d.c. generator driven by either a d.c. or an a.c. motor (usually 3-phase). The series winding produces a magnetic field which opposes that of the shunt winding. On open-circuit, only shunt field is operative and provides maximum voltage for striking the arc. After the arc has been established, current flows through the series winding and sets up a flux which opposes the flux produced by shunt winding. Due to decreases in the net flux, generator voltage is decreased (Art. 48.13).

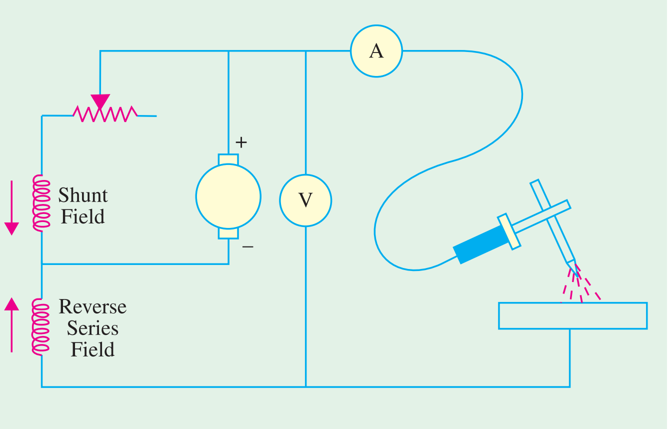

Fig. 48.12

With the help of shunt regulator, generator voltage and current values can be adjusted to the desired level. Matters are so arranged that despite changes in arc voltage due to variations in arc length, current remains practically constant. Fig. 48.12 shows the circuit of a d.c. motor-generator type of welding machine.

Advantages. Such a d.c. welder has the following advantages :

1. It permits portable operation.

2. It can be used with either straight or reverse polarity.

3. It can be employed on nearly all ferrous and non-ferrous materials.

4. It can use a large variety of stick electrodes.

5. It can be used for all positions of welding.

Disadvantages

1. It has high initial cost.
2. Its maintenance cost is higher.
3. Machine is quite noisy in operation.
4. It suffers from arc blow.

48.15. AC Rectified Welding Unit

It consists of a transformer (single-or three-phase) and a rectifier unit as shown in Fig. 48.13. Such a unit has no moving parts, hence it has long life. The only moving part is the fan for cooling the transformer. But this fan is not the basic part of the electrical system. Fig. 48.13 shows a single-phase full-wave rectified circuit of the welder. Silicon diodes are used for converting a.c. into d.c. These diodes are hermetically sealed and are almost ageless because they maintain rectifying characteristics indefinitely.

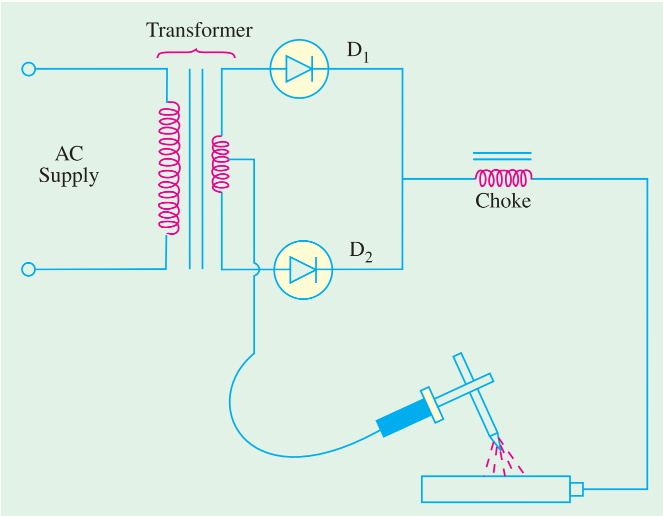

Fig. 48.13

Such a transformer-rectifier welder is most adaptable for shield arc welding because it provides both d.c. and a.c. polarities. It is very efficient and quiet in operation. These welders are particularly suitable for the welding of *(i)* pipes in all positions *(ii)* non-ferrous metals *(iii)* low-alloy and corrosion-heat and creep-resisting steel *(iv)* mild steels in thin gauges.

48.16. AC Welding Machines

As shown in Fig. 48.14, it consists of a step-down transformer with a tapped secondary having an adjustable reactor in series with it for obtaining drooping V/I characteristics. The secondary is tapped to give different voltage/ current settings.

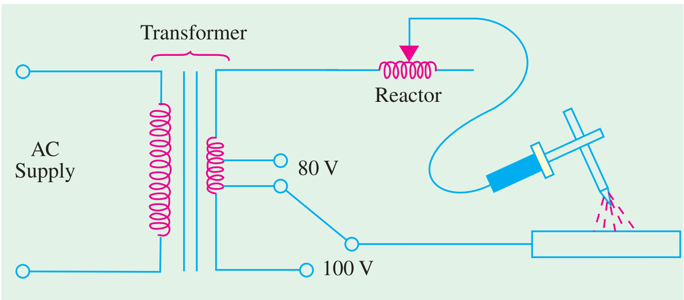

Fig. 48.14

Advantages. This a.c. welder which can be operated from either a single-phase or 3-phase supply has the following advantages :

(i) Low initial cost
(ii) Low operation and maintenance cost
(iii) Low wear
(iv) No arc blow

Disadvantages. *(i)* its polarity cannot be changed *(ii)* it is not suitable for welding of cast iron and non-ferrous metals.

48.17. Duty Cycle of a Welder

The duty cycle of an arc welder is based on a working period of 10 minutes. For example, if a welder is operated for 2 minutes in a period of 10 minutes, then its percentage duty cycle is (2/10) × 100 = 20 percent. Conversely, a 10 percent duty cycle would mean that the welder would be operated for 10 percent of 10 minutes *i.e.* for one minute only in a period of 10 minutes.

Usually, values of maximum amperage and voltage are indicated along with the duty cycle. It is advisable to adhere to these values. Suppose a welding machine has maximum amperage of 300A and voltage of 50 V for a duty cycle of 60 percent. If this machine is operated at higher settings and for periods longer than 6 minutes, then its internal insulation will deteriorate and cause its early failure.

48.18. Carbon Arc Welding

(*a*) General

Carbon arc welding was the first electric welding process developed by a French inventor Auguste de Meritens in 1881. In this process, fusion of metal is accomplished by the heat of an electric arc. No pressure is used and generally, no shielding atmosphere is utilized. Filler rod is used only when necessary. Although not used extensively these days, it has, nevertheless, certain useful fields of application.

Carbon arc welding differs from the more common shield metal arc welding in that ***it uses non-consumable carbon or graphic electrodes*** instead of the consumable flux-coated electrodes.

(*b*) Welding Circuit

The basic circuit is shown in Fig. 48.15 and can be used with d.c. as well as a.c. supply. When direct current is used, the electrode is mostly negative (DCSP). The process is started by adjusting the amperage on the d.c. welder, turning welder ON and bringing the electrode into contact with the workpiece. After the arc column starts, electrode is withdrawn 25 – 40 mm away and the arc is maintained at this distance. The arc can be extinguished by simply removing the electrode from the workpiece completely. The only function of the carbon arc is to supply heat to the base metal. This heat is used to melt the base metal or filler rod for obtaining fusion weld Depending on the type and size of electrodes, maximum current values range from 15 A to 600 A for single-electrode carbon arc welding.

(*c*) Electrodes

These are made of either carbon or graphite, are usually 300 mm long and 2.5 – 12 mm in diameter. Graphite electrodes are harder, more brittle and last longer than carbon electrodes. They can withstand higher current densities but their arc column is harder to control. Though considered non-consumable, they do disintegrate gradually due to vaporisation and oxidisation.

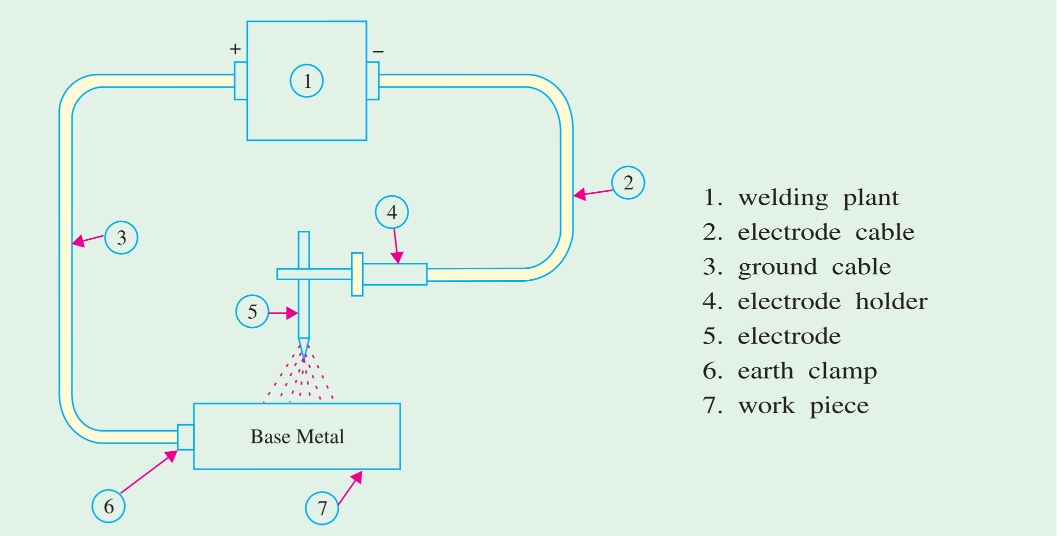

Fig. 48.15

(*d*) Applications

1. The joint designs that can be used with carbon arc welding are butt joints, bevel joints, flange joints, lap joints and fillet joints.
2. This process is easily adaptable for automation particularly where amount of weld deposit is large and materials to be fabricated are of simple geometrical shapes such as water tanks.
3. It is suitable for welding galvanised sheets using copper-silicon-manganese alloy filler metal.
4. It is useful for welding thin high-nickel alloys.
5. Monel metal can be easily welded with this process by using a suitable coated filler rod.
6. Stainless steel of thinner gauges is often welded by the carbon-arc process with excellent results.

(*e*) Advantages and Disadvantages

1. The main advantage of this process is that the temperature of the molten pool can be easily controlled by simply varying the arc length.
2. It is easily adaptable to automation.
3. It can be easily adapted to inert gas shielding of the weld and
4. It can be used as an excellent heat source for brazing, braze welding and soldering etc.

Its disadvantages are as under :

1. A separate filler rod has to be used if any filler material is required.
2. Since arc serves only as a heat source, it does not transfer any metal to help reinforce the weld joint.
3. The major disadvantage of the carbon-arc process is that blow holes occur due to magnetic arc blow especially when welding near edges of the workpiece.

48.19. Submerged Arc Welding

In this ***fusion*** process, welding is done under a blanket of granulated flux which shields the weld from all bad effects of atmospheric gases while a consumable electrode is continuously and mechanically fed into the arc. The arc, the end of the bare metal electrode and the molten weld pool are all submerged under a thick mound of finely-divided granulated powder that contains deoxidisers, cleansers and other fluxing agents. The fluxing powder is fed from a hopper that is carried on the welding head itself (Fig. 48.16). This hopper spread the powder in a continuous mound ahead of the

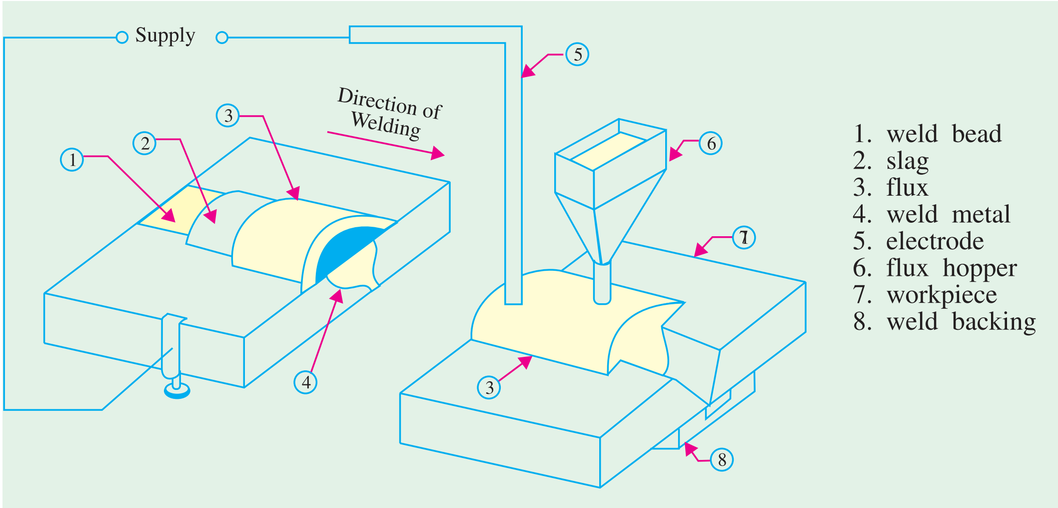

Fig. 48.16

electrode in the direction of welding. Since arc column is completely submerged under the powder, there is no splatter or smoke and, at the same time, weld is completely protected from atmospheric contamination. Because of this protection, weld beads are extremely smooth. The flux adjacent to the arc column melts and floats to the top of the molten pool where it solidifies to form slag. This slag is easy to remove. Often it cracks off by itself as it cools. The unused flux is removed and is reused again and again.

The electrode is either a bare wire or has a slight mist of copper coated over it to prevent oxidation. In automatic or semi-automatic submerged arc welding, wire electrode is fed mechanically through an electrically contacting collet. Though a.c. power supply may be used, yet d.c. supply is more popular because it assures a simplified and positive control of the welding process. This process requires high current densities about 5 to 6 times of those used in ordinary manual stick electrode welding. As a result, melting rate of the electrode as well as welding speed become much higher. Faster welding speed minimizes distortion and warpage.

The submerged arc process is suitable for

1. Welding low-alloy, high-tensile steels.

2. Welding mild, low-carbon steels.

3. Joining medium-carbon steel, heat-resistant steels and corrosion-resistant steels etc.

4. Welding nickel, Monel and other non-ferrous metals like copper.

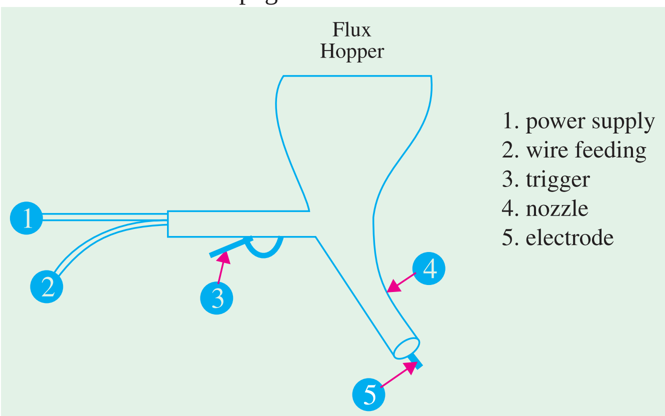

Fig. 48.17

This process has many industrial applications such as fabrication of pipes, boiler pressure vessels, railroad tank cars, structural shapes etc. which demand welding in a straight line. Welds made by this process have high strength and ductility. A major advantage of this process is that fairly thick sections can be welded in a single pass without edge preparation.

Submerged arc welding can be done manually where automatic process is not possible such as on curved lines and irregular joints. Such a welding gun is shown in Fig. 48.17. Both manual and automatic submerged arc processes are most suited for flat and slightly downhill welding positions.

48.20. Twin Submerged Arc Welding

As shown in Fig. 48.18, in this case, two electrodes are used simultaneously instead of one. Hence, weld deposit size is increased considerably. Moreover, due to increase in welding current (upto 1500 A), much deeper penetration of base metal is achieved.

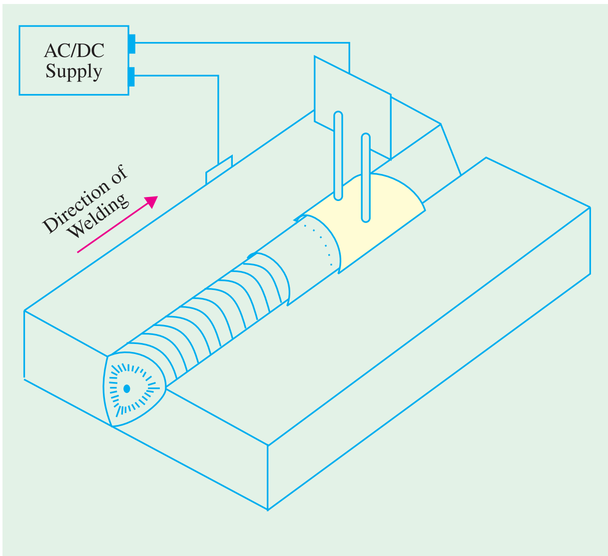

Fig. 48.18

48.21. Gas Shield Arc Welding

In this fusion process, welding is done with bare electrodes but weld zone is shielded from the atmosphere by a gas which is piped to the arc column. Shielding gases used are carbon dioxide, argon, helium, hydrogen and oxygen. No flux is required. Different processes using shielding gas are as follows.

(*a*) Tungsten inert-gas (TIG) Process

In this process, non-consumable tungsten electrode is used and filler wire is fed separately. The weld zone is shielded from the atmosphere by the inert gas (argon or helium) which is ducted directly to the weld zone where it surrounds the tungsten and the arc column.

(*b*) Metal inert-gas (MIG) Process

It is a refinement of the TIG process. It uses a bare consumable (*i.e.* fusible) wire electrode which acts as the source for the arc column as well as the supply for the filler material. The weld zone is shielded by argon gas which is ducted directly to the electrode point.

48.22. TIG Welding

(*a*) Basic Principle

It is an electric process which uses a bare non-consumable tungsten electrode for striking the arc only (Fig. 48.19). Filler material is added separately. It uses an inert gas to shield the weld puddle from atmospheric contamination. This gas is ducted directly to the weld zone from a gas cylinder.

(*b*) Welding Equipment

The usual TIG welding system consists of the following (Fig. 48.20).

1. A standard shield arc welding machine complete with cables etc.
2. A supply of inert gas complete with hose, regulators etc.
3. A source of water supply (in the case of water-cooled torches).
4. A TIG torch with a control switch to which all the above are connected.

(*c*) Electrodes

The electrodes are made of either pure tungsten or zirconiated or thoriated tungsten. Addition of zironium or thorium (0.001 to 2%) improves electron emission tremendously.

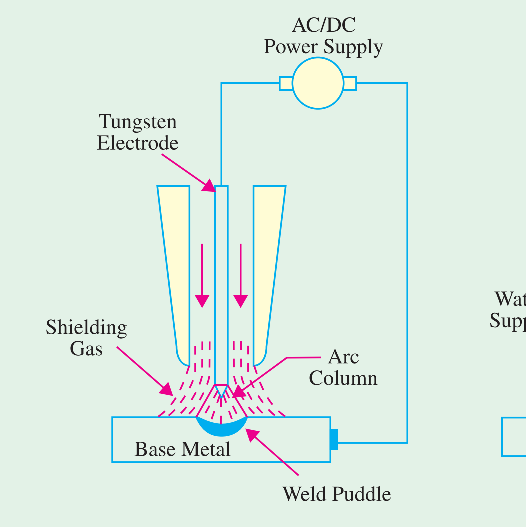

Fig. 48.19

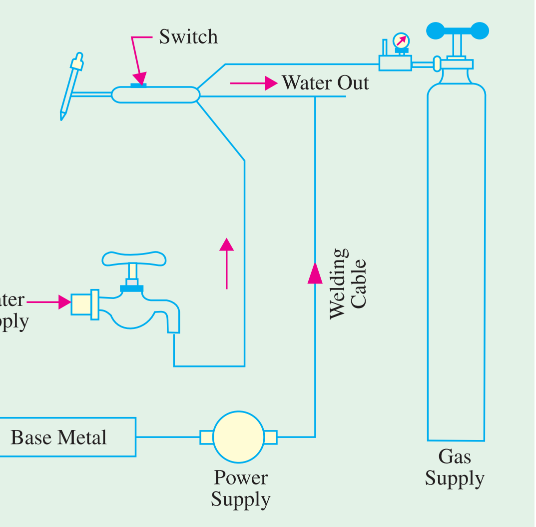

Fig. 48.20

(*d*) Power Supply

The three basic power supplies used in TIG operation are :

1. DCSP power supply–here electrode is negative, runs cooler and, hence, can be thin.
2. DCRP power supply–here electrode is positive and hot. Hence, it has to be large.
3. A.C. high frequency (ACHF) power supply–it is a combination of standard a.c. supply of 50 Hz and high-voltage high-frequency d.c. supply. The function of this d.c. supply is to sustain the arc when a.c. supply is at zero current positions.

(*e*) Advantages of TIG Welding

1. It provides maximum protection to weld bead from atmospheric contamination.
2. TIG welds are stronger, more ductile and more corrosion-resistant than those of shield metal arc welding.
3. Since no flux is used, there is no flux entrapment in the bead.
4. Since no flux is required, a wider variety of joint designs can be used.
5. No post-weld cleansing is necessary.
6. There is no weld splatter or sparks that could damage the surface of the base metal.
7. It gives relatively fast welding speeds.
8. It is suitable for welding food or medical containers where entrapment of any decaying organic matter could be extremely harmful.
9. It is suitable for all welding positions–the flat, horizontal, vertical and overhead positions. The joints suitable for TIG welding process are ***(i)*** butt joint ***(ii)*** lap joint ***(iii)*** T-joint, ***(iv)*** corner joint and ***(v)*** edge joint.

(*f*) Applications

1.	aluminium and its alloys	—	AC/DCRP
2.	magnesium and its alloys	—	ACHF
3.	stainless steel	—	DCSP
4.	mild steel, low-alloy steel, medium	—	DCSP
	-carbon steel and cast iron	—	DCSP
5.	copper and allows	—	DCSP
6.	nickel and alloys	—	DCSP

TIG welding is also used for dissimilar metals, hardfacing and surfacing of metals. Special industrial applications include manufacture of metal furniture and air-conditioning equipment.

Fig. 48.21 shows Phillips 400-D compact fan-cooled DC TIG welding set which has an open-circuit voltage of 80 V and a welding current of 400 A with 60% duty cycle and 310 A with 100% duty cycle.

Fig. 48.21 (TIG welding set)

48.23. MIG Welding

(*a*) Basic Principle

It is also called inert-gas consumable- electrode process. The fusible wire electrode is driven by the drive wheels. Its function is two-fold: to produce arc column and to provide filler material. This process uses inert gas for shielding the weld zone from atmospheric contamination. Argon is used to weld non-ferrous metals though helium gives better control of porosity and arc stability. This process can deposit large quantities of weld metal at a fast welding speed. The process is easily adaptable to semi-automatic or fully automatic operations.

(*b*) Welding Equipment

The basic MIG welding system (Fig. 48.23) consists of the following :

1. Welding power supply
2. Inert gas supply with a regulator and flow meter
3. Wire feed unit containing controls for wire feed, gas flow and the ON/OFF switch for MIG torch
4. MIG torch
5. Depending on amperage, a water cooling unit.

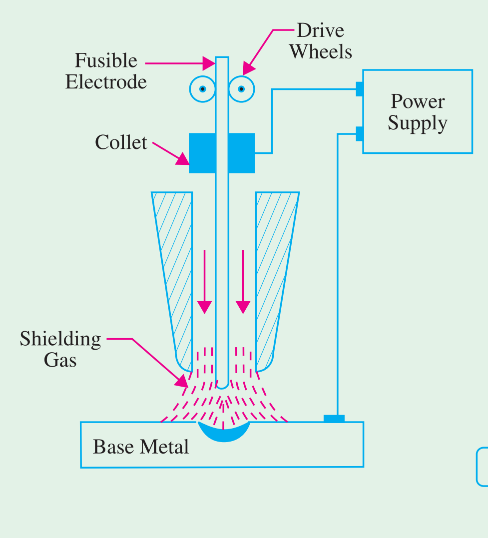

Fig. 48.22

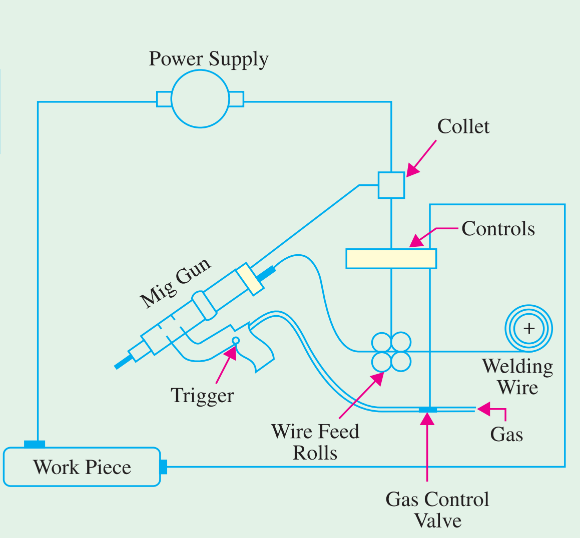

Fig. 48.23

(*c*) Electrode

It is a bare wire fed to the MIG gun by a suitable wire-feed mechanism.

(*d*) Power Supply

The major power supply used for MIG welding is DCRP and the machines which provide this supply are motor-generator sets or a.c. transformers with rectifiers (Art. 48.14). They have either CAV or RAV characteristics (Art. 48.12). The CAV supply gives the operator great latitude in arc length and is helpful in preventing the wire electrode from stubbing. A DCRP current produces deeper penetration and a cleaner weld surface than other types of current.

The RAV machines are more suitable for automatic operation. They are capable of handling large diameter wires than CAV machines.

Fig. 48.24 shows semi-automatic forced-air cooled arc welding set MIG-400. It consists of

(*i*) Indarc 400 MMR rectifier which is basically a 3-phase transformer rectifier with silicon

Fig. 48.24.
MIG-400 Welding Set. (Courtesy : Indian Oxygen Ltd. Calcutta)

diodes and a constant potential output. It provides maximum current of 400 A at 40 V for 75% duty cycle and 350 A at 42 V for 100% duty cycle.

(*ii*) Indarc Wire Feeder which has a twin roll drive system, designed to feed 0.8 to 2.4 mm diameter welding wires to a hand-operated MIG welding torch.

(*iii*) MIG Torches which are available in both air-cooled and water-cooled varieties. Fig. 48.25 (*a*) and (*b*) show light-weight swan-necked torches which are designed to operate upto 360 A and 400 A with CO_2 as shielding gas. Fig. 48.25 (*c*) shows a heavy-duty water-cooled torch designed to operate upto 550 A with CO_2 /mixed shielding gases at 100% duty cycle.

(*iv*) CO_2 Kit for hard wire applications and Argon Kit for soft wire applications.

(*e*) Advantages of MIG Welding

1. Gives high metal deposit rates varying from 2 to 8 kg/h.
2. Requires no flux.
3. Requires no post-welding cleaning.
4. Gives complete protection to weld bead from atmospheric contamination.
5. Is adaptable for manual and automatic operations.
6. Can be used for a wide range of metals both ferrous and non-ferrous.
7. Is easy to operate requiring comparatively much less operating skill.
8. Is especially suited for horizontal, vertical and overhead welding positions.

(*f*) Applications

With inert gas shielding, this process is suitable for fusion welding of **(*i*)** aluminium and its alloys **(*ii*)** nickel and its alloys **(*iii*)** copper alloys **(*iv*)** carbon steels **(*v*)** low-alloy steels **(*vi*)** high strength steels and **(*vii*)** titanium.

48.24. MAG Welding

As discussed earlier, in MIG welding process, the shielding gas used is monoatomic (argon or helium) and is inert *i.e*. chemically inactive and metal transfer takes place by axial pulverization. In MAG (metal-active-gas) process, shielding, gas used is chemically active *i.e*. carbon dioxide or its mixture with other gases. Transfer of metal takes place in big drops.

48.25. Atomic Hydrogen Welding

(*a*) General

It is a non-pressure fusion welding process and the welder set is used only as heat supply for the base metal. If additional metal is required, a filler rod can be melted into the joint. It uses two tungsten electrodes between which an arc column (actually, an arc fan) is maintained by an a.c. supply.

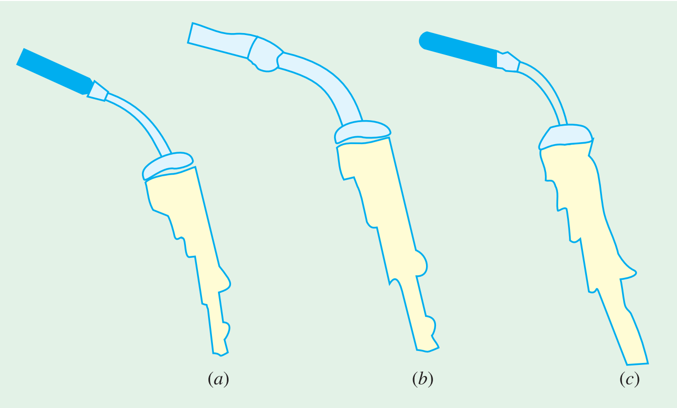

Fig. 48.25

(*b*) Basic Principle

As shown in Fig. 48.26, an arc column is struck between two tungsten electrodes with an a.c. power supply. Soon after, normal molecular hydrogen (H_2) is forced through this arc column. Due to intense heat of the arc column, this diatomic hydrogen is dissociated into atomic hydrogen (H).

However, atomic hydrogen being unstable, recombines to form stable molecular hydrogen. In so doing, it releases intense heat at about 3750°C which is used to fuse the metals.

(*c*) Welding Equipment

The welding equipment essentially consists of the following :

1. Standard welding machine consisting of a step-down transformer with tapped secondary (not shown in Fig. 48.27) energised from normal a.c. supply. Amperage requirement ranges from 15 A to 150 A
2. Hydrogen gas supply with an appropriate regulator
3. Atomic hydrogen welding torch having an ON-OFF switch and a trigger for moving the two tungsten electrodes close together for striking and maintaining the arc column.

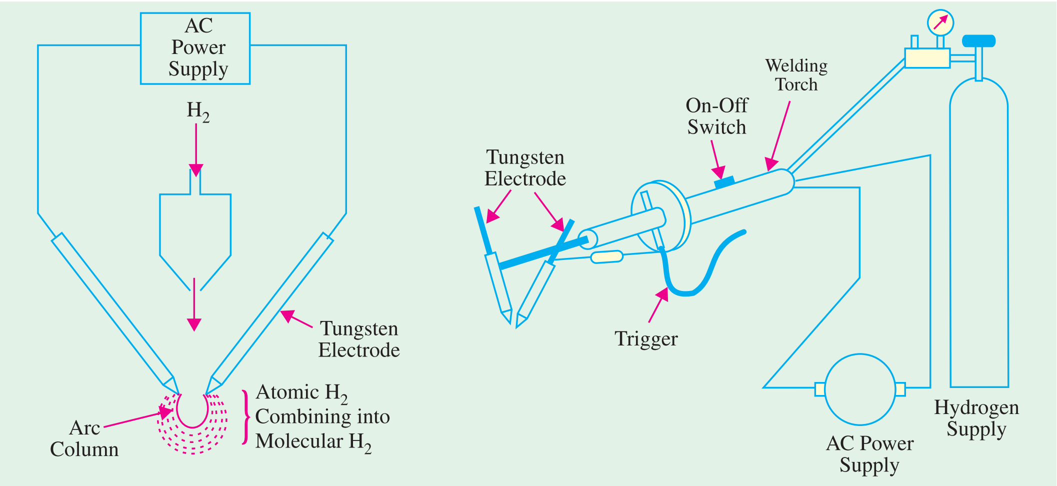

Fig. 48.26 **Fig. 48.27**

(*d*) Method of Welding

The torch is held in the right hand with first finger resting lightly on the trigger. The arc is struck either by allowing the two tungsten electrodes to touch and separate or by drawing the separated electrodes over a carbon block. At the same time, a stream of hydrogen is allowed to flow through the arc. As soon as the arc strikes, an intensely hot flame extends fanwise between the electrodes. When this fan touches the workpiece, it melts it down quickly. If filler material is required, it can be added from the rod held in the left hand as in gas welding.

(*e*) Advantages

1. Arc and weld zone are shrouded by burning hydrogen which, being an active reducing agent, protects them from atmospheric contamination.
2. Can be used for materials too thin for gas welding.
3. Can weld quite thick sections.
4. Gives strong, ductile and sound welds.
5. Can be used for welding of mild steel, alloy steels and stainless steels and aluminium alloys.
6. Can also be used for welding of most non-ferrous metals such as nickel, monel, brass, bronze, tungsten and molybdenum etc.

48.26. Resistance Welding

It is fundamentally a heat and squeeze process. The term ***'resistance welding'*** denotes a group of processes in which welding heat is produced by the resistance offered to the passage of electric current through the two metal pieces being welded. These processes differ from the fusion processes in the sense that no extra metal is added to the joint by means of a filler wire or electrode. According to Joule's law, heat produced electrically is given by $H = I^2Rt/J$. Obviously, amount of heat produced depends on.

(*i*) square of the current (*ii*) the time of current and (*iii*) the resistance offered.

As seen, in simple resistance welding, high-amperage current is necessary for adequate weld. Usually, R is the contact resistance between the two metals being welded together. The current is passed for a suitable length of time controlled by a timer. The various types of resistance welding processes may be divided into the following four main groups :

(*i*) spot welding (*ii*) seam welding (*iii*) projection welding and (*iv*) butt welding which could be further subdivided into flash welding, upset welding and stud welding etc.

Advantages

Some of the advantages of resistance welding are as under :

1. Heat is localized where required
2. Welding action is rapid
3. No filler material is needed
4. Requires comparatively lesser skill
5. Is suitable for large quantity production
6. Both similar and dissimilar metals can be welded
7. Parent metal is not harmed
8. Difficult shapes and sections can be welded.

Only disadvantages are with regard to high initial as well as maintenance cost.

It is a form of resistance welding in which the two surfaces are joined by spots of fused metal caused by fused metal between suitable electrodes under pressure.

48.27. Spot Welding

The process depends on two factors :

1. Resistance heating of small portions of the two workpieces to plastic state and

2. Application of forging pressure for welding the two workpieces.

Heat produced is $H = I^2Rt/J$. The resistance R is made up of (*i*) resistance of the electrodes and metals themselves (*ii*) contact resistance between electrodes and workpieces and (*iii*) contact resistance between the two workpieces. Generally, contact resistance between the two workpieces is the greatest.

Spot welding machine

As shown in 48.28 (*b*), mechanical pressure is applied by the tips of the two electrodes. In fact, these electrodes not only provide the forging pressure but also carry the welding current and concentrate the welding heat on the weld spot directly below them.

Fig. 48.28 (*a*) shows diagrammatically the basic parts of a modern spot welding. It consists of a step-down transformer which can supply huge currents (upto 5,000 A) for short duration of time.

The lower arm is fixed whereas the upper one is movable. The electrodes are made of low-resistance, hard-copper alloy and are either air cooled or butt-cooled by water circulating through the rifled drillings in the electrode. Pointed electrodes [Fig. 48.29 (*a*)] are used for ferrous materials

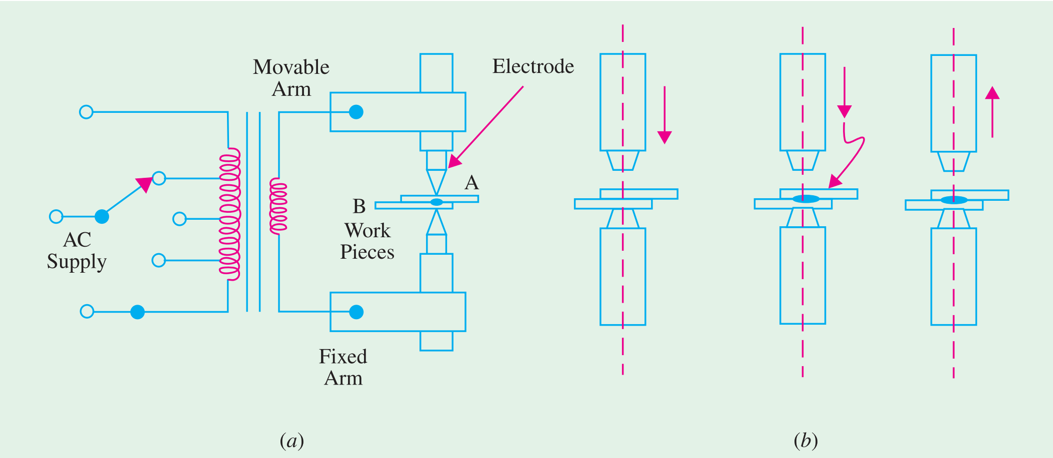

Fig. 48.28

whereas domed electrodes [Fig. 48.25 (*b*)] are used for non-ferrous materials. Flat domes are used when spot-welding deformation is not desired. The weld size is determined by the diameter of the electrode.

The welding machine is cycled in order to produce the required heat timed to coincide with the pressure exerted by the electrodes as shown in Fig. 48.28 (*a*). As the movable electrode comes down and presses the two workpieces *A* and *B* together, current is passed through the assembly. The metals under the pressure zone get heated upto about 950°C and fuse together. As they fuse, their resistance is reduced to zero, hence there is a surge of current. This surge is made to switch off the welding current automatically. In motor-driven machines, speeds of 300 strokes/minute are common.

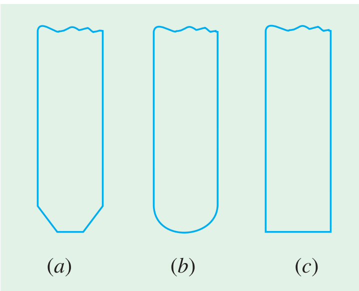

Fig. 48.29

Spot welders are of two different types. One is a stationary welder which is available in different sizes. The other has a stationary transformer but the electrodes are in a gun form.

Electric resistance spot welding is probably the best known and most widely-used because of its low cost, speed and dependability. It can be easily performed by even a semi-skilled operator. This process has a fast welding rate and quick set-up time apart from having low unit cost per weld.

Spot welding is used for galvanized, tinned and lead-coated sheets and mild steel sheet work. This technique is also applied to non-ferrous materials such as brass, aluminium, nickel and bronze etc.

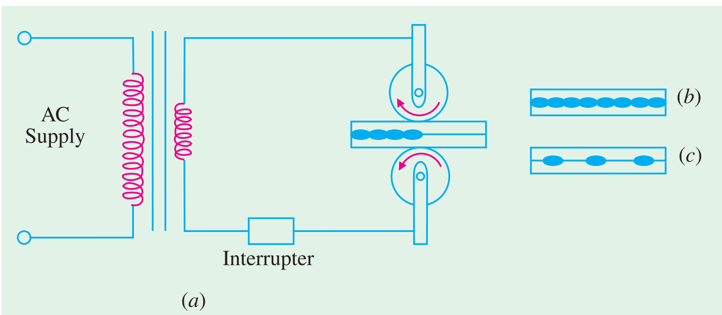

Fig. 48.30

48.28. Seam Welding

The seam welder differs from ordinary spot welder only in respect of its electrodes which are of disc or roller shape as shown in Fig. 48.30 (*a*). These copper wheels are power driven and rotate whilst gripping the work. The current is so applied through the wheels that the weld spots either overlap as in Fig. 48.30 (*b*) or are made at regular intervals as in Fig. 48.30 (*c*). The continuous or overlapped seam weld is also called ***stitch weld*** whereas the other is called roll weld.

Seam welding is confined to welding of thin materials ranging in thickness from 2 mm to 5 mm. It is also restricted to metals having low hardenability rating such as hot-rolled grades of low-alloy steels. Stitch welding is commonly used for long water-tight and gas-tight joints. Roll welding is used for simple joints which are not water-tight or gas-tight. Seam welds are usually tested by pillow test.

48.29. Projection Welding

It can be regarded as a mass-production form of spot welding. Technically, it is a cross between spot welding and butt welding. It uses the same equipment as spot welding. However, in this process, large-diameter flat electrodes (also called platens) are used. This welding process derives its name from the fact that, prior to welding, projections are raised on the surfaces to be welded [Fig. 48.31 (*a*)]. As seen, the upper and lower platens are connected across the secondary of a step-down transformer and are large enough to cover all the projections to be welded at one stroke of the machine. When platen *A* touches the workpiece, welding current flows ***through each projection***.

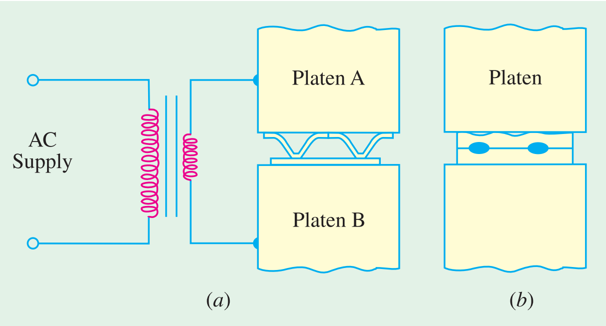

Fig. 48.31

The welding process is started by first lowering the upper platen *A* on to the work-piece and then applying mechanical pressure to ensure correctly-forged welds. Soon after, welding current is switched on as in spot welding. As projection areas heat up, they collapse and union takes place at all projections simultaneously [Fig. 48.31 (*b*)].

It is seen that projections serve many purposes :

1. They increase the welding resistance of the material locally.
2. They accurately locate the positions of the welds.
3. They speed up the welding process by making it possible to perform several small welds simultaneously.
4. They reduce the amount of current and pressure needed to form a good bond between two surfaces.
5. They prolong the life of the electrode considerably because the metal itself controls the heat produced.

Projection welding is used extensively by auto manufactures for joining nuts, bolts and studs to steel plates in car bodies. This process is especially suitable for metals like brass, aluminium and copper etc. manily due to their high thermal conductivity.

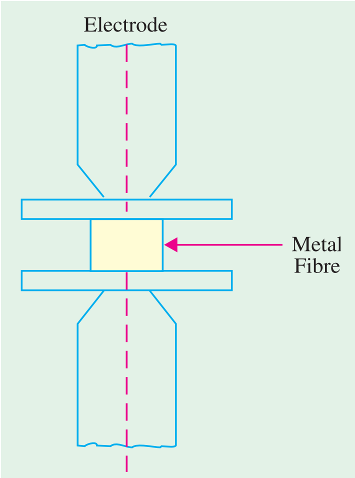

Fig. 48.32

A variation of projection welding is the metal fibre welding which uses a metal fibre rather than a projection point (Fig. 48.32). This metal fibre is generally a felt material. Instead of projections, tiny elements of this felt material are placed between the two metals which are then projection-welded in the usual way.

48.30. Butt Welding

In this case, the two workpieces are brought into contact end-to-end and the butted ends are heated by passing a heavy current through the joint. As in other forms of resistance welding, the weld heat is produced mainly by the electrical resistance of the joint faces. In this case, however, the electrodes are in the form of powerful vice clamps which hold the work-pieces and also convey the forging pressure to the joint [Fig. 48.33].

This process is useful where parts have to be joined end-to-end or edge-to-edge. *i.e*. for welding pipes, wires and rods. It is also employed for making continuous lengths of chain.

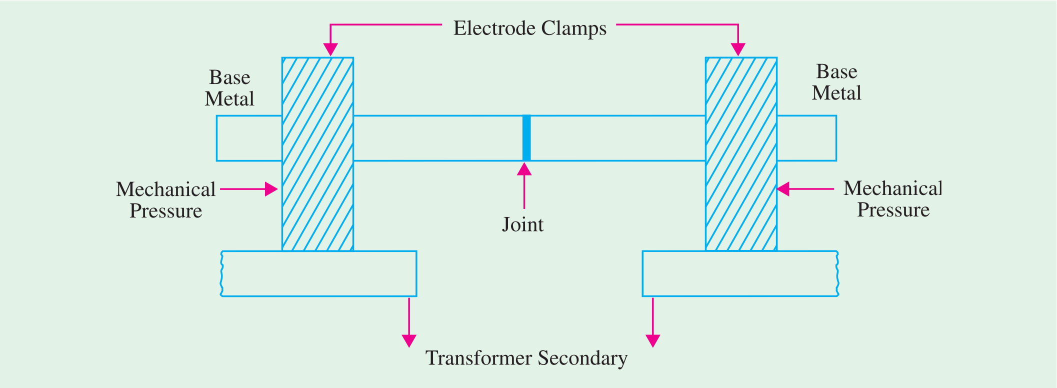

Fig. 48.33

48.31. Flash Butt Welding

It is also called by the simple name of ***flash welding***. It is similar to butt welding but with the difference that here current is applied when ends of the two metal pieces are quite close to each other ***but do not touch intimately***. Hence, an arc or flash is set up between them which supplies the necessary welding heat. As seen, in the process heat is applied ***before*** the two parts are pressed together.

As shown in Fig. 48.34 (*a*), the workpieces to be welded are clamped into specially designed electrodes one of which is fixed whereas the other is movable. After the flash has melted their faces, current is cut off and the movable platen applies the forging pressure to form a fusion weld. As shown in Fig. 48.34 (*b*), there is increase in the size of the weld zone because of the pressure which forces the soft ends together.

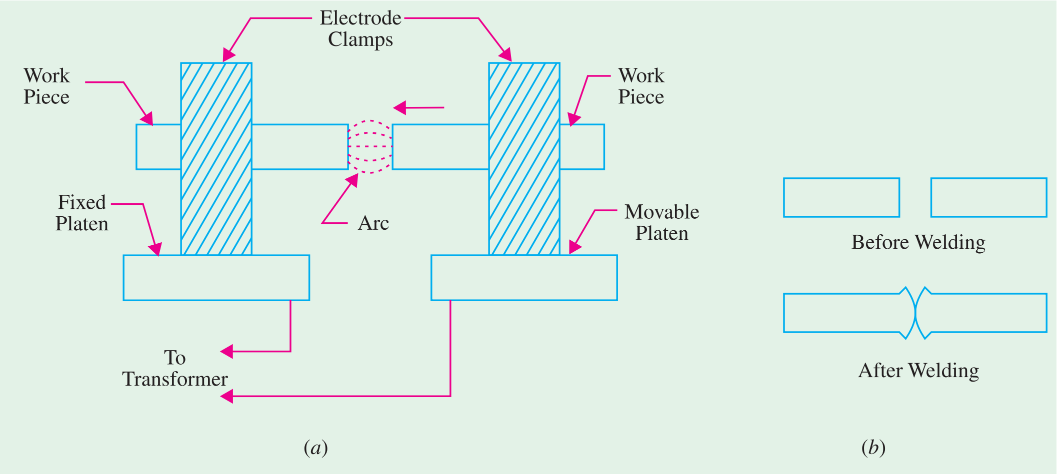

Fig. 48.34

Advantages

1. Even rough or irregular ends can be flash-welded. There is no need to level them by machining and grinding because all irregularities are burnt away during flashing period.
2. It is much quicker than butt welding.
3. It uses considerably less current than butt welding.
4. One of its major advantages is that dissimilar metals with different welding temperatures can be flash-welded.

Applications

1. To assemble rods, bars, tubings, sheets and most ferrous metals.
2. In the production of wheel rims for automobiles and bicycles.
3. For welding tubular parts such as automobile break cross-shafts.
4. For welding tube coils for refrigeration plants etc.

48.32. Upset Welding

In this process, ***no flash is allowed to occur*** between the two pieces of the metals to be welded. When the two base metals are brought together to a single interface, heavy current is passed between them which heats them up. After their temperature reaches a value of about 950°C, the two pieces of base metal are pressed together more firmly. This pressing together is called ***upsetting***. This upsetting takes place ***while current is flowing and continues even after current is switched off***. This upsetting action mixes the two metals homogeneously while pushing out many atmospheric impurities.

48.33. Stud Welding

(*a*) Basic Principle

It is similar to flash welding because it incorporates a method of drawing an arc between the stud (a rod) and the surface of the base metal. Then, the two molten surfaces are brought together under pressure to form a weld. Stud welding eliminates the need for drilling holes in the main structure.

(*b*) Welding Equipment

The stud welding equipment consists of a stud welding gun, a d.c. power supply capable of giving currents upto 400 A, a device to control current and studs and ferrules which are used not only as arc shields but also as containing walls for the molten metal.

(*c*) Applications

It is a low-cost method of fastening extensions (studs) to a metal surface. Most of the ferrous and non-ferrous metals can be stud-welded successfully. Ferrous metals include stainless steel, carbon steel and low-alloy steel. Non-ferrous metals include aluminium, lead-free brass, bronze and chrome-plated metals.

Stud welding finds application in the installations of conduit pipe hangers, planking and corrugated roofings.

This process is also used extensively in shipbuilding, railroad and automotive industries.

48.34. Plasma Arc Welding

(*a*) Basic Principle

It consists of a high-current electronic arc which is forced through a small hole in a water-coled metallic nozzle [Fig. 48.35 (*a*)]. The plasma gas itself is used to protect the nozzle from the extreme heat of the arc. The plasma arc is shielded by inert gases like argon and helium which are pumped through an extra passageway within the nozzle of the plasma torch. As seen, plasma arc consists of electronic arc, plasma gas and gases used to shield the jet column. The idea of using the nozzle is to

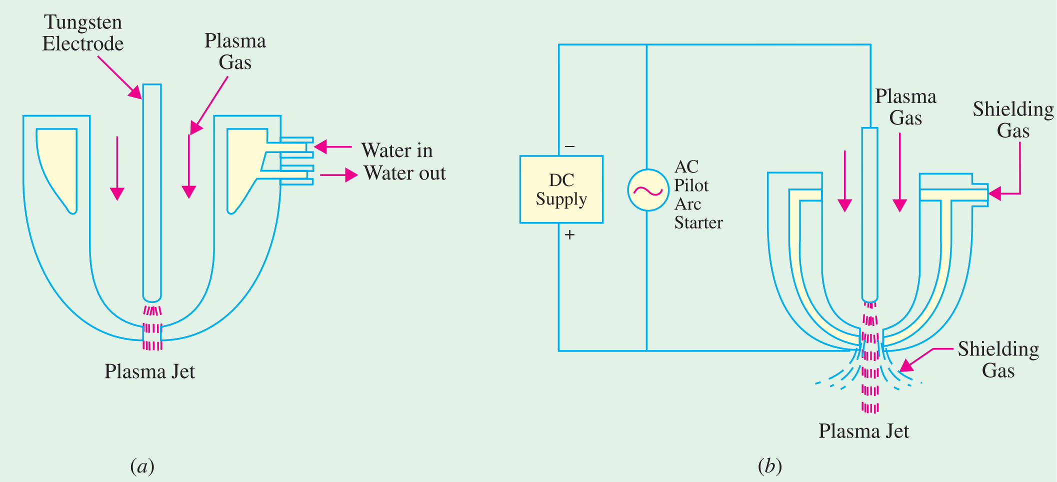

Fig. 48.35

constrict the arc thereby increasing its pressure. Collision of high-energy electrons with gas molecules produces the plasma which is swept through the nozzle and forms the current path between the electrode and the workpiece. Plasma jet torches have temperature capability of about 35,000°C.

(*b*) Electrodes

For stainless steel welding and most other metals, straight polarity tungsten electrodes are used. But for aluminium welding, reverse polarity water-cooled copper electrodes are used.

Plasma arc welding

(*c*) Power Supply

Plasma arc welding requires d.c. power supply which could be provided either by a motor-generator set or transformer-rectifier combination. The latter is preferred because it produces better arc stability. The d.c. supply should have an open-circuit voltage of about 70V and drooping voltage-ampere characteristics. A high-frequency pilot arc circuit is employed to start the arc [Fig. 48.35 (*b*)].

(*d*) Method of Welding

Welding with plasma arc jet is done by a process called 'keyhole' method. As the plasma jet strikes the surface of the workpiece, it burns a hole through it. As the torch progresses along the work-piece, this hole also progresses alongwith but is filled up by the molten metal as it moves along. obviously, 100 percent penetration is achieved in this method of welding. Since plasma jet melts a large surface area of the base metal, it produces a weld bead of wineglass design as shown in Fig. 48.36. The shape of the bead can be changed by changing the tip of the nozzle of the torch. Practically, all welding is done mechanically.

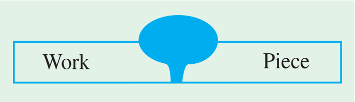

Fig. 48.36

(*e*) Applications

1. Plasma arc welding process has many aerospace applications.
2. It is used for welding of reactive metals and thin materials.
3. It is capable of welding high-carbon steel, stainless steel, maraging steel, copper and copper alloys, brass alloys, aluminium and titanium.
4. It is also used for metal spraying.

5. It can be modified for metal cutting purposes. It has been used for cutting aluminium, carbon steel, stainless steel and other hard-to-cut steels. It can produce high-quality dross-free aluminium cuts 15 cm deep.

(*f*) Disadvantages

1. Since it uses more electrical equipment, it has higher electrical hazards.
2. It produces ***ultra-violet and infra-red*** radiations necessitating the use of tinted lenses.
3. It produces high-pitched noise (100 dB) which makes it necessary for the operator to use ear plugs.

48.35. Electroslag Welding

(*a*) General

It is a metal-arc welding process and may be considered as a further development of sub-merged-arc welding.

This process is used for welding joints of thick sections of ferrous metals in a single pass and without any special joint preparation. Theoretically, there is no upper limit to the thickness of the weld bead. It is usually a vertical uphill process.

It is called ***electroslag*** process because heat is generated by passing current through the molten slag which floats over the top of the metal.

(*b*) Welding Equipment

As shown in Fig. 48.37, two water-cooled copper shoes (or dams) are placed on either side of the joint to be welded for the purpose of confining the molten metal in the joint area. The electrode is fed into the weld joint almost vertically from special wire guides. There is a mechanical device which raises the shoes and wire-feed mechanism as the weld continues upwards till it is completed. An a.c. welding machine has 100 percent duty cycle and which can supply currents upto 1000 A if needed.

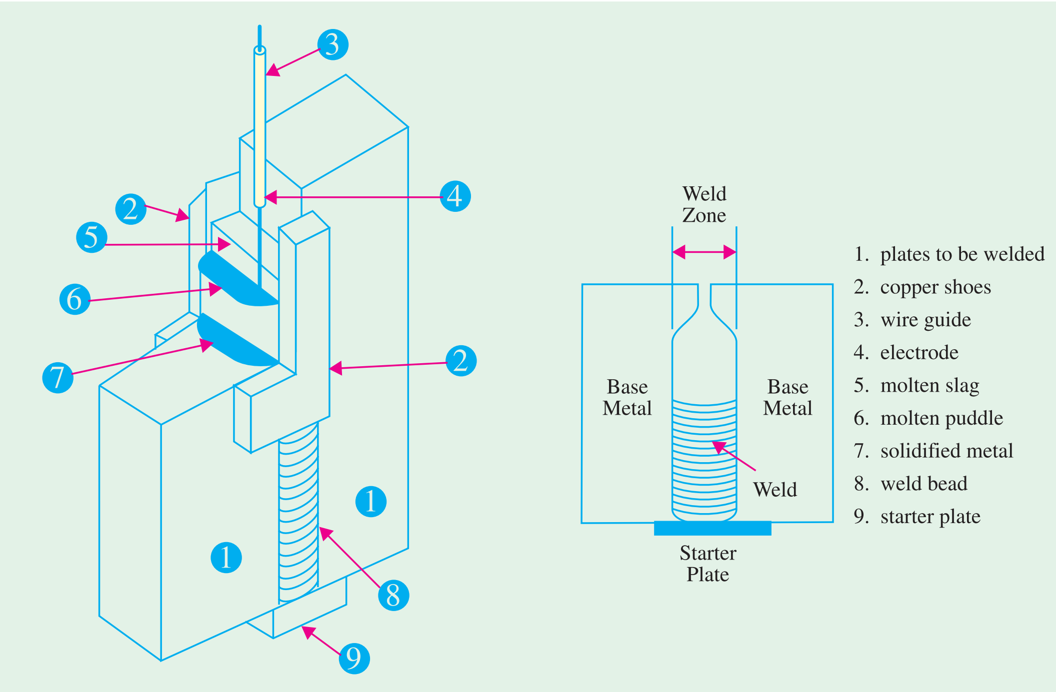

Fig. 48.37

(*c*) Welding Process

The electroslag process is initiated just like submerged arc process by starting an electric arc beneath a layer of granular welding flux. When a sufficient thick layer of hot flux or molten slag is formed, arc action stops and from then onwards, current passes from the electrode to workpiece through the molten slag. At this point, the process becomes truly electroslag welding. A starting plate is used in order to build up proper depth of conductive slag before molten pool comes in contact with the workpieces.

The heat generated by the resistance to the flow of current through the molten slag is sufficient to melt the edges of the workpiece and the filler electrode. The molten base metal and filler metal collect at the bottom of the slag pool forming the weld pool. When weld pool solidfies, weld bead is formed which joins the faces of the base metal as shown in Fig. 48.33 (*b*).

As welding is continued upwards, flux flows to the top in the form of molten slag and cleanses the impurities from the molten metal. A mechanism raises the equipment as the weld is completed in the uphill vertical position.

(*d*) Advantages

1. It needs no special joint preparation.
2. It does welding in a single pass rather than in costly multiple passes.
3. There is theoretically no maximum thickness of the plate it can weld.
4. There is also no theoretical upper limit to the thickness of the weld bead. Weld beads upto 400 mm thick have been performed with the presenttly-available equipment.
5. This process requires less electrical power per kg of deposited metal than either the submerged arc welding process or the shield arc process.
6. It has high deposit rate of upto 20 kg of weld metal per hour.
7. It has lower flux consumption.
8. Due to uniform heating of the weld area, distortion and residual stresses are reduced to the minimal amounts.

However, for electroslag welding, it is necessary to have only a square butt joint or a square edge on the plates to be welded.

(*e*) Applications

It is commonly used in the fabrication of large vessels and tanks. Low-carbon steels produce excellent welding properties with this process.

48.36. Electrogas Welding

This process works on the same basic principle as the electroslag process but has certain additional features of submerged arc welding. Unlike electroslag process, the electrogas process uses an inert gas for shielding the weld from oxidation and there is a continuous arc (as in submerged arc process) to heat the weld pool.

48.37. Electron Beam Welding

In this process, welding operation is performed in a vacuum chamber with the help of a sharply-focussed beam of high-velocity electrons. The electrons after being emitted from a suitable electrode are accelerated by the high anode voltage and are then focussed into a fine beam which is finally directed to the workpiece. Obviously, this process needs no electrodes. The electron beam produces intense local heat which can melt not only the metal but can even boil it. A properly-focussed electron beam can completly penetrate through the base metal thereby creating a small hole whose walls are molten. As the beam moves along the joint, it melt the material coming in contact with it. The molten metal flows back to the previously-melted hole where it fuses to make a perfect weld for the entire depth of penetration.

Electron-beam welding has following advantages :

1. It produces deep penetration with little distortion.
2. Its input power is small as compared to other electrical welding devices.
3. Electron-beam weld is much narrower than the fusion weld.
4. It is especially suitable for reactive metals which become contaminated when exposed to air because this process is carried out in vacuum.
5. It completely eliminates the contamination of the weld zone and the weld bead because operation is performed in a vacuum chamber.

Electron beam welding facility

6. It is especially suited to the welding of beryllium which is being widely used in the fabrication of industrial and aerospace components.
7. Its high deposition rate produces welds of excellent quality with only a single pass.
8. It is the only process which can join high temperature metals such as columbium.

At present, its only serious limitations are that it is extremely expensive and is not available in portable form. However, recently a non-vacuum electron-beam welder has been developed.

48.38. Laser Welding

It uses an extremely concentrated beam of coherent monochromatic light *i.e.* light of only one colour (or wavelength). It concentrates tremendous amount of energy on a very small area of the workpiece to produce fusion. It uses solid laser (ruby, saphire), gas laser (CO_2) and semiconductor laser. Both the gas laser and solid laser need capacitor storage to store energy for later injection into the flash tube which produces the required laser beam.

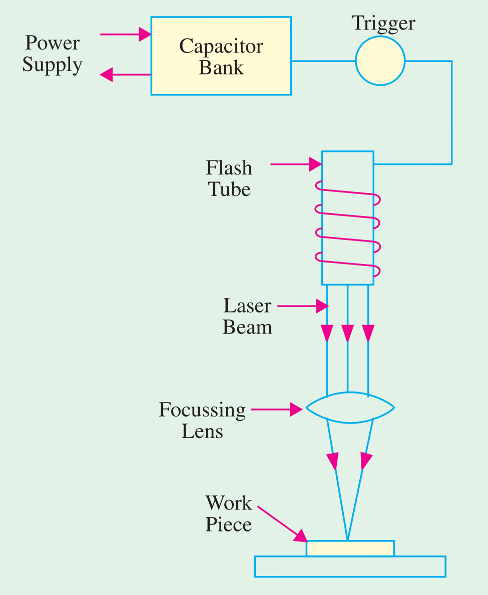

Fig. 48.38

The gas laser welding equipment consists of **(*i*)** capacitor bank for energy storage **(*ii*)** a triggering device **(*iii*)** a flash tube that is wrapped with wire **(*iv*)** lasing material **(*v*)** focussing lens and **(*vi*)** a worktable that can rotate in the three X, Y and Z directions.

When triggered, the capacitor bank supplies electrical energy to the flash tube through the wire. This energy is then converted into short-duration beam of laser light which is pin-pointed on the workpiece as shown in Fig. 48.38. Fusion takes place immediately and weld is completed fast.

Since duration of laser weld beam is very short (2 ms or so), two basic welding methods have been adopted. In the first method, the workpiece is moved so fast that the entire joint is welded in a single burst of the light. The other method uses a number of pulses one after the other to form the weld joint similar to that formed in electric resistance seam welding (Art 48.31).

Laser welding is used in the aircraft and electronic industries for lighter gauge metals.

Some of the advantages of laser welding process are as follows :

1. It does not require any electrode.
2. It can make welds with high degree of precision and on materials as thin as 0.025 mm.
3. It does not heat the workpiece except at one point. In fact, heat-affected zone is virtually non-existent.
4. Liquidus is reached only at the point of fusion.
5. It can produce glass-to-metal seals as in the construction of klystron tubes.
6. Since laser beam is small in size and quick in action, it keeps the weld zone uncontaminated.
7. It can weld dissimilar metals with widely varying physical properties.
8. It produces minimal thermal distortion and shrinkage because area of heat-affected zone is the minimum possible.
9. It can easily bond refractory materials like molybdenum, titanium and tantalium etc.

However, the major disadvantage of this process is its slow welding speed. Moreover, it is limited to welding with thin metals only.

Tutorial Problem No. 48.1

1. Describe various types of electric arc welding processes.
(J.N. University, Hyderabad, December 2002/January 2003)
2. Compare resistance welding and arc welding.
(J.N. University, Hyderabad, December 2002/January 2003)
3. Briefly explain the different methods of electric welding and state their relative merits.
(J.N. University, Hyderabad, December 2002/January 2003)
4. Give the comparison between A.C. and D.C. welding.
(J.N. University, Hyderabad, December 2002/January 2003)
5. Explain the different methods of electric welding and their relative advantages.
(J.N. University, Hyderabad, December 2002/January 2003)

OBJECTIVE TESTS – 48

1. The basic *electrical* requirement in arc welding is that there should be
   (*a*) coated electrodes
   (*b*) high open-circuit voltage
   (*c*) no arc blow
   (*d*) d.c. power supply.
2. Welding is not done directly from the supply mains because
   (*a*) it is customary to use welding machines
   (*b*) its voltage is too high
   (*c*) its voltage keeps fluctuating
   (*d*) it is impracticable to draw heavy currents.
3. A.C. welding machine cannot be used for welding
   (*a*) MIG
   (*b*) atomic hydrogen
   (*c*) resistance
   (*d*) submerged arc.
4. In electric welding, arc blow can be avoided by
   (*a*) using bare electrodes
   (*b*) welding away from earth ground connection
   (*c*) using a.c. welding machines
   (*d*) increasing arc length.

5. In DCSP welding
(*a*) electrode is the hottest
(*b*) workpiece is relatively coole
(*c*) base metal penetration is deep
(*d*) heavily-coated electrodes are used.
6. Overhead welding position is thought to be the most
(*a*) hazardous
(*b*) difficult
(*c*) economical
(*d*) useful.
7. The *ultimate* aim of using electrode coating is to
(*a*) provide shielding to weld pool
(*b*) prevent atmospheric contamination
(*c*) improve bead quality
(*d*) cleanse the base metal.
8. In electrode-positive welding .... of the total heat is produced at the electrode.
(*a*) one-third
(*b*) two-third
(*c*) one-half
(*d*) one-fourth.
9. Submerged arc process is characterised by
(*a*) deep penetration
(*b*) high welding current
(*c*) exceptionally smooth beads
(*d*) all of the above.
10. The major disadvantage of carbon arc welding is that
(*a*) there is occurrence of blow holes
(*b*) electrodes are consumed fast
(*c*) separate filler rod is needed
(*d*) bare electrodes are necessary.
11. In atomic hydrogen welding, electrodes are long-lived because
(*a*) two are used at a time
(*b*) arc is in the shape of a fan
(*c*) of a.c. supply
(*d*) it is a non-pressure process.
12. Unlike TIG welding, MIG welding
(*a*) requires no flux
(*b*) uses consumable electrodes
(*c*) provides complete protection from atmospheric contamination
(*d*) requires no post-weld cleansing.
13. The major power supply used in MIG welding is
(*a*) a.c. supply
(*b*) DCSP
(*c*) electrode-negative
(*d*) DCRP.
14. MIG welding process is becoming increa- singly popular in welding industry mainly because of
(*a*) its easy operation
(*b*) its high metal deposit rate
(*c*) its use in both ferrous and non-ferrous metals.
(*d*) both (*a*) and (*b*).
15. A weld bead of wineglass design is produced in .... welding.
(*a*) plasma arc
(*b*) electron beam
(*c*) laser
(*d*) MAG.
16. Spot welding process basically depends on
(*a*) Ohmic resistance
(*b*) generation of heat
(*c*) application of forging pressure
(*d*) both (*b*) and (*c*).
17. Electric resistance seam welding uses .... electrodes.
(*a*) pointed
(*b*) disc
(*c*) flat
(*d*) domed.
18. Projection welding can be regarded as a mass production form of .... welding.
(*a*) seam
(*b*) butt
(*c*) spot
(*d*) upset.
19. In the process of electroslag welding, theoretically there is no upper limit to the
(*a*) thickness of weld bead
(*b*) rate of metal deposit
(*c*) slag bath temperature
(*d*) rate of slag consumption.
20. High temperature metals like columbium can be easily welded by..................... welding.
(*a*) flash
(*b*) MIG
(*c*) TIG
(*d*) electron beam.
21. During resistance welding heat produced at the joint is proportional to
(*a*) I^2R
(*b*) kVA
(*c*) current
(*d*) voltage
22. Grey iron in usually welded by .... welding
(*a*) gas
(*b*) arc

(c) resistance
(d) MIG

23. The metal surfaces, for electrical resistance welding must be.......
(a) lubricated
(b) cleaned
(c) moistened
(d) rough

24. In a welded joint poor fusion is due to which of the following?
(a) Improper current
(b) High welding speed
(c) Uncleaned metal surface
(d) Lack of flux

25. For arc welding, D.C. is produced by which of the following?
(a) motor-generator set
(b) regulator
(c) transformer
(d) none of the above

26. ..... welding process uses consumable electrodes.
(a) TIG
(b) MIG
(c) Laser
(d) All of the above

27. Which of the following equipment is generally used for arc welding?
(a) single phase alternator
(b) two phase alternator
(c) three phase alternator
(d) transformer

28. Which of the following is not an inert gas?
(a) argon
(b) carbon dioxide
(c) helium
(d) all of the above

29. Electronic components are joined by which of the following methods?
(a) brazing
(b) soldering
(c) seam welding
(d) spot welding
(e) none of the above

30. Resistance welding cannot be used for
(a) dielectrics
(b) ferrous materials
(c) non-ferrous metals
(d) any of the above

31. Electric arc welding process produces temperature upto
(a) 1000^oC
(b) 1500^oC
(c) 3500^oC
(d) 5550^oC

32. Increased heat due to shorter arc is harmful on account of
(a) under-cutting of base material
(b) burn through
(c) excessive porosity
(d) all of the above

33. Arc blow results in which of the following?
(a) Non-uniform weld beads
(b) Shallow weld puddle given rise to weak weld
(c) Splashing out of metal from weld puddle
(d) All of the above defects

34. Inseam welding
(a) the work piece is fixed and disc electrodes move
(b) the work piece moves but rotating electrodes are fixed
(c) any of the above
(d) none of the above

35. In arc welding major personal hazards are
(a) flying sperks
(b) weld spatter
(c) harmful infrared and ultra-violet rays from the arc
(d) all of the above

36. In spot welding composition and thickness of the base metal decides
(a) the amount of squeeze pressure
(b) hold time
(c) the amount of weld current
(d) all above

37. Helium produces which of the following?
(a) deeper penetration
(b) faster welding speeds
(c) narrower heat affected zone in base metal
(d) all of the above

38. Due to which of the following reasons aluminium is difficult to weld?
(a) it has an oxide coating
(b) it conducts away heat very rapidly
(c) both (a) and (b)
(d) none of the above

39. Welding leads have
(a) high flexibility
(b) high current handling capacity
(c) both (a) and (b)
(d) none of the above

40. Air craft body is
(a) spot welded

(*b*) gas welded
(*c*) seam welded
(*d*) riveted

41. For arc welding current range is usually
(*a*) 10 to 15 A
(*b*) 30 to 40 A
(*c*) 50 to 100 A
(*d*) 100 to 350 A

42. Spot welding is used for
(*a*) thin metal sheets
(*b*) rough and irregular surfaces
(*c*) costings only
(*d*) thick sections

43. Galvanising is a process of applying a layer of
(*a*) aluminium
(*b*) lead
(*c*) copper
(*d*) zinc

44. A seamless pipe has
(*a*) steam welded joint
(*b*) spot welded joint
(*c*) arc welded joint
(*d*) no joint

45. Motor-generator set for D.C. arc welding has generator of
(*a*) series type
(*b*) shunt type
(*c*) differentially compound type
(*d*) level compound type

46. Plain and butt welds may be used on materials upto thickness of nearly
(*a*) 5 mm
(*b*) 10 mm
(*c*) 25 mm
(*d*) 50 mm

47. In argon arc welding argon is used as a
(*a*) flux
(*b*) source of heat
(*c*) agent for heat transfer
(*d*) shield to protect the work from oxidation

48. During arc welding as the thickness of the metal to be welded increases
(*a*) current should decrease, voltage should increase
(*b*) current should increases, voltage remaining the same
(*c*) current should increase, voltage should decrease
(*d*) voltage should increase, current remaining the same

49. In D.C. arc welding
(*a*) electrode is made positive and workpiece negative
(*b*) electrode is made negative and workpiece positive
(*c*) both electrode as well as workpiece are made positive
(*d*) both electrode as well as workpiece are made negative

50. The purpose of coating on arc welding electrodes is to
(*a*) stabilise the arc
(*b*) provide a protecting atmosphere
(*c*) provide slag to protect the molten metal
(*d*) all of the above

51. 50 percent duty cycle of a welding machine means
(*a*) machine input is 50 percent of rated input
(*b*) machine efficiency is 50 percent
(*c*) machine work on 50 percent output
(*d*) machine works for 5 minutes in a duration of 10 minutes

ANSWERS

1. (*b*)	**2.** (*d*)	**3.** (*a*)	**4.** (*c*)	**5.** (*c*)	**6.** (*a*	**7.** (*c*)	**8.** (*b*)	**9.** (*d*)
10. (*a*)	**11.** (*c*)	**12.** (*b*)	**13.** (*d*)	**14.** (*d*)	**15.** (*a*)	**16.** (*d*)	**17.** (*b*)	**18.** (*c*)
19. (*a*)	**20.** (*d*)	**21.** (*a*)	**22.** (*a*)	**23.** (*b*)	**24.** (*a*)	**25.** (*a*)	**26.** (*b*)	**27.** (*d*)
28. (*b*)	**29.** (*b*)	**30.** (*a*)	**31.** (*d*)	**32.** (*d*)	**33.** (*d*)	**34.** (*c*)	**35.** (*d*)	**36.** (*d*)
37. (*d*)	**38.** (*c*)	**39.** (*d*)	**40.** (*c*)	**41.** (*d*)	**42.** (*a*)	**43.** (*d*)	**44.** (*d*)	**45.** (*d*)
46. (*c*)	**47.** (*d*)	**48.** (*b*)	**49.** (*b*)	**50.** (*d*)	**51.** (*d*)			

GO To FIRST

CONTENTS

CHAPTER 49

ILLUMINATION

Learning Objectives

- Radiations from a Hot Body
- Solid Angle
- Definitions
- Calculation of Luminance (L)
- Laws of Illumination or Illuminance
- Polar Curves of C.P. Distribution
- Uses of Polar Curves
- Determination of M.S.C.P and M.H.C.P. from Polar Diagrams
- Integrating Sphere or Photometer
- Diffusing and Reflecting Surfaces
- Lighting Schemes
- Illumination Required for Different Purposes
- Space / Height Ratio
- Design of Lighting Schemes and Layouts
- Utilisation Factor ([h]
- Depreciation Factor (P)
- Floodlighting
- Artificial Source of Light
- Incandescent Lamp
- Filament Dimensions
- Incandescent Lamp Characteristics
- Clear and Inside
- Frosted Gas-filled Lamps
- Discharge Lamps
- Sodium Vapour Lamp

When some materials are heated above certain temperatures, they start radiating energy in the form of light. This phenomenon is called luminance. Electric lamps are made based on this phenomenon.

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49.1. Radiations From a Hot Body

The usual method of producing artificial light consists in raising a solid body or vapour to incandescence by applying heat to it. It is found that as the body is gradually heated above room temperature, it begins to radiate energy in the surrounding medium in the form of electromagnetic waves of various wavelengths. The ***nature*** of this radiant energy depends on the temperature of the hot body. Thus, when the temperature is low, radiated energy is in the form of heat waves only but when a certain temperature is reached, light waves are also radiated out in addition to heat waves and the body becomes luminous. Further increase in temperature produces an increase in the amount of both kinds of radiations but the colour of light or visible radiation changes from bright red to orange, to yellow and then finally, if the temperature is high enough, to white. As temperature is increased, the wavelength of the visible radiation goes on becoming shorter. It should be noted that heat waves are identical to light waves except that they are of longer wavelength and hence produce no impression on the retina. Obviously, from the point of view of light emission, heat energy represents so much wasted energy.

The ratio $\dfrac{\text{energy radiated out in the form of light}}{\text{total energy radiated out by the hot body}}$ is called the ***radiant*** efficiency of the luminous source and, obviously, depends on the temperature of the source. As the temperature is increased beyond that at which light waves were first given off, the radiant efficiency increases, because light energy will increase in greater proportion than the total radiated energy. When emitted light becomes white *i.e.*, it includes all the visible wavelengths, from extreme red to extreme violet, then a further increase in temperature produces radiations which are of wavelength smaller than that of violet radiations. Such radiations are invisible and are known as ultra-violet radiations. It is found that maximum radiant efficiency would occur at about 6200°C and even then the value of this maximum efficiency would be 20%. Since this temperature is far above the highest that has yet been obtained in practice, it is obvious that the actual efficiency of all artificial sources of light *i.e.* those depending on ***temperature incandescence***, is low.

As discussed above, light is radiant energy which is assumed to be propagated in the form of transverse waves through an invisible medium known as ether. These light waves travel with a velocity of 2.99776×10^8 m/s or 3×10^8 m/s approximately but their wavelengths are different. The wavelength of red light is nearly 0.000078 cm and that of violet light 0.000039 cm. Since these wavelengths are very small, instead of using 1 cm as the unit for their measurement, a submultiple 10^{-8} cm is used. This submultiple is known as Angstrom Unit (A.U.)

$$1 \text{ A.U.} = 10^{-8} \text{ cm} = 10^{-10} \text{ m}$$

Hence, the wave-length of red light becomes $\lambda_r = 7800 \times 10^{-10}$ m or 7800 A.U. and $\lambda_v = 3900 \times 10^{-10}$ m or 3900 A.U. The sensation of colour is due to the difference in the wavelengths and hence frequencies of the light radiations.

49.2. Solid Angle

Consider an area A which is part of a sphere of radius r (Fig. 49.1). Let us find the solid angle ω subtended by this area at the centre C of the sphere. For this purpose, let point C be joined to every point on the edges of the area A. Then, the angle enclosed by the cone at point C gives the solid angle. Its value is

$$\omega = \frac{A}{r^2} \text{ steradian}$$

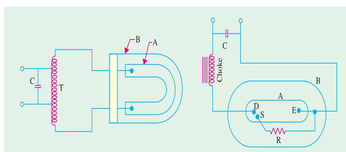


Fig. 49.1

The unit of solid angle is ***steradian*** (sr). If, in the above equation, $A = r^2$, then $\omega = 1$ steradian. Hence, steradian is defined as the angle subtended at the centre of a sphere by a part of its surface having an area equal to $(\text{radius})^2$.

Obviously, the solid angle subtended at the centre by whole of the spherical surface = $4\pi\, r^2/r^2 = 4\pi$ steradian (sr).

Fireworks radiate light energy of different frequencies, which appear in different colours

49.3. Definitions

Before proceeding further, definitions of a few principal terms employed in connection with illumination, are given below :

1. Candela. It is the unit of luminous intensity of a source. It is defined as 1/60th of the luminous intensity per cm^2 of a black body radiator at the temperature of solidification of platinum (2045°K).

A source of one candela (*cd*) emits one lumen per steradian. Hence, total flux emitted by it allround is $4\pi \times 1 = 4\pi$ lumen.

2. Luminous Flux (F or Φ). It is the light energy radiated out per second from the body in the form of luminous light waves.

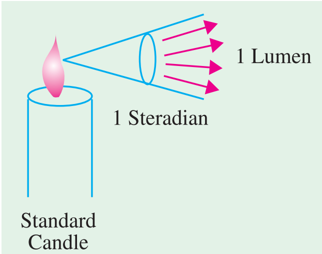


Fig. 49.2

Since, it is a rate of flow of energy, it is a sort of *power* unit. Unit of luminous flux is *lumen* (lm). It is defined as the ***flux contained per unit solid angle of a source of one candela or standard candle*** (Fig. 49.2).

Approximate relation between lumen and electric unit of power *i.e.* watt is given as

$$1 \text{ lumen} = 0.0016 \text{ watt (approx.)}$$

3. Lumen-hour. It is the quantity of light delivered in one hour by a flux of one lumen.*

4. Luminous Intensity (I) or Candle-power of a point source in any particular direction is given by the ***luminous flux radiated out per unit solid angle in that direction***. In other words, it is solid angular flux density of a source in a specified direction.

If $d\Phi$ is the luminous flux radiated out by a source within a solid angle of $d\omega$ steradian in any particular direction, then $I = d\Phi/d\omega$.

If flux is measured in lumens and solid angle in steradian, then its unit is lumen/steradian (lm/sr) or candela (*cd*).

If a source has an average luminous intensity of I lm/sr (or I candela), then total flux radiated by it all around is $\Phi = \omega I = 4\pi I$ lumen.

Generally, the luminous intensity or candle power of a source is different in different directions. The average candle-power of a source is the average value of its candle power in all directions. Obviously, it is given by total flux (in lm) emitted in all directions in all planes divided by 4π. This average candle-power is also known as ***mean spherical candle-power*** (M.S.C.P.).

$$\therefore \quad \text{M.S.C.P.} = \frac{\text{total flux in lumens}}{4\,\pi}$$

If the average is taken over a hemisphere (instead of sphere), then this average candle power is known as ***mean hemispherical candle-power*** (M.H.S.C.P.).

It is given by the total flux emitted in a hemisphere (usually the lower one) divided by the solid angle subtended at the point source by the hemisphere.

$$\therefore \quad \text{M.H.S.C.P.} = \frac{\text{flux emitted in a hemisphere}}{2\,\pi}$$

* It is similar to watt-hour (Wh)

5. Reduction Factor of a source is given by the ratio, f = M.S.C.P./M.H.C.P. where M.H.C.P. is the mean horizontal candle power.

It is also referred to as spherical reduction factor.

6. Illuminance or Illumination (E). When the luminous flux falls on a surface, it is said to be illuminated. The illumination of a surface is measured by the normal luminous flux per unit area received by it.

If $d\Phi$ is the luminous flux incident normally on an area dA, then $E = d\Phi/dA$ or $E = \Phi/A$.

Unit. Since flux Φ is measured in lumens and area in m^2, unit of E is lm/m^2 or lux. The alternative name is metre-candle (m-cd). Let us see why ? Imagine a sphere of radius of one metre around a point source of one candela. Flux radiated out by this source is 4π lumen. This flux falls normally on the curved surface of the sphere which is $= 4\pi$m^2. Obviously, illumination at every point on the inner surface of this sphere is $4\pi\ lm/4\pi\ \text{m}^2 = 1\ lm/\text{m}^2$. However, the term lm/m^2 is to be perferred to metre-candle.

7. Luminance (L) of an Extended Source. Suppose ΔA is an element of area of an ***extended*** source and Δt its luminous intensity when viewed in a direction making an angle ϕ with the perpendicular to the surface of the source (Fig. 49.3), then luminance of the source element is given by

$$L = \frac{\Delta I}{\Delta A \cos\phi} = \frac{\Delta I}{\Delta A'}\ cd/\text{m}^2 \quad ...(i)$$

where $\Delta A' = \Delta A \cos\phi$

= area of the source element projected onto a plane perpendicular to the specified direction.

As will be seen from Art. 49.5.

$$E = \frac{I\cos\theta}{d^2} \quad \text{or} \quad \Delta E = \frac{\Delta I}{d^2}\cos\theta$$

Substituting the value of ΔI from Eq. (i) above, we get

$$\Delta E = \frac{L.\Delta A'}{d^2}\cos\theta = L\cos\theta.\ d\omega$$

where $d\omega = \Delta A'/d^2$ steradian

$$E = \int L\cos\theta\ .\ d\omega = L\int\cos\theta\ .\ d\omega$$

—if L is constant.

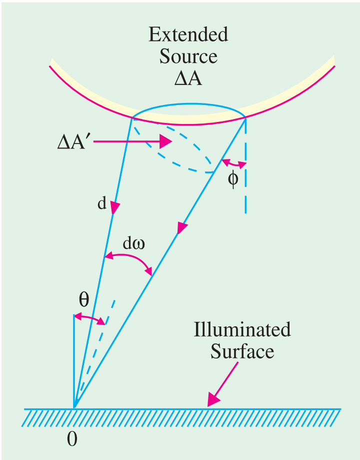


Fig. 49.3

8. Luminous Exitance (M) of a Surface. The luminous exitance (M) at a point on a surface is defined as luminous flux emitted per unit area in all directions. If an element of an illuminated area ΔA emits a total flux of $\Delta\Phi$ in all directions (over a solid angle of 2π steradian) then

$$M = \Delta\Phi/\Delta A \qquad lm/\text{m}^2$$

It can be proved that $M = \pi L$ in the case of a uniform diffuse ***source***.

9. Transmittance (T) of an Illuminated Diffuse Reflecting Surface. It is defined as the ratio of the total luminous flux transmitted by it to the total flux incident on it.

The relation between luminous exitance (M) of a surface transmitting light and illuminance (E) on the other side of it is

$$M = TE \quad \text{or} \quad T = M/E$$

Since light falling on a surface is either transmitted, reflected or absorbed the following relation holds good

$$T + \rho + \alpha = 1$$ where α is the absorptance of the surface.

10. Reflection Ratio or Coefficient of Reflection or Reflectance (ρ). It is given by the luminous flux reflected from a small area of the surface to the total flux incident upon it

$$\rho = M/E \text{ i.e. ratio of luminous exitance and illuminance.}$$

It is always less than unity. Its value is zero for ideal 'black body' and unity for a perfect reflector.

11. Specific Output or Efficiency of a lamp is the ratio of luminous flux to the power intake. Its unit is lumen/watt (lm/W). Following relations should be taken note of :

(*a*) $\dfrac{\text{lumen}}{\text{watt}} = \dfrac{4\pi \times \text{M.S.C.P.}}{\text{watt}}$

or $\dfrac{\text{lm}}{\text{W}} = \dfrac{4\,\pi}{\text{watt/M.S.C.P.}}$

(*b*) since $f = \text{M.S.C.P./M.H.C.P.}$ $\therefore$ $\text{lm/W} = \dfrac{4\,\pi\, f}{\text{watt/M.S.C.P.}}$

(*c*) Obviously, $\text{watts/M.S.C.P.} = \dfrac{4\,\pi}{\text{lm/W}} = \dfrac{\text{watt/M.H.C.P.}}{f}$

(*d*) Also $\text{watts/M.H.C.P.} = \dfrac{4\,\pi\, f}{\text{lm/W}} = f \times \text{watts/M.S.C.P.}$

12. Specific Consumption. It is defined as the ratio of the power input to the average candle-power. It is expressed in terms of watts per average candle or watts/M.S.C.P.

The summary of the above quantities along with their units and symbol is given in Table 49.1.

Table 49.1

Name of Qty	*Unit*	*Symbols*
Luminous Flux	Lumen	*F* or Φ
Luminous Intensity (candle-power)	Candela	*I*
Illumination or Illuminance	lm/m^2 or lux	*E*
Luminance or Brightness	cd/m^2	*L* or *B*
Luminous Exitance	lm/m^2	*M*

49.4. Calculation of Luminance (L) of a Diffuse Reflecting Surface

The luminance (or brightness) of a surface largely depends on the character of the surface, if it is itself not the emitter. In the case of a polished surface, the luminance depends on the angle of viewing. But if the surface is matt and diffusion is good, then the luminance or brightness is practically independent of the angle of viewing. However, the reflectance of the surface reduces the brightness proportionately. In Fig. 49.4 is shown a perfectly diffusing surface of small area *A*. Suppose that at point *M* on a hemisphere with centre *O* and radius *R*, the illuminance is *L* cd/m^2. Obviously, ***luminous intensity*** at point *M* is $= L \times A \cos\theta$ candela (or lumen/steradian). Now, the hemisphere can be divided into a number of zones as shown. Consider one such zone *MN* between θ and (θ + *d*θ). The width of this zone is $R.d\theta$ and length $2\pi R \sin\theta$ so that its area (shown shaded) is $= 2\pi R^2 \sin\theta.\, d\theta$. Hence, it subtends a solid angle $= 2\pi R^2 \sin\theta . d\theta / R^2 = 2\pi \sin\theta.\, d\theta$ steradian at point *O*. The luminous flux passing through this zone is

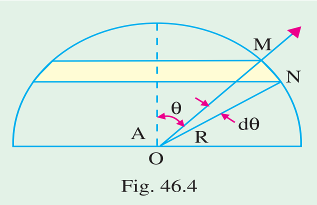


Fig. 49.4

$$d\Phi = L\ A\cos\theta \times 2\pi \sin\theta . d\,\theta = \pi L\ A \times 2 \sin\theta \cos\theta\ d\,\theta = \pi\, L\, A \sin 2\,\theta\ d\theta \text{ lumen}$$

Total flux passing through the whole hemisphere is

$$\Phi = \int_0^{\pi/2} \pi L A \sin 2\,\theta\, .\, d\theta = \pi\, L\, A \text{ lumen}$$

If the ***illumination*** of the surface (produced by a light source) is E lm/m^2 and ρ is its reflection coefficient, then $\Phi = \rho\, A\, E$ lumen.

Equating the two values of flux, we have $\pi L A = \rho A\, E$

or $\quad \boldsymbol{L} = \rho E/\pi$ cd/m^2 = ρE lm/m^2

For example, consider a perfectly diffusing surface having $\rho = 0.8$ and held at a distance of 2 metres from a source of luminous intensity 100 candela at right angles to the direction of flux. Then

$$\boldsymbol{E} = 100/22 = 25 \text{ lm/m}^2$$
$$\boldsymbol{L} = \rho E/\pi = 25 \times 0.8/\pi = 6.36 \text{ cd/m}^2 = 636 \times \pi = 20 \text{ lm/m}^2$$

49.5. Laws of Illumination or Illuminance

The illumination ***(E)*** of a surface depends upon the following factors. The source is assumed to be a point source or is otherwise sufficiently away from the surface to be regarded as such.

(i) ***E*** is directly proportional to the luminous intensity ***(I)*** of the source or $E \propto I$

(ii) ***Inverse Square Law***. The illumination of a surface is inversely proportional to the square of the distance of the surface from the source.

In other words, $E \propto 1/r^2$

Proof

In Fig. 49.5 are shown portions of the surfaces of three spheres whose radii are in the ratio 1 : 2 : 3. All these portions, obviously, subtend the same solid angle at the source and hence receive the same amount of total flux. However, since their areas are in the ratio of 1 : 4 : 9, their illuminations are in the ratio $1 : \frac{1}{4} : \frac{1}{9}$.

(iii) ***Lambert's Cosine Law***. According to this law, E is directly proportional to the cosine of the ***angle made by the normal to the illuminated surface with the direction of the incident flux***.

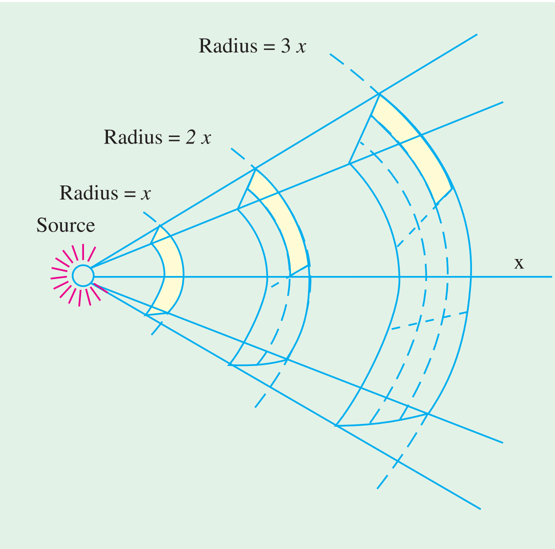


Fig. 49.5

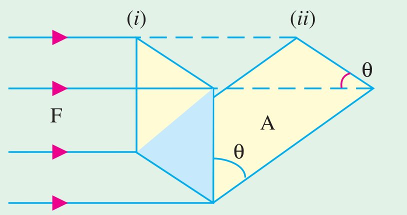


Fig. 49.6

Proof

As shown in Fig. 49.6, let Φ be the flux incident on the surface of area A when in position 1. When this surface is turned back through an angle θ, then the flux incident on it is $\Phi \cos \theta$. Hence, illumination of the surface when in position 1 is $E_1 = \Phi/A$. But when in position 2.

$$E_2 = \frac{\Phi \cos\theta}{A} \qquad \therefore \qquad E_2 = E_1 \cos\theta$$

Combining all these factors together, we get $E = I \cos\theta/r^2$. The unit is lm/m^2.

The above expression makes the determination of illumination possible at a given point provided the position and the luminous intensity or candle power (in the given direction) of the source (or sources) by which it is illuminated, are known as illustrated by the following examples.

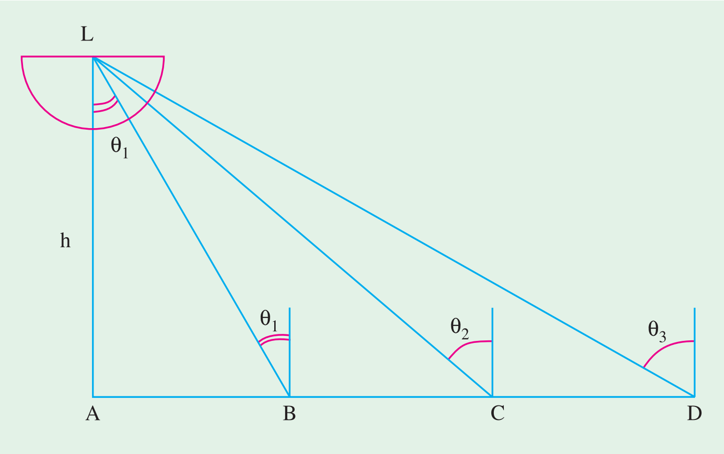


Fig. 49.7

Consider a lamp of uniform luminous intensity suspended at a height h above the working plane as shown in Fig. 49.7. Let us consider the value of illumination at point A immediately below the lamp and at other points B,C,D etc., lying in the working plane at different distances from A.

$$E_A = \frac{I}{h^2} \text{ —since } \theta = 0 \text{ and } \cos\theta = 1$$

$$E_B = \frac{I}{LB^2} \times \cos\theta_1. \quad \text{Since,} \quad \cos\theta_1 = h/LB$$

$$\therefore \quad E_B = \frac{I}{LB^2} \times \frac{h}{LB} = I \times \frac{h}{LB^2} = \frac{1}{h^2} \cdot \frac{h^3}{LB^3} = \frac{1}{h^2}\left(\frac{h}{LB}\right)^3$$

Now $\frac{1}{h^2} = E_A$ and $\left(\frac{h}{LB}\right)^3 = \cos^3\theta_1$

$$\therefore \quad E_B = E_A \cos^3\theta_1$$

Similarly, $E_C = E_A \cdot \cos^3\theta_2$ and $E_D = E_A \cos^3\theta_3$ and so on.

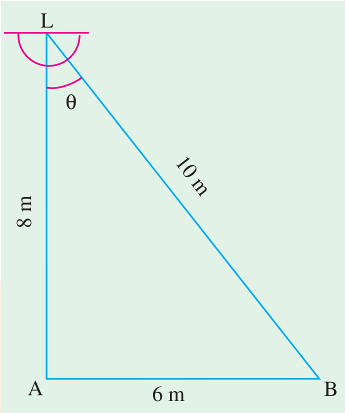


Fig. 49.8

Example 49.1. *A lamp giving out 1200 lm in all directions is suspended 8 m above the working plane. Calculate the illumination at a point on the working plane 6 m away from the foot of the lamp.* **(Electrical Technology, Aligarh Muslim Univ.)**

Solution. Luminous intensity of the lamp is

$$I = 1200/4\pi = 95.5 \text{ cd}$$

As seen from Fig. 49.8.

$$L_B = \sqrt{8^2 + 6^2} = 10 \text{ m} ; \cos\theta = 8/10 = 0.8$$

Now, $E = I \cos\theta/r^2$

$$\therefore \quad E_B = 95.5 \times 0.8/10^2 = \mathbf{0.764 \ lm/m^2}$$

Example 49.2. *A small light source with intensity uniform in all directions is mounted at a height of 10 metres above a horizontal surface. Two points A and B both lie on the surface with point*

A directly beneath the source. How far is B from A if the illumination at B is only 1/10 as great as at A ? **(A.M.I.E.)**

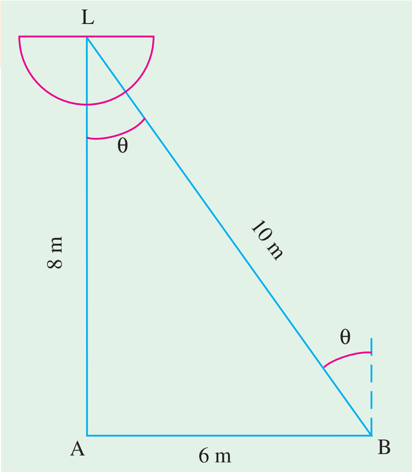


Fig. 49.9

Solution. Let the intensity of the lamp be I and the distance between A and B be x metres as shown in Fig. 49.9.

Illumination at point $A, E_A = I/10^2 = I/100$ lux

Illumination at point B,

Illumination at point B,

$$E_B = \frac{I}{10^2} \times \left[\frac{10}{\sqrt{10^2 + x^2}}\right]^2 = \frac{10\,I}{(100 + x^2)^{3/2}}$$

Since $E_B = \dfrac{E_A}{10}$,

$$\therefore \quad \frac{10\,I}{(100 + x^2)^{3/2}} = \frac{1}{10} \times \frac{I}{100}, \qquad \therefore \quad x = \mathbf{19.1\ m}$$

Example 49.3. *A corridor is lighted by 4 lamps spaced 10 m apart and suspended at a height of 5 m above the centre line of the floor. If each lamp gives 200 C.P. in all directions below the horizontal, find the illumination at the point on the floor mid-way between the second and third lamps.*

(Electrical Engineering, Bombay Univ.)

Solution. As seen from 49.10, illumination at point C is due to all the four lamps. Since point C is symmetrically situated between the lamps, illumination at this point is twice that due to L_1 and L_2.

(*i*) illumination due to $L_1 = I \cos\theta_1 / L_1C^2 \quad L_1C = \sqrt{5^2 + 15^2} = 15.8$ m

$\cos\theta = 5/15.8$

illumination due to $L_1 = \dfrac{220 \times (5/15.8)}{250} = \mathbf{0.253\ lm/m^2}$

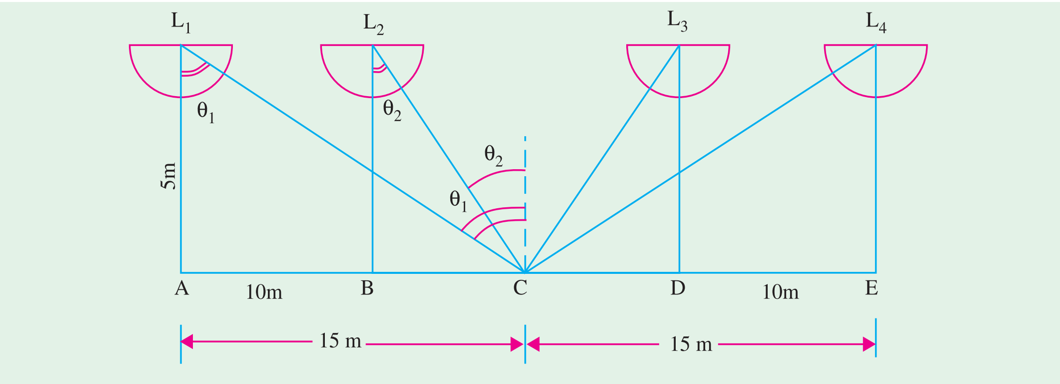


Fig. 49.10

(*ii*) $L_2C = 5/\sqrt{2}$ m ; $\theta_2 = 45°$; $\cos\theta_2 = 1/\sqrt{2}$

Illumination due to $L_2 = \dfrac{200 \times 1/\sqrt{2}}{50} = 2.83$ lm/m^2

$\therefore$ illumination at C due to L_1 and $L_2 = 3.08$ lm/m^2

Illumination at C due to all the four lamps, $E_C = 2 \times 3.08 = \mathbf{6.16\ lm/m^2}$

Example 49.4. *Two lamps A and B of 200 candela and 400 candela respectively are situated 100 m apart. The height of A above the ground level is 10 m and that of B is 20 m. If a photometer is placed at the centre of the line joining the two lamp posts, calculate its reading.*

(Electrical Technology, Gujarat Univ.)

Solution. When the illumination photometer is placed at the centre point, it will read the value of combined illumination produced by the two lamps (Fig. 49.11).

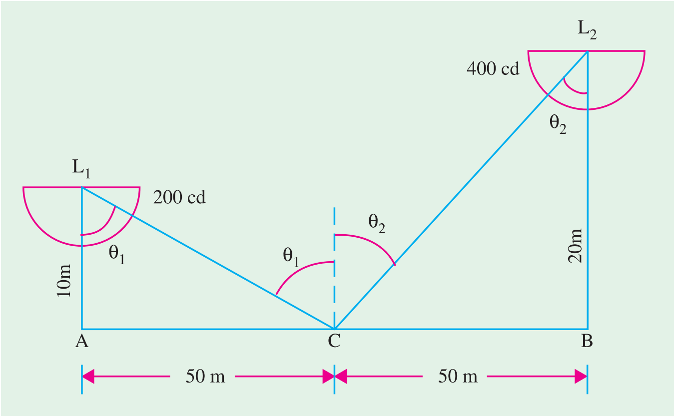


Fig. 49.11

Now, $L_1C = \sqrt{10^2 + 50^2}$

$= 51$ m

$L_2C = \sqrt{20^2 + 50^2}$

$= 53.9$ m

$\cos\theta_1 = 10/51$;

$\cos\theta_2 = 20/53.9$

Illumination at point C due to lamp L_1

$$= \frac{200 \times 10}{51 \times 2600}$$

$= \mathbf{0.015\ lm/m^2}$

Similarly, illumination due to lamp L_2

$$= \frac{400 \times 20/53.9}{2900} = \mathbf{0.051\ lm/m^2}$$

$\therefore \quad E_C = 0.015 + 0.051$

$= \mathbf{0.066\ lm/m^2}$ **or lux**

Example 49.5. *The average luminous output of an 80-W fluorescent lamp 1.5 metre in length and 3.5 cm diameter is 3300 lumens. Calculate its average brightness. If the auxiliary gear associated with the lamp consumes a load equivalent to 25 percent of the lamp, calculate the cost of running a twin unit for 2500 hours at 30 paise per kWh.*

Solution. Surface area of the lamp $= \pi \times 0.035 \times 1.5 = 0.165\ m^2$

Flux emitted per unit area $= 3300/0.165 = 2 \times 10^4\ lm/m^2$

$$\therefore \quad B = \frac{\text{flux emitted per unit erea}}{\pi}\ cd/m^2 = 2 \times \frac{10^2}{\pi} = \mathbf{6,382\ cd/m^2}$$

Total load of twin fitting $= 2[80 + 0.25 \times 80] = 200$ W

Energy consumed for 2500 hr $= 2500 \times 200 \times 10^{-3} = 500$ kWh

cost $=$ Rs. $500 \times 0.3 =$ **Rs. 150.**

Example 49.6. *A small area 7.5 m in diameter is to be illuminated by a lamp suspended at a height of 4.5 m over the centre of the area. The lamp having an efficiency of 20 lm/w is fitted with a reflector which directs the light output only over the surface to be illuminated, giving uniform candle power over this angle. Utilisation coefficient = 0.40. Find out the wattage of the lamp. Assume 800 lux of illumination level from the lamp.* **(A.M.I.E.)**

Solution. $A = \pi d^2/4 = 44.18\ m^2$, $E = 800$ lux

Luminous flux reaching the surface $= 800 \times 44.18 = 35,344$ lm

Total flux emitted by the lamp = 35,344/0.4 = 88,360 lm

Lamp wattage = 88,360/20 = **4420 W**

Example 49.7. *A lamp of 100 candela is placed 1 m below a plane mirror which reflects 90% of light falling on it. The lamp is hung 4 m above ground. Find the illumination at a point on the ground 3 m away from the point vertically below the lamp.*

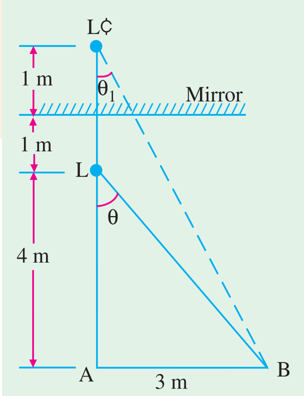


Fig. 49.12

Solution. The lamp and mirror arrangement is shown in Fig. 49.12. The lamp L produces an image L' as far behind the mirror as it is in front. Height of the image from the ground is (5 + 1) = 6m. L' acts as the secondary source of light and its candle power is = 0.9 × 100 = 90 candela.

Illumination at point B equals the sum of illumination due to L and that due to L'.

$$\therefore \quad E_B = \frac{100}{(LB)^2} \times \cos\theta + \frac{90}{(L'B)^2}\cos\theta_1$$

$$= \frac{100}{5^2} \times \frac{4}{5} \times \frac{90}{45} \times \frac{6}{\sqrt{45}} = \textbf{5 lux}$$

Example. 49.8. *A light source having an intensity of 500 candle in all directions is fitted with a reflector so that it directs 80% of its light along a beam having a divergence of 15°. What is the total light flux emitted along the beam? What will be the average illumination produced on a surface normal to the beam direction at a distance of 10 m?* **(A.M.I.E.)**

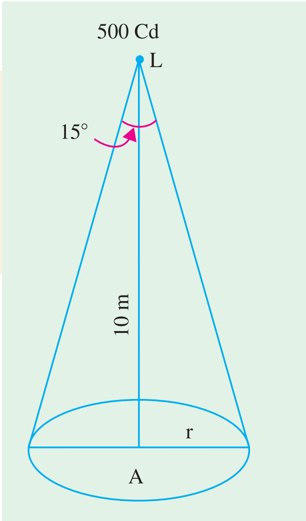


Fig. 49.13

Solution. Total flux emitted along the beam = 0.8 (4π × 500) = **5,227 lm**

Beam angle, θ = 15°, l = 10 m

Radius of the circle to be illuminated, $r = l \tan\theta/2$

= 10 tan 15°/2 = 1.316 m

Area of the surface to be illuminated, $A = \pi r^2 = \pi \times 1.316^2$

= 5.44 m^2

∴ Average illumination = 5227/5.44 = **961 lux**

Example 49.9. *A lamp has a uniform candle power of 300 in all directions and is fitted with a reflector which directs 50% of the total emitted light uniformly on to a flat circular disc of 20 m diameter placed 20 m vertically below the lamp. Calculate the illumination (a) at the centre and (b) at the edge of the surface without the reflector. Repeat these two calculations with the reflector provided.* **(Electrical Engg., Grad I.E.T.E.)**

Solution. It should be noted that the formula $E = I \cos\theta/r^2$ will not be applicable when the reflector is used. Moreover, with reflector, illumination would be uniform.

Without Reflector

(*a*) $E = 300 \times 1/20^2$ = **0.75 lm/m²**

(*b*) Here, $\theta = \tan^{-1}(10/20) = 26.6°$, $\cos\theta = 0.89$; $x^2 = 10^2 + 20^2 = 500$

∴ $E = 300 \times 0.89/500$ = **0.534 lm/m²**

With Reflector

Luminous output of the lamp = 300 × 4π lumen

Flux directed by the reflector $= 0.5 \times 1200\,\pi = 600\,\pi$ lm

Illumination produced on the disc $= 600\,\pi/100\,\pi =$ **6 lm/m²**

It is the same at every point of the disc.

Example 49.10. *A light is placed 3 m above the ground and its candle power is 100 cos θ in any downward direction making an angle q with the vertical. If P and Q are two points on the grond, P being vertically under the light and the distance PQ being 3 m, calculate.*

(a) the illumination of the ground at P and also at Q.

(b) the total radiations sent down by the lamp.

Solution. With reference to Fig. 49.14

(*a*) C.P. along $LP = 100 \times \cos 0° = 100$ cd $\quad\therefore\quad E_P = 100/3^2 =$ **11.1 lm/m²**

C.P. along $LQ = 100 \times \cos 45° = 70.7 \quad\therefore\quad E_Q = 70.7 \times \cos 45°/18 =$ **1.39 lm/m²**

(*b*) Consider an imaginary hemisphere of radius *r* metre at whose centre lies the given lamp (Fig. 49.15).

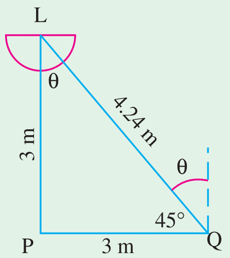


Fig. 49.14

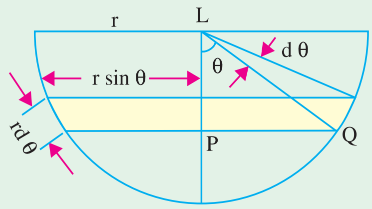


Fig. 49.15

C.P. along $LQ = 100 \cos\theta \quad\therefore\quad E_Q = 100 \cos\theta/r^2$

The area of the elementary strip at an angular distance θ from the vertical and of width $PQ = r.\,d\theta$ is $= (2\pi\, r \sin\theta) \times r.d\theta = 2\pi r^2 \sin\theta\, d\theta$.

Flux incident on the shaded area

$$= \frac{100\cos\theta}{r^2} 2\pi\, r^2 \sin\theta.\, d\theta = 100\,\pi.\, 2\sin\theta.\cos\theta\, d\theta = 100\,\pi \sin 2\theta\, d\theta.$$

Total flux over the hemisphere can be obtained by integrating the above expression between proper limits.

$$\therefore \qquad \text{total flux} = \int_0^{\pi/2} 100\,\pi \sin 2\theta\, d\theta = 100\,\pi \left| -\frac{\cos 2\theta}{2} \right|_0^{\pi/2} = \frac{100\,\pi}{2}$$

$= 100\,\pi =$ **314.lm.**

Example 49.11. *A drawing office containing a number of boards and having a total effective area of 70 m² is lit by a number of 40 W incandescent lamps giving 11 lm/W. An illumination of 80 lux is required on the drawing boards. Assuming that 60% of the total light emitted by the lamps is available for illuminating the drawing boards, estimate the number of lamps required.*

Solution. Let *N* be the number of 40 W lamps required.

Output/lamp $= 40 \times 11 = 440$ lm ; Total flux $= 440\,N$ lm

Flux actually utilized $= 0.6 \times 440\,N = 264\,N$ lm

Illumination required $= 80$ lux $= 80$ lm/m²

Total flux required at the rate of 80 lm/m² $= 80 \times 70$ lm $= 5600$ lm

$264\,N = 5600 \quad\therefore\quad N =$ **21**

Example 49.12. *A perfectly diffusing surface has a luminous intensity of 10 candles at an angle of 60º to the normal. If the area of the surface is 100 cm^2, determine the brightness and total flux radiated.*

Solution. Brightness B is defined as the luminous intensity divided by the projected area (Fig. 49.16).

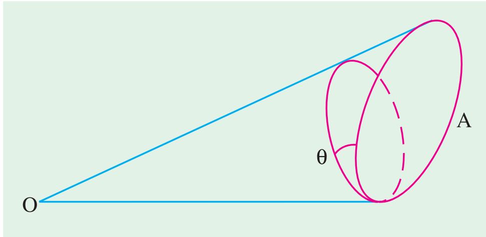


Fig. 49.16

$$\text{Luminous intensity} = 10 \text{ candela}$$
$$\text{Projected area} = A \cos\theta$$
$$= 100 \times \cos 60^\circ$$
$$= 50 \text{ cm}^2$$
$$\therefore \quad B = 10/50 \times 10^{-4}$$
$$= 2 \times 10^3 \text{ cd/m}^2$$
$$= \mathbf{2\pi \times 10^3 \text{ lm/m}^2} \text{—Art. 46.4}$$
$$\text{Total flux radiated} = 2\pi \times 10^3 \times 100 \times 10^{-4}$$
$$= \mathbf{68.2 \text{ lm}}$$

Example 49.13. *Calculate the brightness (or luminance) of snow under an illumination of* ***(a)*** *44,000 lux and* ***(b)*** *0.22 lux. Assume that snow behaves like a perfect diffusor having a reflection factor of 85 per cent.*

Solution. **(*a*)** $L = \rho E/\pi = 44{,}000 \times 0.85/\pi \text{ cd/m}^2 = \mathbf{1.19 \times 10^4 \text{ cd/m}^2}$

(*b*) $L = \dfrac{0.22 \times 0.85}{\pi} = \mathbf{5.9 \times 10^{-2} \text{ cd/m}^2}$

Example 49.14. *A 21 cm diameter globe of dense opal glass encloses a lamp emitting 1000 lumens and has uniform brightness of 4×10^3 lumen/m^2 when viewed in any direction. What would be the luminous intensity of the globe in any direction? Find what percentage of the flux emitted by the lamp is absorbed by the globe.*

Solution. Surface area of the globe $= \pi d^2 = \pi \times 21^2 = 1{,}386 \text{ cm}^2 = 0.1386 \text{ m}^2$

Flux emitted by the globe is $= 0.1386 \times 4 \times 10^3 = 554.4$ lumen

Now, 1 candela $= 4\pi$ lumens

Hence, luminous intensity of globe is $= 554.4/4\pi =$ **44 cd**

Flux absorbed by the globe is $= 1000 - 554.4 = 445.6$ lm.

$\therefore$ percentage absorption $= 445.6 \times 100/1000 =$ **44.56%**

Example 49.15. *A 2.5 cm diameter disc source of luminance 1000 cd/cm^2 is placed at the focus of a specular parabolic reflector normal to the axis. The focal length of the reflector is 10 cm, diameter 40 cm and reflectance 0.8. Calculate the axial intensity and beam-spread. Also show diagrammatically what will happen if the source were moved away from the reflector along the axis in either direction.* **(A.M.I.E. Sec. B, Winter 1991)**

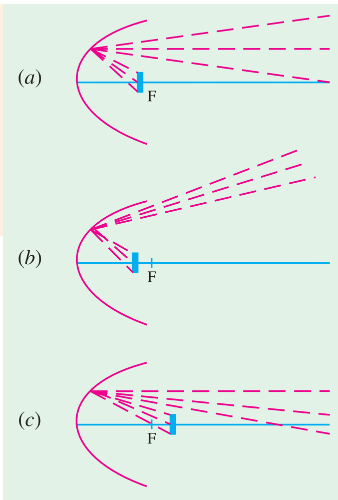


Fig. 49.17

Solution. The axial or beam intensity I depends upon

(*i*) luminance of the disc source *i.e.* L

(*ii*) aperture of the reflector *i.e.* A

(*iii*) reflectivity of the reflector *i.e.* r

$\therefore \quad I = \rho A L$ candela

Now, $\quad L = 1000 \text{ cd/cm}^2 = 10^7 \text{ cd/m}^2$

$A = \pi d^2/4 = \pi \times 0.4^2/4 = 125.7 \times 10^{-3} \text{ m}^2$

$\therefore \quad I = 0.8 \times 125.7 \times 10^{-3} \times 10^7 =$ **1,005,600 cd**

To a first approximation, the beam spread for disc source is determined by reflector focal length and the size of the disc source. If θ is the total beam spread when the source is at the focus of the reflector [Fig. 49.17] (*a*)] then

$$\theta = 2 \tan^{-1} (r/f)$$

Here, $r = 2.5/2 = 1.25$ cm ; $f = 10$ cm

$\therefore$ $\theta = 2 \tan^{-1}(1.25/10) = 2 \times 7°7' = $ **14°14′**

The effect of axial movement of the source is shown in Fig. 49.17 (*b*) and (*c*).

Example 49.16. *A 22 diameter globe of opal glass encloses a lamp of uniform luminous intensity 120 C.P. Thirty per cent of light emitted by the lamp is absorbed by globe. Determine* ***(a)*** *luminance of globe* ***(b)*** *C.P. of globe in any direction.*

Solution.

(*a*) Flux emitted by source $= 120 \times 4\pi$ lm ; flux emitted by globe $= 0.7 \times 480\ \pi$ lm

$$\therefore \quad L = \frac{0.7 \times 480\ \pi}{\pi \times 0.22^2} = \mathbf{6{,}940\ lm/m^2}$$

(*b*) Since 1 candela $= 4\ \pi$ lm

$\therefore$ candle-power or luminous intensity of the globe is

$$= \frac{\text{flux in lumens}}{4\ \pi} = \frac{0.7 \times 480\ \pi}{\pi \times 4} = \mathbf{84\ cd}$$

Example 49.17. *A 0.4 m diameter diffusing sphere of opal glass (20 percent absorption) encloses an incandescent lamp with a luminous flux of 4850 lumens. Calculate the average luminance of the sphere.* **(A.M.I.E. Sec. B, Summer 1993)**

Solution. Flux emitted by the globe 80% or 4850 = 3880 lm

Surface area of the globe $= 4\pi r^2 = \pi d^2 \text{m}^2$

$$B = \frac{\text{flux emitted}}{\text{surfacr area}} = \frac{3880}{\pi \times 0.4^2} = \mathbf{7{,}720\ lm/m^2}$$

Tutorial Problem No. 49.1

1. A high-pressure mercury-vapour lamp is mounted at a height of 6 m in the middle of a large road crossing. A special reflector directs 100 C.P. maximum in a cone of 70º to the vertical line. Calculate the intensity of illumination on the road surface due to this beam of 100 C.P. ***(Electrical Engineering, Bombay Univ.)***
2. A room 6m × 4 m is illuminated by a single lamp of 100 C.P. in all directions suspended at the centre 3 m above the floor level. Calculate the illumination (*i*) below the lamp and (*ii*) at the corner of the room. ***(Mech. & Elect. Engg. : Gujarat Univ.)***
3. A lamp of 100 candle-power is placed at the centre of a room 10 m × 6m × 4 m high. Calculate the illumination in each corner of the floor and at a point in the middle of a 6 m wall at a height of 2 m from the floor. ***(Utilization of Elect. Power A.M.I.E.)***
4. A source of 5000 lumen is suspended 6.1 m. above ground. Find out the illumination (*i*) at a point just below the lamp and (*ii*) at a point 12.2 m away from the first, assuming uniform distribution of light from the source. **[(*i*) 10.7 lux (*ii*) 0.96 lux]** ***(A.M.I.E. Sec. B)***
5. Determine the average illumination of a room measuring 9.15 m by 12.2 m illuminated by a dozen 150 W lamps. The luminous efficiency of lamps may be taken as 14 lm/W and the co-efficient of utilisation as 0.35. **[79 lux]** ***(A.M.I.E. Sec. B)***
6. Two lamps are hung at a height of 9 m from the floor level. The distance between the lamps is one metre. Lamp one is of 500 candela. If the illumination on the floor vertically below this lamp is 20 lux, find the candle power of the lamp number two. **[1140 candela]** ***(Utili. of Elect. Power A.M.I.E.)***

7. Two powerful street lamps of 1,000 candela and 800 candela (assumed uniform in all directions) are mounted 12.5 m above the road level and are spaced 25 metres apart. Find the intensity of horizontal illumination produced at a point on the ground in-between the lamp posts and just below the lamp posts. **[4.07 lux, 6.86 lux, 5.69 lux]** ***(A.M.I.E.)***
8. It is required to provide an illumination of 100 lux in a factory hall 30 m by 15 m. Assume that the depreciation factor is 0.8, coefficient of utilisation is 0.4 and efficiency of lamp is 14 lm/W. Suggest the number of lamps and their ratings. The sizes of the lamps available are 100, 250, 400 and 500 W. **[40 lamps of 250 W in 5 rows]**
9. It is required to provide an illumination of 100 lm/m^2 in a workshop hall 40m $\times$ 10m and efficiency of lamp is 14 lm/W. Calculate the number and rating of lamps and their positions when trusses are provided at mutual distance of 5m. Take coefficient of utilisation as 0.4 and depreciation factor as 0.8. **[14 lamps of 750 W each]**
10. A drawing hall 30 m by 15 m with a ceiling height of 5 m is to be provided with a general illumination of 120 lux. Taking a coefficient of utilization of 0.5 and depreciation factor of 1.4, determine the number of flourscent tubes required, their spacing, mounting height and total wattage.

    Taking luminous efficiency of flourescent tube as 40 Im/W for 80 W tubes.

[48, 24 twin-tube units each tube of 80 W; row spacing of 5 m and unit spacing of 3.75m, 3840 W] ***(Utilisation of Elect. Power, A.M.I.E.)***

49.6. Laws Governing Illumination of Different Sources

The laws applicable to the illumination produced by the following three types of sources will be considered.

(*i*) Point Source

As discussed in Art. 49.5, the law governing changes in illumination due to point source of light is $E = I \cos\theta/d^2$.

(*ii*) Line Source

Provided the line source is of infinite length and of uniform intensity, the illumination at a point lying on a surface parallel to and facing the line source is given by

$$E = \frac{\pi I}{2d} \text{ lm/m}^2$$

where I = luminous intensity normal to the line source (in candles per-meter length of the sources)

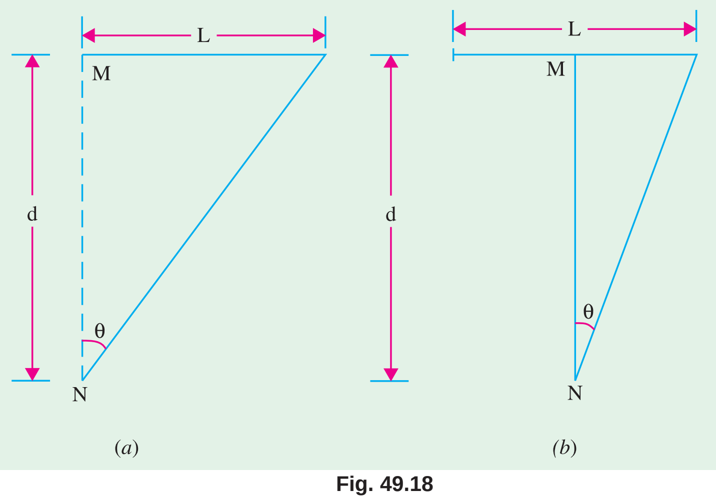


Fig. 49.18

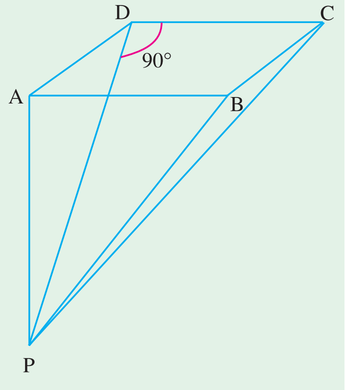


Fig. 49.19

However, in practice, the line sources are of finite length, so that the following law applies

$$E = \frac{I}{4\,d}\,(\sin 2\theta + 2\theta)\ \text{lm/m}^2 \qquad \text{—Fig. 49.18 }(a)$$

$$= \frac{I}{2\,d}\,(\sin 2\theta + 2\theta)\ \text{lm/m}^2 \qquad \text{—Fig. 49.18 }(b)$$

where I = candle power per metre length in a direction normal to the line source

$$= \frac{\Phi}{\pi^2 L}\ \text{cd/m}$$

where Φ is the total flux of the source in lumens and L is the length of the line source in metres.

(iii) Surface Source

Provided the surface source is of infinite area and of uniform brightness, illumination at any point facing the source is independant of the distance between the point and the surface source. Mathematically, its value is $E = \pi L$ lm/m^2 where L is the luminance of the surface source in cd/m^2.

In case the surface source is limited and rectangular in shape (Fig. 49.19), the law governing the illumination at a point P is

$$E = \frac{L}{2}\,(\alpha' \sin \beta + \beta' \sin a)$$

where $\alpha = \angle APD$; $\alpha' = \angle BPC$; $\beta = \angle APB$; $\beta' = \angle DPC$

Note. *(i)* In case, distance d is more than 5 times the greatest dimension of the source, then irrespective of its shape, the illumination is found to obey inverse square law. This would be the case for illumination at points 5 metres or more away from a fluorescent tube of length one metre.

(ii) In the case of surface sources of large area, such as luminous daylights covering the whole ceiling of a large room, illumination is found to be practically constant irrespective of the height of the working place.

It may be noted that a point source produces deep shadows which may, however, be cancelled by installing a large number of fittings. Usually, glare is present. However, point sources are of great practical importance where accurate light control is required as in search-lights.

Line sources give more diffusion but cast shadows of objects lying parallel to them thus hindering vision.

Large-area surface sources though generally of low brightness, produce minimum glare but no shadows. However, the final effect is not liveliness but one of deadness.

Example 49.18. *A show case is lighted by 4 metre of architectural tubular lamps arranged in a continuous line and placed along the top of the case. Determine the illumination produced on a horizontal surface 2 metres below the lamps in a position directly underneath the centre of the 4 m length of the lamps on the assumption that in tubular lamps emit 1,880 lm per metre run. Neglect the effect of any reflectors which may be used.*

Solution. As seen in Art 49.10

$$E = \frac{I}{2d}\,(\sin 2\theta + 2\theta)\ \text{lm/m}^2 \text{ and } I = \frac{\Phi}{\pi^2 L}\ \text{cd/m}$$

As seen from Fig. 49.15

$$\theta = \tan^{-1}(L/2\,d)$$

$$= \text{tan–1}(4/2) = 45°$$

$$I = 4 \times 1{,}880/\pi^2 \times 4$$

$$= \mathbf{188\ cd/m}$$

$$\therefore \quad E = \frac{188}{2 \times 2}\left(\sin 90° + \frac{\pi}{2}\right) = \mathbf{121\ lm/m^2}$$

49.7. Polar Curves of C.P. Distribution

All our calculations so far were based on the tacit assumption that the light source was of equal luminous intensity or candle-power in all directions. However, lamps and other sources of light, as a rule, do not give uniform distribution in the space surrounding them.

If the actual luminous intensity of a source in various directions be plotted to scale along lines radiating from the centre of the source at corresponding angles, we obtain the polar curve of the candle power.

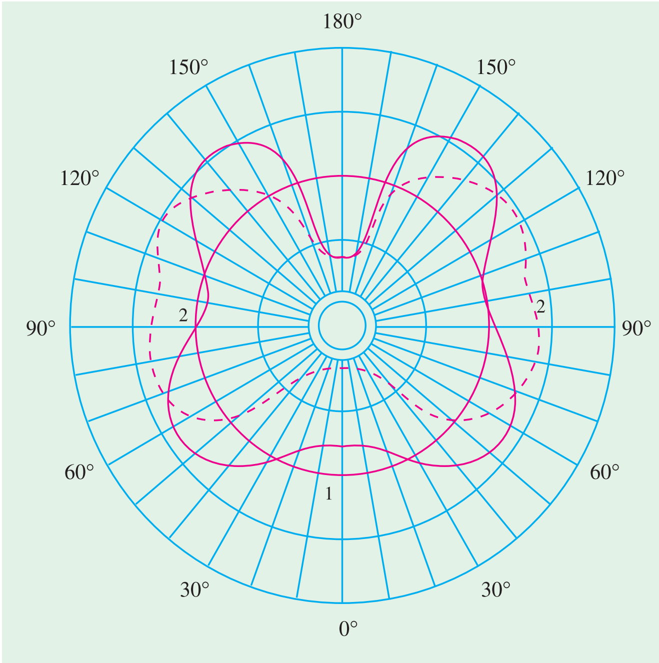


Fig. 49.20

Suppose we construct a figure consisting of large number of spokes radiating out from a point—the length of each spoke representing to some scale the candle power or luminous intensity of the source in that particular direction. If now we join the ends of these spokes by some suitable material, say, by linen cloth, then we get a surface whose shape will represent to scale the three dimensional candle power distribution of the source placed at the centre. In the ideal case of a point source having equal distribution in all directions, the surface would be spherical.

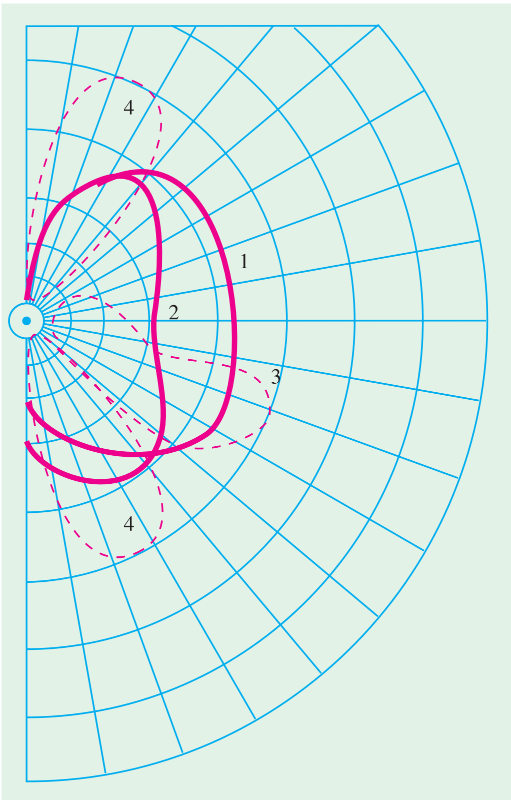


Fig. 49.21

It would be realized that it is difficult to give a graphic representation of such a 3-dimensional model in a plane surface. Therefore, as with engineering drawings, it is usual to draw only one or more elevations and a plan of sections through the centre of the source. Elevations represent c.p. distribution in the ***vertical*** plane and the plans represent c.p. distribution in ***horizontal*** plane. The number of elevations required to give a complete idea of the c.p. distribution of the source in all directions depends upon the shape of the plan *i.e.* on the horizontal distribution. If the distribution is uniform in every horizontal plane *i.e.* if the polar curve of horizontal distribution is a circle, then only one vertical curve is sufficient to give full idea of the space distribution.

In Fig. 49.20 are shown two polar curves of c.p. distribution in a vertical plane. Curve 1 is for vacuum type tungsten lamp with zig-zag filament whereas curve 2 is for gas filled tungsten lamp with filament arranged as a horizontal ring.

If the polar curve is symmetrical about the vertical axis as in the figures given below, then it is sufficient to give only the polar curve within one semicircle in order to completely define the distribution of c.p. as shown in Fig. 49.21.

The curves 1 and 2 are as in Fig. 49.20, curves 3 is for d.c. open arc with plain carbons and curve 4 is for a.c. arc with plain carbons. However, if the source and/or reflector are not symmterical about vertical axis, it is impossible to represent the space distribution of c.p. by a single polar diagram and even polar diagrams for two planes at right angles to each other give no definite idea as to the distribution in the intermediate planes.

Consider a filament lamp with a helmet-type reflector whose axis is inclined and cross-section elliptical—such reflectors are widely used for lighting shop windows. Fig. 49.22 represents the distribution of luminous intensity of such source and its reflector in two planes at right angles to each other.

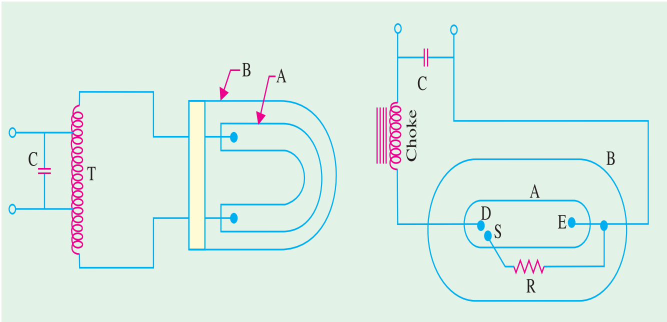


Fig. 49.22

The importance of considering the polar curves in different planes when the c.p. distribution in asymmetrical is even more strikingly depicted by the polar curves in *YY* plane and *XX* plane of a lamp with a special type of reflector designed for street lighting purposes (Fig. 49.23).

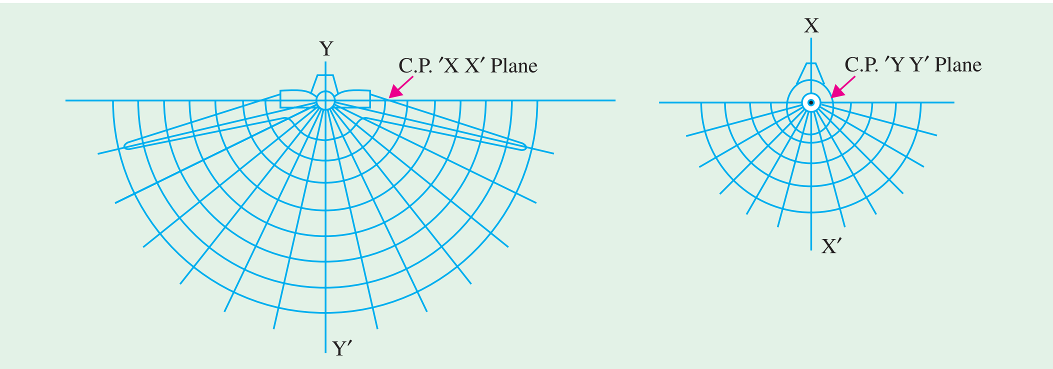


Fig. 49.23

It would be realized from above that the polar distribution of light from any source can be given any desired form by using reflectors and/or refractors of appropriate shape.

In Fig. 49.24 is shown the polar curve of c.p. distribution of a straight type of lamp in a horizontal plane.

49.8. Uses of Polar Curves

Polar curves are made use of in determining the M.S.C.P. etc. of a source. They are also used in determining the actual illumination of a surface *i.e.* while calculating the illumination in a particular direction, the c.p. in that particular direction as read from the vertical polar curve, should be employed.

49.9. Determination of M.S.C.P. and M.H.C.P. from Polar Diagrams

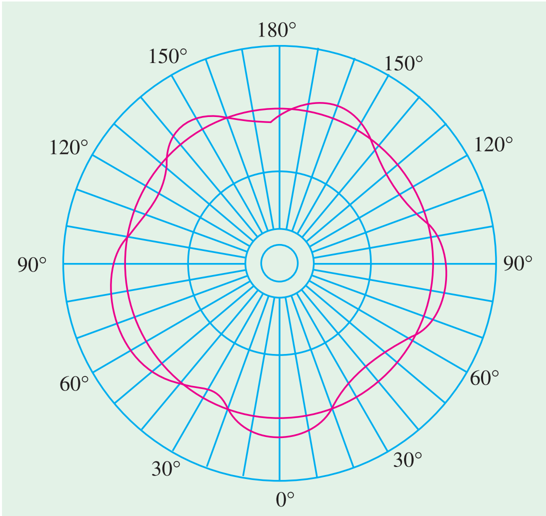


Fig. 49.24

In Fig. 49.25 (*a*) is shown the polar distribution curve of a filament lamp in a horizontal plane and the polar curve in Fig. 49.25 (*b*) represents the c.p. distribution in a vertical plane. It will be seen that the horizontal candle-power is almost uniform in all directions in that plane. However, in the vertical plane, there is a large variation in the candle power which falls to zero behind the cap of the lamp. The curve in Fig. 49.25 (*a*) has been drawn with the help of a photometer while the lamp is rotated about a vertical axis, say, 10° at a time. But the curve in Fig. 49.25 (*b*) was drawn while the lamp was rotated in a vertical plane about a horizontal axis passing through the centre of the filament.

The M.H.C.P. is taken as the mean of the readings in Fig. 49.25 (*a*). However, a more accurate result can be obtained by plotting candle power on an angular base along the rectangular axes and by determining the mean height of the curve by the mid-ordinate or by Simpson's rule.

The M.S.C.P. of the lamp can be obtained from the vertical polar curve of Fig. 49.25 (*b*) by Rousseau's construction as explained below :

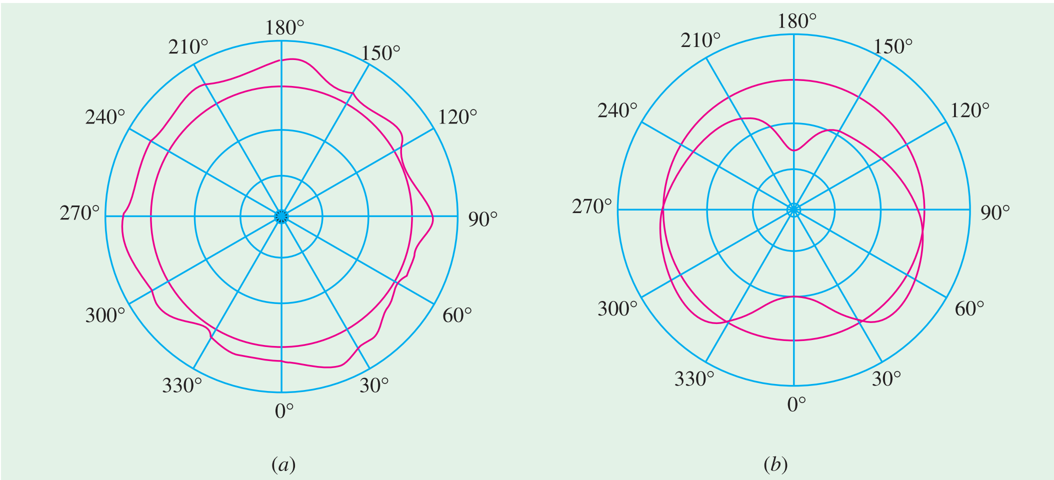


Fig. 49.25

Only half of the vertical polar curve is shown in the figure (Fig. 49.26) since it is symmetrical about the vertical axis. With *O* is the centre and radius *OR* equal to the maximum radius of the polar curve, a semi-circle *LRM* is drawn. A convenient number of points on this semi-circle (say 10° points) are projected onto any vertical plane as shown. For example, points *a*,*b*,*c* etc. are projected to *d*,*e*,*f* and so on. From point *d*, the horizontal line *dg* is drawn equal to the intercept *OA* of the polar diagram on the radius *oa*. Similarly, *eh* = *OB*, *fk* = *OC* and so on. The points *g*, *h*, *k* etc., define the Rousseau figure. The average width *w* of this figure represents the M.S.C.P. to the same scale as that of the candle powers in the polar curve. The average width is obtained by dividing the Rousseau area by the base of the Rousseau figure *i.e.* length lm which is the projection of the semi-circle *LM* on the vertical axis. The area may be determined by Simpson's rule or by using a planimeter.

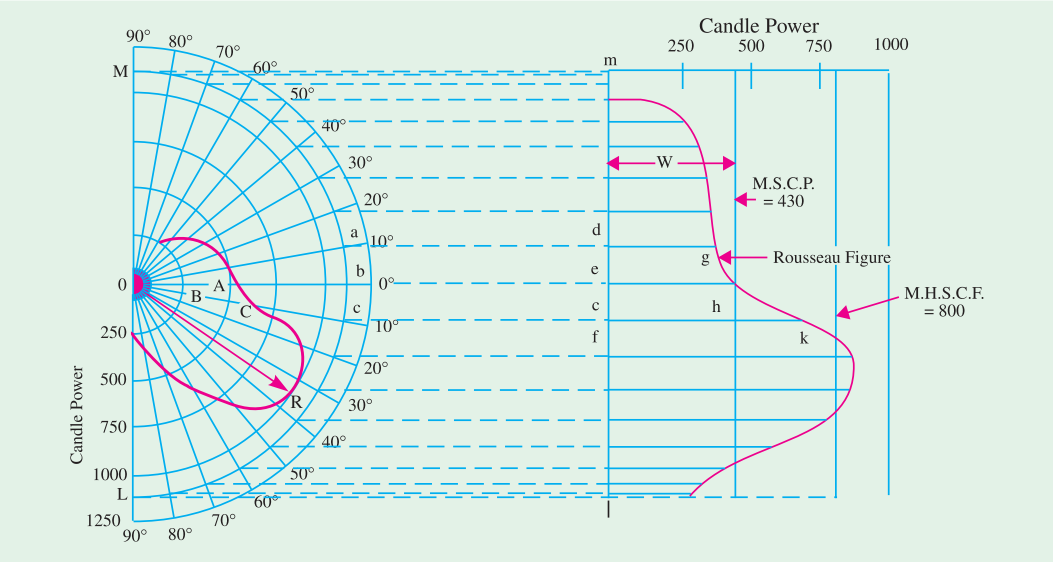


Fig. 49.26

$$\therefore \qquad \text{M.S.C.P.} = \frac{\text{area of Rousseau figure}}{\text{length of the base}}$$

As explained earlier, the M.H.C.P. of an incandescent lamp can be easily obtained by mounting the lamp with its axis vertical and taking photometer readings in the horizontal plane while the lamp is rotated about its axis in steps of 10° or so. A definite ratio exists between the M.H.C.P. and M.S.C.P. of each particular type of filament. M.S.C.P. of a lamp can be found by multiplying M.H.C.P. by a factor known as spherical reduction factor which, as defined earlier, is

$$\text{Spherical reduction factor} \quad f = \frac{\text{M.S.C.P.}}{\text{M.H.C.P.}} \qquad \therefore \quad \text{M.S.C.P.} = f \times \text{M.H.C.P.}$$

For the particular lamp considered, $f = 430/80 = 0.54$ (approx.)

Typical values of this factor are :

Ordinary vacuum-type tungsten lamp having zig-zag filament	0.76 – 0.78
Gas-filled tungsten lamp with filament in the form of broad shallow *V*'s	0.85 – 0.94
Gas-filled tungsten lamp with filament in the shape of a horizontal ring	1.0 – 1.2

The total lumen output is given by the relation ; lumen output = $4\pi \times$ M.S.C.P.

In the present case, lumen output = $4\pi \times 430$

= 5,405 lm

49.10. Integrating Sphere or Photometer

The M.S.C.P. is usually measured by means of an integrating photometer, the most accurate form of which consists of a hollow sphere (as originally proposed by Ulbricht) whose diameter is large (at least 6 times) as compared to that of the lamp under test. The interior surface of the hollow sphere is whitened by means of a special matt white paint. When the lamp is placed inside the sphere (not

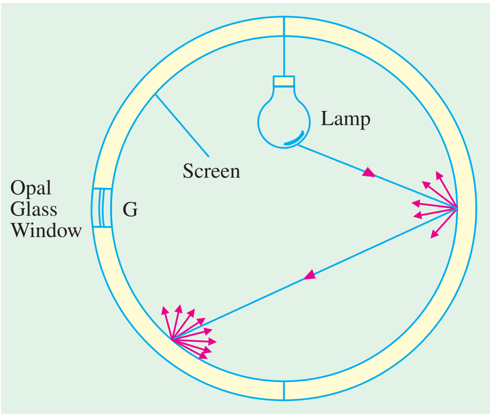


Fig. 49.27

necessarily at its centre) then due to successive reflections, its light is so diffused as to produce a uniform illumination over the whole surface. At some point, a small matt opal-glass window, shaded from the direct rays of the source, is made in the hollow sphere.

The brightness of the matt opal glass is proportional to that of the interior surface of the sphere. By using a suitable illumination photometer, the illumination of the window can be measured which can be used to find out the total flux emitted by the source.

$$\text{Total flux} = \text{illumination (1m/m}^2) \times \text{surface area of the sphere (m}^2)$$

$$\therefore \quad \text{M.S.C.P.} = \frac{\text{total flux}}{4\pi} \text{ candela}$$

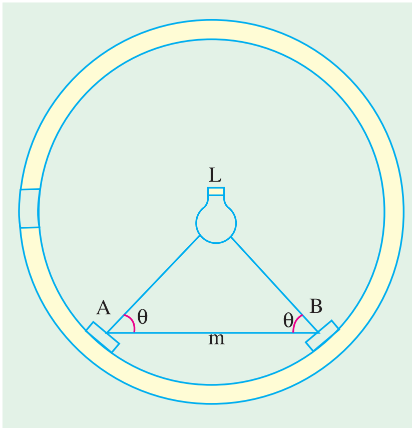


Fig. 49.28

Theory. In Fig. 49.28 is shown a light source L of luminous intensity I candela and having a total flux output of F_L placed at the centre of an integrating sphere of radius r and reflection factor ρ. Let E_A and E_B represent the illuminations at two points A and B, each of infinitely small area da and db respectively and distance m apart. We will now consider total illumination (both direct and reflected) at point A.

Obviously, E_A directly due to $L = I/r^2$

E_B directly due to $L = I/r^2$

Luminous intensity of B in the direction of A is

$$I_B = \frac{\rho . E_B . A_B}{\pi} \text{ candela} \qquad \text{—Art. 45.4}$$

where A_B = projected area of B at right angles to the line $BA = db.\cos\theta$

$$\therefore \quad I_B = \frac{\rho . I . db \cos\theta}{\pi r^2} \text{ candela}$$

Hence, illumination of A due to B is $= \dfrac{I_B \cos\theta}{m^2} = \dfrac{\rho I . db \cos^2\theta}{\pi r^2 \times m^2}$

Now, as seen from Fig. 49.28, $m = 2r\cos\theta$

∴ illumination of A due to B becomes

$$= \frac{\rho . I . db \cos^2\theta}{\pi r^2 \times 4r^2 \cos^2\theta} = \frac{\rho}{4\pi r^2} \times \frac{I}{r^2} \times db = \frac{\rho}{S} . E_B . db = \frac{\rho F_B}{D}$$

where F_B = flux incident on B and A = surface area of the sphere

Hence, total illumination due to first reflection $= \sum \dfrac{\rho F_B}{S} = \dfrac{\rho F_B}{S}$

Now, consider any other point C. Illumination on B due to point $C = \rho F_L/S$. The illumination on A due to C as reflected from B.

$$= \left[\rho . \left(\frac{\rho F_L}{S}\right) \times \frac{db \cos\theta}{\pi}\right] \times \frac{\cos\theta}{m^2} = \frac{\rho F_L}{S} \times \frac{\rho . db \cos\theta}{\pi} \times \frac{\cos\theta}{4r^2 \cos^2\theta}$$

$$= \frac{\rho F_L}{S} \times \frac{\rho . db}{S}$$

Total illumination due to two reflections $= \sum \dfrac{\rho F_L}{S} \times \dfrac{\rho . db}{S} = \dfrac{\rho^2 F_L}{S} \qquad (\because \Sigma ab = S)$

Continuing this way, it can be proved that total illumination at point A from all reflections from all points

$$= \frac{\rho F_L}{S} (1 + \rho^2 + \rho^3 + \ldots\ldots\ldots + \rho^{n-1}) = \frac{\rho F_L}{S} \left(\frac{1}{1-\rho}\right)$$

Hence, total illumination at A from direct and reflected lights is

$$= E_A + \frac{\rho F_L}{S}\left(\frac{1}{1-\rho}\right)$$

If A is shielded from lamp L, then its illumination is proportional to F_L because $\frac{\rho}{S}\left(\frac{1}{1-\rho}\right)$ is a constant factor. Obviously, if either brightness or illumination at one point in the sphere is measured, it would be proportional to the light output of the source. This fact is made use of while using this sphere as a globe photometer.

Example 49.19. *If an integrating sphere 0.6 m in diameter whose inner surface has a reflection coefficient of 0.8 contains a lamp producing on the portion of the sphere, screened from direct radiation, a luminance of 1000 cd/m², what is the luminous flux yield of the source ?*

(A.M.I.E. Sec. B. Summer 1986)

Solution. Obviously, the screened portion of the sphere receives light by reflection only. Reflection illumination of the screened portion is

$$E = \frac{\rho F_1}{S}\left(\frac{1}{1-\rho}\right) = \frac{0.8\, F_L}{\pi \times 0.6^2}\left(\frac{1}{1-0.8}\right) = \frac{100\, F_L}{9\pi}\ \text{lm/m}^2$$

Also $\quad L = \rho E/\pi$ — Art. 49.4

$$\therefore \quad 1000 = \frac{100\, F_L}{9\pi} \times \frac{0.8}{\pi} \quad \text{or} \quad F_L = \mathbf{1/25\ lm}$$

49.11. Diffusing and Reflecting Surfaces : Globes and Reflectors

When light falls on polished metallic surfaces or silvered surfaces, then most of it is reflected back according to the laws of reflection *i.e.* the angle of incidence is equal to the angle of reflection. Only a small portion of the incident light is absorbed and there is always the image of the source. Such reflection is known as ***specular*** reflection.

However, as shown in Fig. 49.29 (*b*), if light is incident on coarse surfaces like paper, frosted glass, painted ceiling etc., then it is scattered or diffused in all directions, hence no image of the source is formed. Such reflection of light is called ***diffuse reflection***. A perfect diffuser is one that scatters light uniformly in all directions and hence appears equally bright from whatever direction it is viewed. A white blotting paper is the nearest approach to a diffuser.

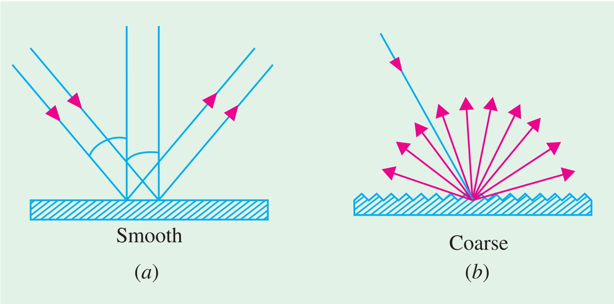


Fig. 49.29

By reflecting factor of a surface is meant the ratio = $\dfrac{\text{reflected light}}{\text{incident light}}$

It is also known as reflection ratio or coefficient of reflection of a surface.

If the light is incident on a transparent surface, then some of it is absorbed and greater percentage of it passes through and emerges on the other side.

To avoid direct glare from electric arcs and incandescent filament lamps, they are surrounded more or less completely by diffusing shades or globes. In addition, a reflector may also be embodied to prevent the escape of light in directions where it serves no useful purpose. In that case, so far as the surroundings are concerned, the diffusing globe is the source of light. Its average brilliancy is lower the more its diffusing area. Depending on the optical density, these globes absorb 15 to 40% of light emitted by the encircled bulb. The bulbs may also be frosted externally by etching or sand-blasting but internal frosting is better because there is no sharp scratching or cracks to weaken the glass.

In domestic fittings, a variety of shades are used whose main purpose is to avoid glare. Properly designed and installed prismatic glass shades and holophane type reflectors have high efficiency and are capable of giving accurate predetermined distribution of light.

Regular metallic reflection is used in search-light mirrors and for general lighting purposes. But where it is used for general lighting, the silvered reflectors are usually fluted to make the illumination as uniform as possible.

Regular cleaning of all shades, globes, and reflectors is very important otherwise the loss of light by absorption by dust etc., collected on them becomes very serious.

Various types of reflectors are illustrated in Figs. 49.30 to 49.34. Fig. 49.30 shows a holophane stiletto reflector used where extensive, intensive or focussing light distribution is required.

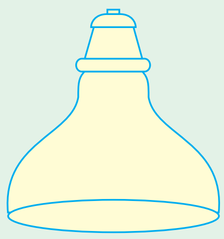


Fig. 49.30

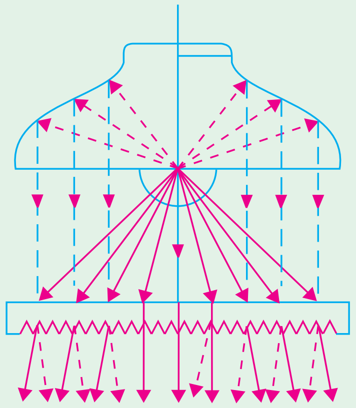


Fig. 49.31

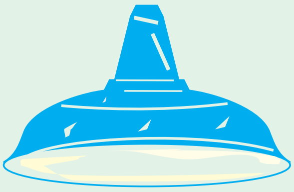


Fig. 49.32

The optical combination of a lamp, reflector and a lens plate, as shown in Fig. 49.31, provides a high degree of light control. Multiple panels can be conveniently incorporated in fittings suited to different architectural schemes.

The dispersive reflector of Fig. 49.32 is suitable for practically all classes of industrial installations. The reflector is a combination of concave and cylindrical reflecting surfaces in the form of a deep bowl of wide dispersive power. It gives maximum intensity between 0º and 45º from the vertical.

The concentrating reflector of parabolic form shown in Fig. 49.33 is suitable for situations requiring lofty installations and strongly-concentrated illumination as in public halls, foundries and power stations etc. They give maximum intensity in zones from 0º to 25º from the vertical.

Fig. 49.33

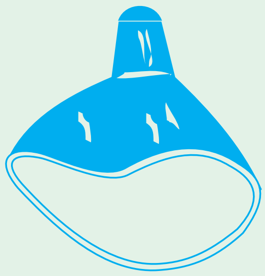


Fig. 49.34

The elliptical angle reflector shown in Fig. 49.34 is suitable for the side lighting of switchboards, show windows etc., because they give a forward projection of light in the vertical plane and spread the light in the horizontal plane.

49.12. Lighting Schemes

Different lighting schemes may be classfied as *(i)* direct lighting *(ii)* indirect lighting and *(iii)* semi-direct lighting *(iv)* semi-indirect lighting and *(v)* general diffusing systems.

(i) Direct Lighting

As the name indicates, in the form of lighting, the light from the source falls directly on the object or the surface to be illuminated (Fig. 49.35). With the help of shades and globes and reflectors of various types as disscussed in Art. 49.11, most of the light is directed in the lower hemisphere and also the brilliant source of light is kept out of the direct line of vision. Direct illumination by lamps in suitable reflectors can be supplemented by standard or bracket lamps on desk or by additional pendant fittings over counters.

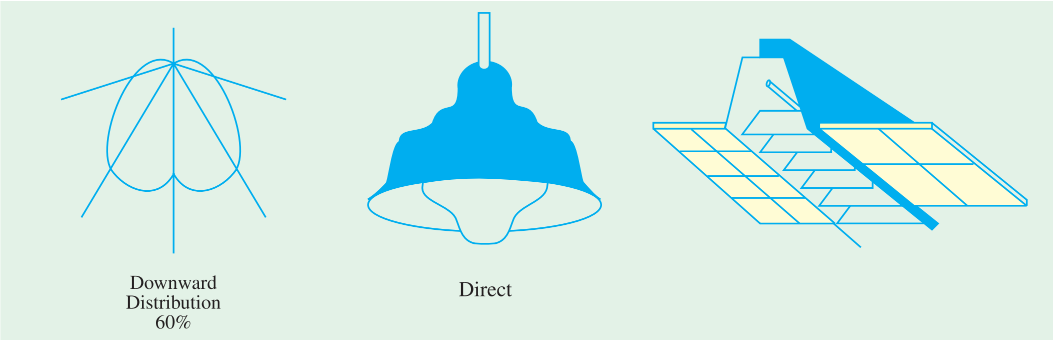


Fig. 49.35

The fundamental point worth remembering is planning any lighting installation is that sufficient and sufficiently uniform lighting is to be provided at the working or reading plane. For this purpose, lamps of suitable size have to be so located and furnished with such fittings as to give correct degree and distribution of illumination at the required place. Moreover, it is important to keep the lamps and fittings clean otherwise the decrease in effective illumination due to dirty bulbs or reflectors may amount to 15 to 25% in offices and domestic lighting and more in industrial areas as a result of a few weeks neglect.

Direct lighting, though most efficient, is liable to cause glare and hard shadows.

(ii) Indirect Lighting

In this form of lighting, light does not reach the surface directly from the source but indirectly by diffuse reflection (Fig. 49.36). The lamps are either placed behind a cornice or in suspended ***opaque*** bowls. In both cases, a silvered reflector which is corrugated for eliminating striations is placed beneath the lamp.

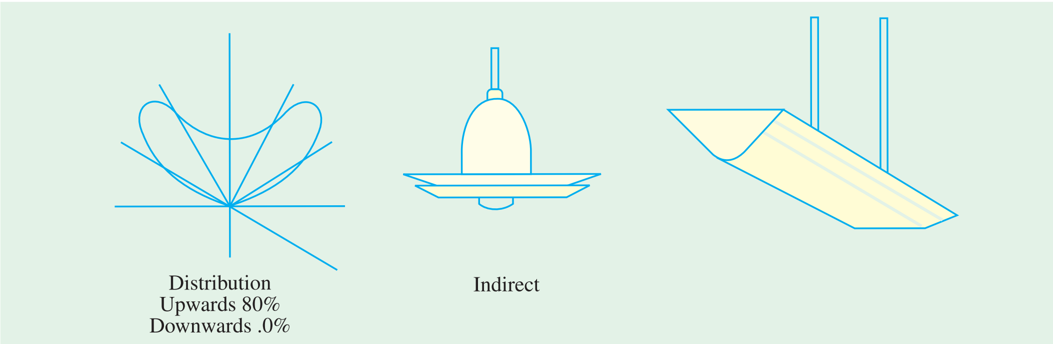


Fig. 49.36

In this way, maximum light is thrown upwards on the ceiling from which it is distributed all over the room by diffuse reflection. Even gradation of light on the ceiling is secured by careful adjustement of the position and the number of lamps. In the cornice and bowl system of lighting, bowl fittings are generally suspended about three-fourths the height of the room and in the case of cornice lighting, a frieze of curved profile aids in throwing the light out into the room to be illuminated. Since in indirect lighting whole of the light on the working plane is received by diffuse reflection, it is important to keep the fittings clean.

One of the main characteristics of indirect lighting is that it provides shadowless illumination which is very useful for drawing offices, composing rooms and in workshops especially where large machines and other obstructions would cast troublesome shadows if direct lighting were used.

However, many people find purely indirect lighting flat and monotonous and even depressive. Most of the users demand 50 to 100% more light at their working plane by indirect lighting than with direct lighting. However, for appreciating relief, a certain proportion of direct lighting is essential.

(*iii*) Semi-direct System

This system utilizes luminaries which send most of the light downwards directly on the working plane but a considerable amount reaches the ceilings and walls also (Fig. 49.37).

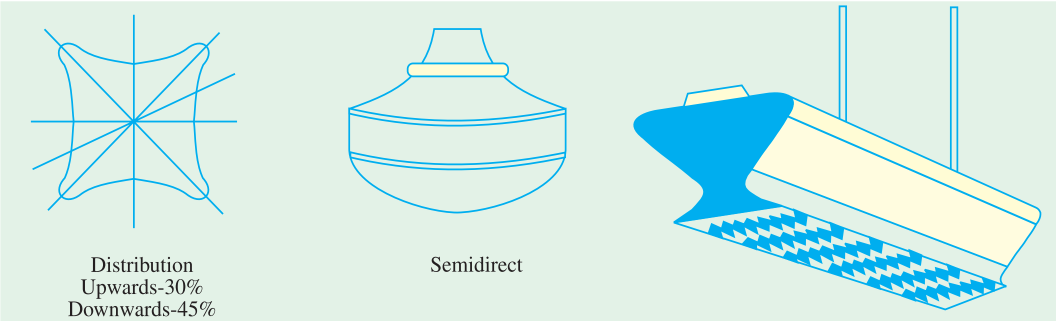


Fig. 49.37

The division is usually 30% upwards and 45% downwards. Such a system is best suited to rooms with high ceilings where a high level of uniformly-distributed illumination is desirable. Glare in such units is avoided by using diffusing globes which not only improve the brightness towards the eye level but improve the efficiency of the system with reference to the working plane.

(*iv*) Semi-indirect Lighting

In this system which is, in fact, a compromise between the first two systems, the light is partly received by diffuse reflection and partly direct from the source (Fig. 49.38). Such a system, therefore, eliminates the objections of indirect lighting mentioned above. Instead of using opaque bowls with reflectors, translucent bowls without reflector are used. Most of the light is, as before, directed upwards to the ceiling for diffuse reflection and the rest reaches the working plane directly except for some absorption by the bowl.

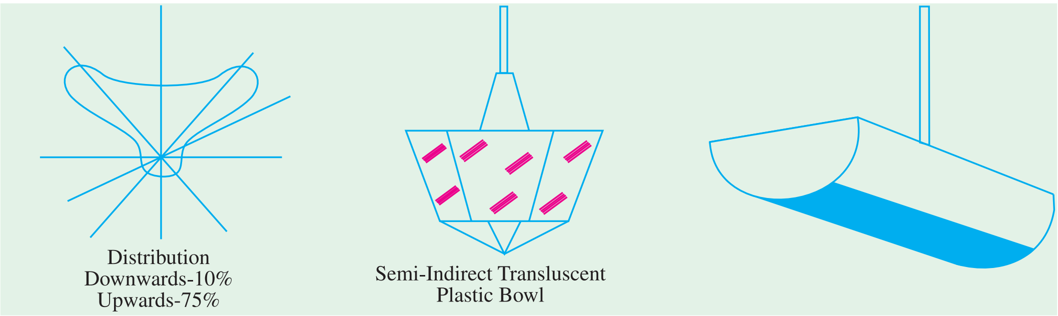


Fig. 49.38

(v) General Diffusing System

In this system, luminaries are employed which have almost equal light distribution downwards and upwards as shown in Fig. 49.39.

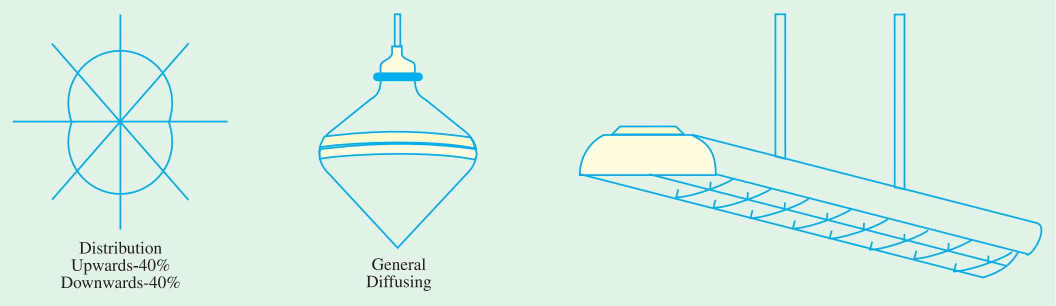


Fig. 49.39

49.13. Illumination Required for Different Purposes

There has been a steady movement towards higher intensities for artificial illumination during the last few decades. The movement is likely to continue because the highest intensities in average installations are much less than those of the diffused daylight. The human eye posses a tremendous power of accommodation and it can work comfortably within an enormous range of illuminations.

For example, at full noon, sun provides about 120,000 lm/m^2, diffuse day-light near a window is of the order of 600 lm/m^2 (value varying widely) and full moon-light gives 0.1 to 0.3 lm/m^2. For reading, usually 20 to 30 lm/m^2 is generally considered sufficient, though daylight illumination is much higher.

Some persons can read without much strain even when illumination is as low as 3 lm/m^2. Because of this, it is difficult to lay down definite values of illumination for various purposes but the following summary will be found useful :

Purpose and Places	*lm/m^2*
Precision work, displays, tasks requiring rapid discrimination	above 500
Extra fine machine work, around needles of sewing machines, fine engraving, inspection of fine details having low contrast	200-500
Proof-reading, drawing, sustained reading, fine assembling, skilled bench-work	100-200
Drawing offices, art exhibition, usual reading	60-100
In museums, drill halls, for work of simple nature not involving close attention to fine details	40-60
Usual observation as in bed-rooms, waiting rooms, auditoriums and general lighting in factories	20-40
Hospital wards, yards, railway platforms and corridors	5-10

49.14. Space/Height Ratio

It is given by the ratio : $\dfrac{\text{horizontal distance between two lamps}}{\text{mounting height of lamps}}$

This ratio depends on the nature of the polar curve of a lamp ***when used along with its reflector***. A reflector has tremendous influence on the shape of the polar curve of the lamp, hence the value of space/height ratio, in fact, depends entirely on the type of reflector used. For obtaining uniform illumination on the working plane, it is essential to choose a correct value for this ratio.

In other words, it means that a reflector gives uniform illumination for a definite value of this ratio only. The ratio may be found easily if the polar curve of the type of fixture used is known. For reflectors normally used in indoor lighting, the value of this ratio lies between 1 and 2.

49.15. Design of Lighting Schemes and Lay-outs

A well-designed lighting scheme is one which

(*i*) provides adequate illumination (*ii*) avoids glare and hard shadows (*iii*) provides sufficiently uniform distribution of light all over the working plane.

Before explaining the method of determining the number, size and proper arrangement of lamps in order to produce a given uniform illumination over a certain area, let us first consider the following two factors which are of importance in such calculations.

49.16. Utilization Factor or Coefficient of Utilization (η)

It is the ratio of the lumens actually received by a particular surface to the total lumens emitted by a luminous source.

$$\therefore \qquad \eta = \frac{\text{lumens actually received on working plane}}{\text{lumens emitted by the light source}}$$

The value of this factor varies widely and depends on the following factors :

1. the type of lighting system, whether direct or indirect etc.
2. the type and mounting height of the fittings
3. the colour and surface of walls and ceilings and
4. to some extent on the shape and dimensions of the room.

For example, for direct lighting, the value of η varies between 0.4 and 0.6 and mainly depends on the shape of the room and the type and mounting height of fittings but very little on the colour of walls and ceiling. For indirect lighting, its value lies between 0.1 and 0.35 and the effect of walls and ceiling, from which light is reflected on the working plane, is much greater. Exact determination of the value of utilization factor is complicated especially in small rooms where light undergoes multiple reflections.

Since the light leaving the lamp in different directions is subjected to different degrees of absorption, the initial polar curve of distribution has also to be taken into account. Even though manufacturers of lighting fittings supply tables giving utilization factors for each type of fitting under specified conditions yet, since such tables apply only to the fittings for which they have been compiled, a good deal of judgment is necessary while using them.

49.17. Depreciation Factor (*p*)

This factor allows for the fact that effective candle power of all lamps or luminous sources deteriorates owing to blackening and/or accumulation of dust or dirt on the globes and reflectors etc. Similarly, walls and ceilings etc., also do not reflect as much light as when they are clean. The value of this factor may be taken as 1/1.3 if the lamp fittings are likely to be cleaned regularly or 1/1.5 if there is much dust etc.

$$p = \frac{\text{illumination under actual condtions}}{\text{illumination when everything is perfectly clean}}$$

Since illumination is specified in lm/m^2, the area in square metre multiplied by the illumination required in lm/m^2 gives the total useful luminous flux that must reach the working plane. Taking into consideration the utilization and depreciation or maintenance factors, the expression for the gross lumens required is

$$\text{Total lumens,} \qquad \Phi = \frac{E \times A}{\eta \times p}$$

where E = desired illumination in lm/m^2 ; A = area of working plane to be illuminated in m^2

p = depreciation or maintenance factor ; η = utilization factor.

The size of the lamp depends on the number of fittings which, if uniform distribution is required, should not be far apart. The actual spacing and arrangement is governed by space/height values and by the layout of ceiling beams or columns. Greater the height, wider the spacing that may be used, although the larger will be the unit required. Having settled the number of units required, the lumens per unit may be found from (total lumens/number of units) from which the size of lamp can be calculated.

Example 49.20. *A room 8 m × 12 m is lighted by 15 lamps to a fairly uniform illumination of 100 lm/m^2. Calculate the utilization coefficient of the room given that the output of each lamp is 1600 lumens.*

Solution. Lumens emitted by the lamps = 15 × 1600 = 24,000 lm

Lumens received by the working plane of the room = 8 × 12 × 100 = 9600 lm

Utilization coefficient = 9600/24,000 = **0.4 or 40%.**

Example 49.21. *The illumination in a drawing office 30 m × 10 m is to have a value of 250 lux and is to be provided by a number of 300-W filament lamps. If the coefficient of utilization is 0.4 and the depreciation factor 0.9, determine the number of lamps required. The luminous efficiency of each lamp is 14 lm/W.* **(Elect. Drives & Utilization, Punjab Univ. Dec. 1994)**

Solution. $\Phi = EA/\eta p$; E = 250 lm/m^2, A = 30 × 10 = 300 m^2; η = 0.4, p = 0.9

$\therefore \quad \Phi = 250 \times 300/0.4 \times 0.9 = 208{,}333$ lm

Flux emitted/lamp = 300 × 14 = 4200 lm; No. of lamps reqd.= 208,333/4200 = **50.**

Example 49.22. *Find the total saving in electrical load and percentage increase in illumination if instead of using twelve 150 W tungsten-filament lamps, we use twelve 80 W fluorescent tubes. It may be assumed that **(i)** there is a choke loss of 25 per cent of rated lamp wattage **(ii)** average luminous efficiency throughout life for each lamp is 15 lm/W and for each tube 40 lm/W and **(iii)** coefficient of utilization remains the same in both cases.*

Solution. Total load of filament lamps = 12 × 150 = 1800 W

Total load of tubes = 12 (80 + 0.25 × 80) = 1200 W

Net saving in load = 1800 – 1200 = **600 W**

If A is the room area and η the corfficient of utilization, then

illumination with lamps, $E_1 = \dfrac{12 \times 150 \times 15\eta}{A}$ = **27,000 η/A lm/m^2**

illumination with tubes, $E_2 = \dfrac{12 \times 80 \times 40\eta}{A}$ = **38,400 η/A lm/m^2**

increase in illumination $= \dfrac{38{,}400 - 27{,}000}{27{,}000}$ = **0.42 or 42%**

Example 49.23. *A football pitch 120 m × 60 m is to be illuminated for night play by similar banks of equal 1000 W lamps supported on twelve towers which are distributed around the ground to provide approximately uniform illumination of the pitch. Assuming that 40% of the total light emitted reaches the playing pitch and that an illumination of 1000 lm/m^2 is necessary for television purposes, calculate the number of lamps on each tower. The overall efficiency of the lamp is to be taken as 30 lm/W.* **(Elect. Technology-I, Bombay Univ.)**

Solution. Area to be illuminated = 120 × 60 = 7,200 m^2

Flux required $= 7{,}200 \times 1{,}000 = 7.2 \times 10^6$ lm

Since only 40% of the flux emitted reaches the ground, the total luminous flux required to be produced is $= 7.2 \times 10^6/0.4 = 18 \times 10^6$ lm

Flux contributed by each tower bank $= 18 \times 10^6/12 = 1.5 \times 10^6$ lm

Output of each 1000-W lamp $= 30 \times 1000 = 3 \times 10^4$ lm

Hence, number of such lamps on each tower is $= 1.5 \times 10^6/3 \times 10^4 =$ **50**

Example 49.24. *Design a suitable lighting scheme for a factory 120 m × 40 m with a height of 7 m. Illumination required is 60 lux. State the number, location and mounting height of 40 W fluorescent tubes giving 45 lm/W.*

Depreciation factor = 1.2 ; utilization factor = 0.5

(Electric Drives & Util. Punjab Univ. 1993)

Solution. $\Phi = \dfrac{60 \times 120 \times 40}{0.5 \times (1/1.2)} = 691{,}200$ lm; Flux per tube $= 45 \times 40 =$ **1800 lm.**

No. of fluorescent tubes reqd. = 691,200/1800 = 384. If twin-tube fittings are employed, then number of such fittings required. = 384/2 = **192.**

These can be arranged in 8 rows of 24 fittings each. Assuming that the working plane is 1 metre above the floor level and the fittings are fixed 1 metre below the ceiling, we get a space/height factor of unity both along the length as well as width of the factory bay.

Example 49.25. *A drawing hall in an engineering college is to be provided with a lighting installation. The hall is 30 m × 20 m × 8 m (high). The mounting height is 5 m and the required level of illumination is 144 lm/m². Using metal filament lamps, estimate the size and number of single lamp luminaries and also draw their spacing layout. Assume :*

Utilization coefficient = 0.6; maintenance factor = 0.75; space/height ratio=1

lumens/watt for 300-W lamp = 13,lumens/watt for 500-W lamp = 16.

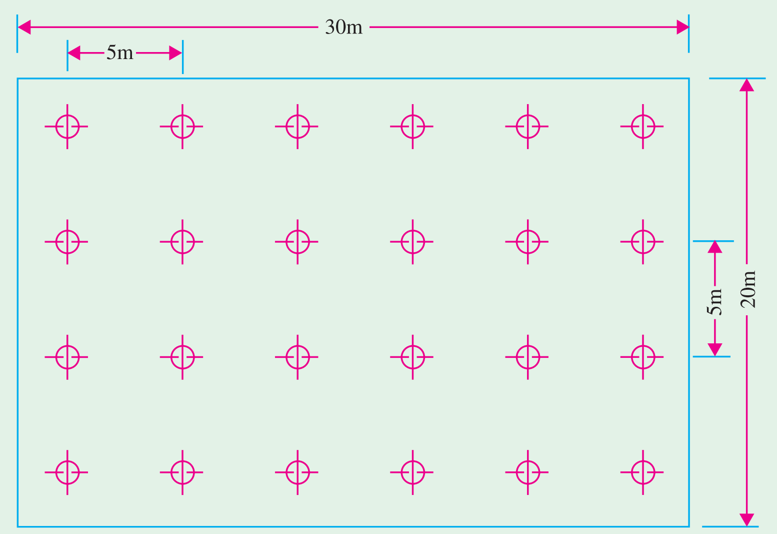


Fig. 49.40

Solution. Flux is given by $\Phi = EA/\eta p = 30 \times 20 \times 144/0.6 \times 0.75 = 192{,}000$ lm

Lumen output per 500-W lamp $= 500 \times 16 = 8{,}000$

∴ No. of 500-W lamps required = 192,000/8000 = **24**

Similarly, No. of 300-W lamps required = 192,000/3900 = **49**

The 300-W lamps cannot be used because their number cannot be arranged in a hall of 30 m × 20 m with a space/height ratio of unity. However, 500-W lamps can be arranged in 4 rows of 6 lamps each with a spacing of 5 m both in the width and the length of the hall as shown in Fig. 49.40.

Example 49.26. *Estimate the number and wattage of lamps which would be required to illuminate a workshop space 60 × 15 metres by means of lamps mounted 5 metres above the working plane. The average illumination required is about 100 lux.*

Coefficient of utilization=0.4 ; Luminous efficiency=16 lm/W.

Assume a spacing/height ratio of unity and a candle power depreciation of 20%.

(Utilization of Electrical Energy, Madras Univ.)

Solution. Luminous flux is given by $\Phi = \dfrac{EA}{\eta p} = \dfrac{100 \times (60 \times 15)}{0.4 \times 1/1.2} = 27 \times 10^4$ lm

Total wattage reqd. $= 27 \times 10^4/16 =$ **17,000 W**

For a space/height ratio of unity, only three lamps can be mounted along the width of the room. Similarly, 12 lamps can be arranged along the length of the room. Total number of lamps required is $12 \times 3 =$ **36**. Wattage of each lamp is = 17,000/36 = 472 W. We will take the nearest standard lamp of **500 W**. These thirty-six lamps will be arranged as shown in Fig. 49.41.

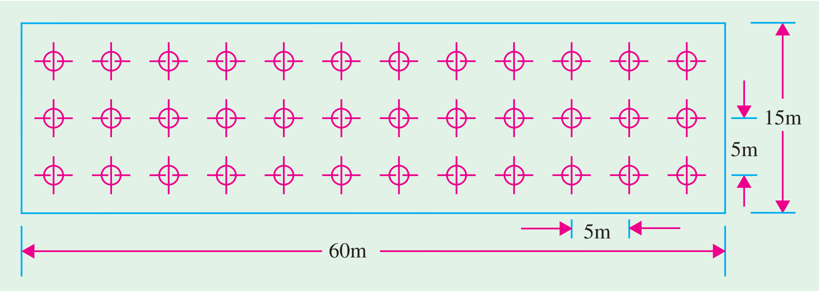


Fig. 49.41

Example 49.27. *A drawing hall 40 m × 25 m × 6 high is to be illuminated with metal-filament gas-filled lamps to an average illumination of 90 lm/m² on a working plane 1 metre above the floor. Estimate suitable number, size and mounting height of lamps. Sketch the spacing layout.*

Assume coefficient of utilization of 0.5, depreciation factor of 1.2 and spacing/height ratio of 1.2

Size of lamps	*200 W*	*300 W*	*500 W*
Luminous efficiency (in lm/W)	*16*	*18*	*20*

(Elect. Technology, Bombay Univ.)

Solution. Total flux required is $\Phi = \dfrac{40 \times 25 \times 90}{0.5 \times 1/1.2} = 216{,}000$ lm

Lumen output of each 200-W lamp is 3200 lm, of 300-W lamp is 5,400 lm and of 500-W lamp is 10,000 lm.

No. of 200-W lamps reqd. $= \dfrac{216{,}000}{3{,}200} =$ **67**; No. of 300-W lamps reqd.$= \dfrac{216{,}000}{5{,}400} =$ **40**

No. of 500-W lamps reqd. = 216,000/10,000 = **22**

With a spacing/height ratio of 1.2, it is impossible to arrange both 200-W and 300-W lamps. Hence, the choice falls on 500-W lamp. If instead of the calculated 22, we take 24 lamps of 500 wattage, they can be arranged in four rows each having six lamps as shown in Fig. 49.42. Spacing along the length of the hall is 40/6 = 6.67 m and that along the width is 25/4 = 6.25 m.

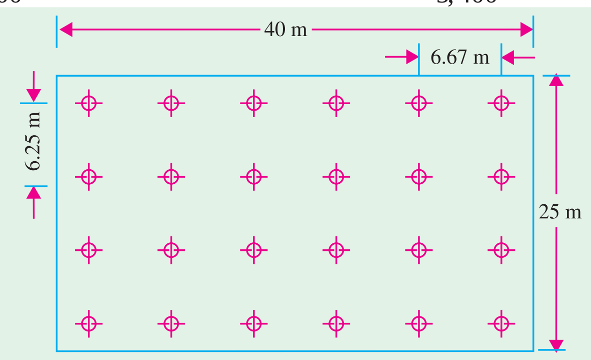


Fig. 49.42

Since mounting height of the lamps is 5 m above the working plane, it gives a space/height ratio of 6.7/5 = 1.5 along the length and 6.25/5 = 1.4 along the width of the hall.

Example 49.28. *A school classroom, 7 m × 10 m × 4 m high is to be illuminated to 135 lm/m² on the working plane. If the coefficient of utilization is 0.45 and the sources give 13 lumens per watt, work out the total wattage required, assuming a depreciation factor of 0.8 . Sketch roughly the plan of the room, showing suitable positions for fittings, giving reasons for the positions chosen.*

Solution. Total flux required is $\Phi = EA/\eta p$

Now $E = 135$ lm/m^2 ;

$A = 7 \times 10 = 70$ m^2 ;

$\eta = 0.45$; $p = 0.8$

$F = 135 \times 7 \times 10/0.45 \times 0.8 = 26{,}250$ lm

Total wattage reqd. = 26,250/13 = **2020 W**

Taking into consideration the dimensions of the room, light fitting of 200 W would be utilized.

No. of fittings required = 2020/200 = **10**

As shown in Fig. 49.43, the back row of fittings has been located 2/3 m from the rear wall so as to (*i*) provide adequate illumination on the rear desk and (*ii*) to minimise glare from paper because light would be incident practically over the shoulders of the students. The two side fittings help eliminate shadows while writing. One fitting has been provided at the chalk board end of the classroom for the benefit of the teacher. The fittings should be of general diffusing pendant type at a height of 3 m from the floor.

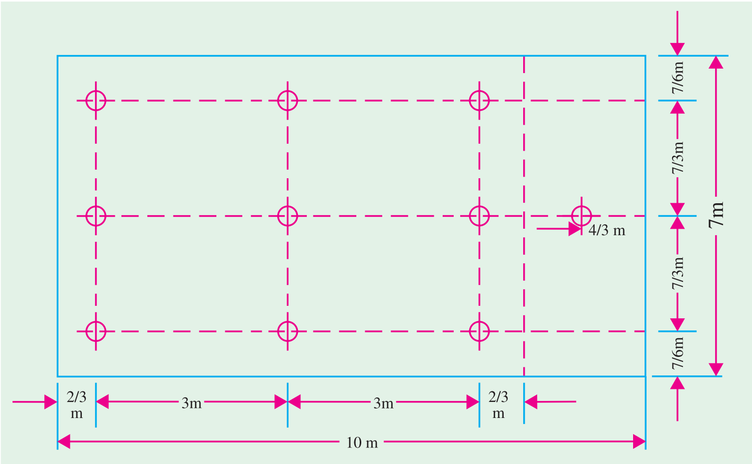


Fig. 49.43

Example 49.29. *A hall 30 m long and 12 m wide is to be illuminated and the illumination required is 50 lm/m². Calculate the number, the wattage of each unit and the location and mounting height of the units, taking a depreciation factor of 1.3 and utilization factor of 0.5, given that the outputs of the different types of lamp are as under :*

Watts :	*100*	*200*	*300*	*500*	*1000*
Lumens :	*1615*	*3650*	*4700*	*9950*	*21500*

(Utili. of Elect. Power, A.M.I.E.)

Solution. Area to be illuminated, $A = 30 \times 12 = 360$ m^2

Illumination required, $E = 50$ lm/m^2, $p = 1/1.3$, $\eta = 0.5$

Now, $\Phi = EA/\eta p = 50 \times 360/0.5 \times (1/1.3) = 46{,}800$ lm

If 100-W lamps are used, No. reqd. = 46,800/1615 = 29

If 200-W lamps are used, No. reqd. = 46,800/3650 = 13

If 300-W lamps are used, No. reqd. = 6,800/4700 = 10

If 500-W lamps are used, No. reqd. = 46,800/9950 = 5

If 1000-W lamps are used, No. reqd. = 46,800/21500 = 2

If we take the mounting height of 5 m, then 300 W lamps would be suitable. The No. of lamps required would be 10, arranged in two rows, each row having 5 lamps thus giving a spacing of 6 m in lengths as well as width and space/height ratio of 6/5 = 1.2.

If we use lamps of low power, their number would be large thereby increasing the number of fittings and hence cost. Lamps of higher voltage would be few in number but will not give a desirable space/height ratio.

Tutorial Problem No. 49.2

1. A room 30 m × 15 m is to be illuminated by 15 lamps to give an average illumination of 40 lm/m^2. The utilization factor is 4.2 and the depreciation factor is 1.4. Find the M.S.C.P. of each lamp.
[561 cd] ***(Elect. Technology-1. Bombay Univ.)***

2. A factory space of 33 m × 13 m is to be illuminated with an average illumination of 72 lm/m^2 by 200 W lamps. The coefficient of utilization is 0.4 and the depreciation factor is 1.4. Calculate the number of lamps required. The lumen output of a 200-W lamp is 2,730 lm.
[40] ***(Elect. Technology-I, Bombay Univ.)***

3. A drawing hall 30 m × 15 m with a ceiling height of 5 m is to be provided with an illumination of 120 lux. Taking the coefficient of utilization of 0.5, depreciation factor of 1.4, determine the No. of fluorescent tubes required and their spacing, mounting height and total wattage. Take luminous efficiency of fluorescent tube as 40 lm/W for 80-W tube. ***(A.M.I.E. Sec. B, Summer)***

4. A room 40 m × 15 m is to be illuminated by 1.5 m 80-W fluorescent tubes mounted 3.5 m above the working plane on which an average illumination of 180 lm/m^2 is required. Using maintenance factor of 0.8 and the utilization factor of 0.5, design and sketch a suitable layout. The 80-W fluorescent tube has an output of 4,500 lm. ***(Electrical Technology, Bombay Univ.)***

5. A hall is to be provided with a lighting installation. The hall is 30 m × 20 m × 8 m (high). The mounting height is 5 m and the required level of illumination is 110 lux. Using metal filament lamps, estimate the size and number of single lamp luminaries and draw their spacing layout. Assume depreciation factor = 0.8, utilization coefficient = 0.6 and space/height ratio = 1.

Watt :	200	300	500
Lumen/watt :	10	12	12.3

[24 lamps, 500 W] ***(Services & Equipment-II, Calcutta Univ.)***

6. It is required to provide an illumination of 100 lux in a factory hall 30 m × 12 m. Assume that the depreciation factor is 0.8, the coefficient of utilization 0.4 and the efficiency of proposed lamps 14 lm/W. Calculate the number of lamps and their disposition.
(Utilization of Elect. Energy, Madras Univ.)

7. Define the terms : (*i*) Lux (*ii*) Luminous Flux (*iii*) Candle Power.

A workshop 100 m × 50 m is to be illuminated with intensity of illumination being 50 lux. Design a suitable scheme of lighting if coefficient of utilization = 0.9 ; Depreciation factor = 0.7 and efficiency of lamps = 80 lm/W. Use 100-W lamps. ***(Electrical Engineering-III, Poona Univ.)***

49.18. Floodlighting

It means 'flooding' of large surfaces with the help of light from powerful projectors. Flooding is employed for the following purposes :

1. For aesthetic purposes as for enhancing the beauty of a building by night *i.e.* flood lighting of ancient monuments, religious buildings on important festive occassions etc.
2. For advertising purposes *i.e.* flood lighting, huge hoardings and commercial buildings.
3. For industrial and commercial purposes as in the case of railway yards, sports stadiums and quarries etc.

Usually, floodlight projectors having suitable reflectors fitted with standard 250-, 500-, or 1,000-watt gas-filled tungsten lamps, are employed. One of the two typical floodlight installations often used is as shown in Fig. 49.44 (*a*). The projector is kept 15 m to 30 m away from the surface to be floodlighted and provides approximately parallel beam having beam spread of 25° to 30°. Fig. 49.44 (*b*) shows the case when the projector cannot be located away from the building. In that case, an asymmetric reflector is used which directs more intense light towards the top of the building.

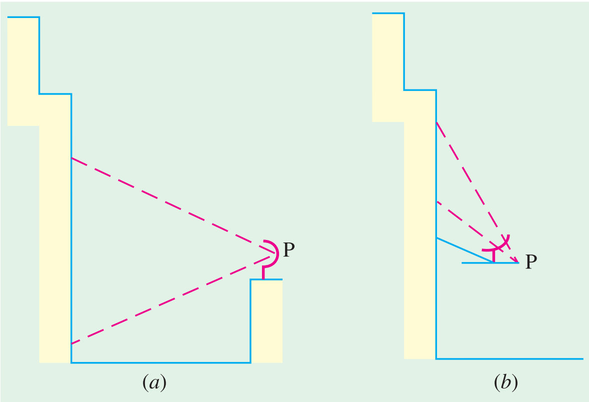


Fig. 49.44

The total luminous flux required to floodlight a building can be found from the relation, $\Phi = EA/\eta \times p$.

However, in the case of flood-lighting, one more factor has to be taken into account. That factor is known as waste-light factor (W). It is so because when several projectors are used, there is bound to be a certain amount of overlap and also because some light would fall beyond the edges of the area to be illuminated. These two factors are taken into account by multiplying the theoretical value of the flux required by a waste-light factor which has a value of nearly 1.2 for regular surfaces and about 1.5 for irregular objects like statues etc. Hence, the formula for calculation of total flux required for floodlighting purposes is

$$\Phi = \frac{EAW}{\eta p}$$

Example 49.30. *It is desired to floodlight the front of a building 42 m wide and 16 m high. Projectors of 30° beam spread and 1000-W lamps giving 20 lumen/watt are available. If the desired level of illumination is 75 lm/m² and if the projectors are to be located at ground level 17 m away, design and show a suitable scheme. Assume the following :*

Coefficient of utilization = 0.4 ; Depreciation factor = 1.3; Waste-light factor = 1.2.

(Electrical Power-II ; M.S. Univ. Baroda)

Solution. $\Phi = \dfrac{EAW}{\eta p}$

Here $E = 75 \text{ lm/m}^2$; $A = 42 \times 16 = 672 \text{ m}^2$; $W = 1.2$; $\eta = 0.4$; $p = 1/1.3$

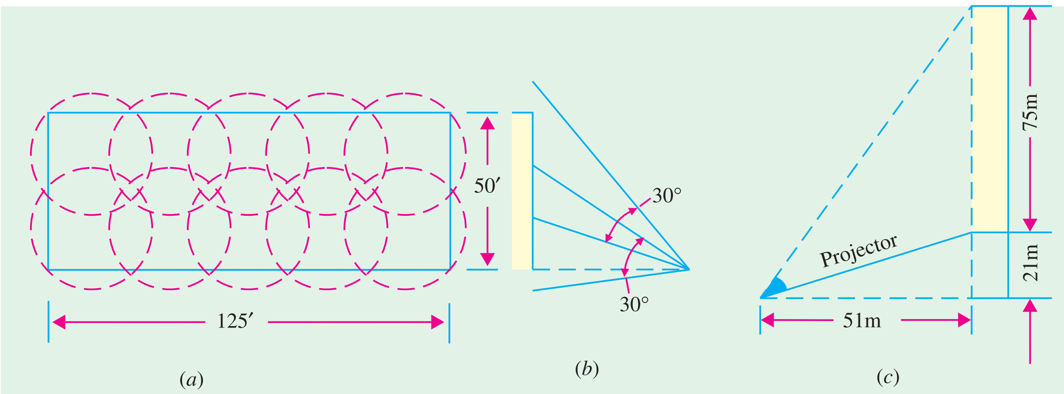


Fig. 49.45

$\therefore \qquad \Phi = \dfrac{7.5 \times 672 \times 1.2}{0.4 \times 1/1.3} = 196{,}500$ lm

Lumen output of each 1,000-W lamp = 1,000 × 20 = 20,000 lm

No. of lamps required = 196,500/20,000 = **10.**

With a beam spread* of 30°, it is possible to cover the whole length and width of the building by arranging the 10 projectors in two rows as shown in Fig. 49.45 (*a*).

Example 49.31. *Estimate the number of 1000-W floodlight projectors required to illuminate the up per 75 m of one face of a 96 m tower of width 13 m if approximate initial average luminance is to be 6.85 cd/m^2. The projectors are mounted at ground level 51m from base of the tower. Utilization factor is = 0.2; reflection factor of wall = 25% and efficiency of each lamp = 18 lm/W.*

(A.M.I.E. Sec. B., Winter 1992)

Solution. $B = 6.85$ cd/m^2 Now, $B = \rho E/\pi$ cd/m^2 —Art 49.4

$\therefore \quad E = pB/\rho = 6.85\ \pi/0.25 = 27.4\ \pi$ lm/m^2

Area to be floodlighted $= 13 \times 75 = 975$ m^2

$\therefore$ flux required $= 27.4\ \pi \times 975$ lm

Taking utilization factor into account, the flux to be emitted by all the lamps

$= 27.4\ \pi \times 975/0.2$ lm

Flux emitted by each lamp $= 18 \times 1000 = 18{,}000$ lm

$\therefore$ No. of lamps reqd. $= \dfrac{27.4\ \pi \times 975}{0.2 \times 18{,}000} =$ **24 (approx.)**

49.19. Artificial Sources of Light

The different methods of producing light by electricity may, in a board sense, be divided into three groups.

1. By ***temperature incandescence***. In this method, an electric current is passed through a filament of thin wire placed in vacuum or an inert gas. The current generates enough heat to raise the temperature of the filament to luminosity.

Incandescent tungsten filament lamps are examples of this type and since their output depends on the temperature of their filaments, they are known as ***temperature radiators***.

2. By establishing an arc between two carbon electrodes. The source of light, in their case, is the incandescent electrode.

3. ***Discharge Lamps***. In these lamps, gas or vapour is made luminous by an electric discharge through them. The colour and intensity of light *i.e.* candle-power emitted depends on the nature of the gas or vapour only. It should be particularly noted that these discharge lamps are luminiscent-light lamps and do not depend on temperature for higher efficiencies. In this respect, they differ radically from incandescent lamps whose efficiency is dependent on temperature. Mercury vapour lamp, sodium-vapour lamp, neon-gas lamp and fluorescent lamps are examples of light sources based on discharge through gases and vapours.

49.20. Incandescent Lamp

An incandescent lamp essentially consists of a fine wire of a high-resistance metal placed in an evacuated glass bulb and heated to luminosity by the passage of current through it. Such lamps were

* It indicates the divergence of a beam and may be defined as the angle within which the minimum illumination on a surface normal to the axis of the beam is 1/10th of the maximum.

produced commercially for the first time by Edison in 1879. His early lamps had filaments of carbonized paper which were, later on, replaced by carbonized bamboo. They had the disadvantage of negative temperature coefficient of resistivity. In 1905, the metallized carbon-filament lamps were put in the market whose filaments had a positive temperature coefficient of resistivity (like metals). Such lamps gave 4 lm/W.

An incandescent lamp

At approximately the same time, osmium lamps were manufactured which had filaments made of osmium which is very rare and expensive metal. Such lamps had a very fair maintenance of candle-power during their useful life and an average efficiency of 5 lm/W. However, osmium filaments were found to be very fragile.

In 1906 tantalum lamps having filaments of tantalum were produced which had an initial efficiency of 5 lm/watt.

All these lamps were superseded by tungsten lamps which were commercially produced in about 1937 or so. The superiority of tungsten lies mainly in its ability to withstand a high operating temperature without undue vaporisation of the filament. The necessity of high working temperature is due to the fact that the amount of visible radiation increases with temperature and so does the radiant efficiency of the luminous source. The melting temperature of tungsten is 3655°K whereas that of osmium is 2972°K and that of tantalum is 3172°K. Actually, carbon has a higher melting point than tungsten but its operating temperature is limited to about 2073°K because of rapid vaporization beyond this temperature.

In fact, the ideal material for the filament of incandescent lamps is one which has the following properties :

1. A high melting and hence operating temperature

2. A low vapour pressure

3. A high specific resistance and a low temperature coefficient

4. Ductility and

5. Sufficient mechanical strength to withstand vibrations.

Since tungsten possesses practically all the above mentioned qualities, it is used in almost all modern incandescent lamps. The earlier lamps had a square-cage type filament supported from a central glass stem enclosed in an evacuated glass bulb. The object of vacuum was two fold :

(a) to prevent oxidation and

(b) to minimize loss of heat by convection and the consequent lowering of filament temperature. However, vacuum favoured the evaporation of the filament with the resulting blackening of the lamp so that the operating temperature had to be kept as low as 2670° K with serious loss in luminous efficiency.

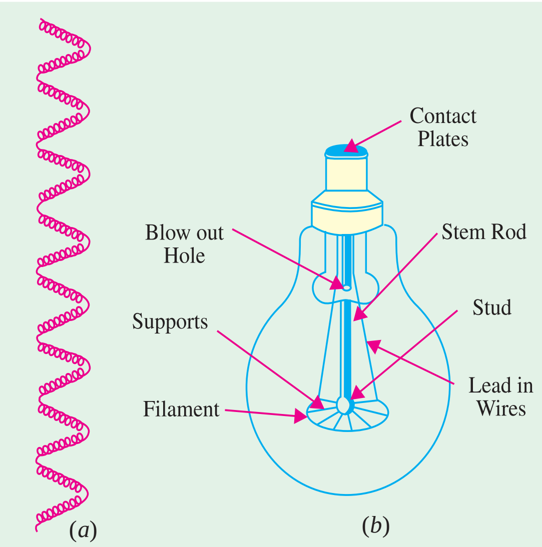


Fig. 49.46

It was, later on, found that this difficulty could be solved to a great extent by inserting a chemically inert gas like nitrogen or argon. The presence of these gases within the glass bulb decreased the evaporation of the filament and so lengthened its life. The filament could now be run at a relatively higher temperature and hence higher luminous efficiency could be realized. In practice, it was found that an admixture of 85% argon and about 15 percent nitrogen gave the best results.

However, introduction of gas led to another difficulty *i.e.* loss of heat due to convection which offsets the additional increase in efficiency. However, it was found that for securing greater efficiency, a concentrated filament having a tightly-wound helical construction was necessary. Such a coiled filament was less exposed to circulating gases, its turns supplying heat to each other and further the filament was mechanically stronger. The latest improvement is that the coiled filament is itself 'coiled' resulting in 'coiled-coil' filament Fig. 49.46 (*a*) which leads to further concentrating the heat, reducing the effective exposure to gases and allows higher temperature operation, thus giving greater efficiency. The construction of a modern coiled-coil gas-filled filament lamp is shown in Fig. 49.46 (*b*). The lamp has a 'wreath' filament *i.e.* a coiled filament arranged in the form of a wreath on radial supports.

A electric filament is a metallic wire, usually made of tungsten, when heated to luminance by passing electricity, radiates light

49.21. Filament Dimensions

There is found to be a definite relation between the diameter of a given filament and the current. Consider a filament operating at a fixed temperature and efficiency. Then since no heat is being utilized for further raising the temperature, all the heat produced in a given time is mostly lost by radiation (if vacuum is good). In other words,

Heat produced per second = heat lost per second by radiation.

$$\text{Now, power intake} = I^2 R = I^2 \times \frac{\rho l}{A} = \frac{I^2 \rho l}{\pi d^2/4} = I^2 \left(\frac{4\rho l}{\pi d^2}\right)$$

where, I = filament current in amperes, l = filament length

A = filament cross-section, d = filament diameter

r = resistivity of filament material at the working temperature.

Heat radiated per second from the surface is proportional to the area of the surface and emissivity of the material

$\therefore$ heat lost/second $\propto$ surface area $\times$ emissivity σ

$\therefore$ $I^2 (4rl/\pi d^2) \propto l \times \pi d \times \sigma$ or $I^2 \propto d^3$ ...(*i*)

$\therefore$ $I \propto d^{1.5}$ or $d \propto I^{2/3}$

In general, for two filaments of the same material working at the same temperature and efficiency, the relation as seen from (*i*) above is

$$\left(\frac{I_1}{I_2}\right)^2 = \left(\frac{d_1}{d_2}\right)^3$$

It would be noticed that the above expressions are similar to those concerning fusing current of a given material under stated conditons (Preece's Rule).

Moreover, for two filaments working at the same temperature, the flux per unit area is the same. Denoting their lengths by l_1 and l_2 and their diameters by d_1 and d_2 respectively, we have, Lumen output $\propto l_1\, d_1 \propto l_2\, d_2$ or $l_1 d_1 = l_2 d_2$ = constant.

Example 49.32. *If the filament of a 32 candela, 100-V lamp has a length l and diameter d, calculate the length and diameter of the filament of a 16 candela 200-V lamp, assuming that the two lamps run at the same intrinsic brilliance.*

Solution. Using the above relation $32 \propto l_1 d_1$ and $16 \propto l_2 d_2$

$$\therefore \quad l_2 d_2 = \frac{1}{2} l_1 d_1$$

Assuming that the power intakes of the two lamps are proportional to their outputs, we have

$$32 \propto 100\, I_1 \text{ and } 16 \propto 200\, I_2 \qquad \therefore \; I_2 = I_1 \times (16/200) \times (100/32) = \frac{1}{4} I_1$$

$$\text{Also } I_1 \propto d_1^{3/2} \text{ and } I_2 \propto d_2^{3/2} \qquad \therefore \; (d_2/d_1)^{3/2} = I_2/I_1 = \frac{1}{4}$$

$$\therefore \quad d_2 = 0.4\, d_1 \text{ (approx.)}$$

$$\therefore \quad l_2 = \frac{1}{2} l_1 \times (d_1/d_2) = \frac{1}{2} \times (1/0.3968) \times l_1 = 1/26\, l_1.$$

Actually, this comparison is not correct because a thicker filament can be worked at a somewhat higher temperature than a thinner one.

Example 49.33. *An incandescent lamp has a filament of 0.005 cm diameter and one metre length. It is required to construct another lamp of similar type to work at double the supply voltage and give half the candle power. Assuming that the new lamp operates at the same brilliancy, determine suitable dimensions for its filament.* **(Elect. Technology, Utkal Univ.)**

Solution. Let I_1 and I_2 be the luminous intensities of the two lamps. Then

$$I_1 \propto l_1 d_1 \text{ and } I_2 \propto l_2 d_2 \qquad \therefore \; \frac{l_2 d_2}{l_1 d_1} = \frac{I_2}{I_1} = \frac{1}{2} \text{ or } l_2 d_2 = \frac{1}{2} l_1 d_1$$

Assuming that the power intakes of the two lamps are proportional to their outputs, we have

$$I_1 \propto V_1 i_1 \text{ and } I_2 \propto V_2 i_2 \qquad \therefore \; \frac{V_2 i_2}{V_1 i_1} = \frac{I_2}{I_1}$$

$$\therefore \quad i_2 = i_1 (V_1/V_2)(I_2/I_1) = i_1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} i_1 \qquad \text{Now, } i_1 \propto d_1^{3/2} \text{ and } i_2 \propto d_2^{3/2}$$

$$\therefore \quad (d_2/d_1)^{3/2} = (i_2/i_1) = \frac{1}{4} \qquad \therefore \; d_2 = 0.3968\, d_1$$

$$\therefore \quad l_2 = \frac{1}{2} l_1 \frac{d_1}{d_2} = \frac{1}{2} l_1 \times \left(\frac{1}{0.3968}\right) = 1.26\, l_1$$

Now, d_1 = **0.005 cm ;** l_1 = **100 cm**

$\therefore$ d_2 = 0.3968 × 0.005 = **0.001984 cm.** l_2 = 1.26 × 100 = **126 cm.**

Example 49.34. *A 60 candle power, 250-V metal filament lamp has a measured candle power of 71.5 candela at 260 V and 50 candela at 240 V.*

(*a*) *Find the constant for the lamp in the expression $C = aV^b$ where C = candle power and V = voltage.*

(b) *Calculate the change of candle power per volt at 250 V. Determine the percentage variation of candle power due to a voltage variation of æ 4% from the normal value.* **(A.M.I.E. Sec. B)**

Solution. (a) $C = aV^b \quad \therefore \quad 71.5 = a \times 260^b$ and $50 = a \times 240^b$

$\therefore \quad 71.5/50 = (260/240)^b$, $b = $ **4.5**

Substituting this value of b in the above equation, we get

$a = 50/240^{4.5}$, $\quad a = $ **0.98×10^{-9}**

Hence, the expression for the candle power of the lamp becomes $C = 0.98 \times 10^{-9}\, V^{4.5}$ candela

(b) Differentiating the above expression and putting $V = 250$ V, we get

$$\frac{dC}{dV} = 0.98 \times 10^{-9} \times 4.5 \times 250^{3.5} = \textbf{4.4 candela per volt}$$

When voltage increases by 4%, $C_2/C_1 = 1.04^{4.5}$

% change in candle power $= \dfrac{C_2 - C_1}{C_1} \times 100 = (1.04^{4.5} - 1) \times 100 = 19.3$

When voltage falls by 4%, $C_2/C_1 = 0.96^{4.5}$

$\therefore$ % change in candle power $= (0.96^{4.5} - 1) \times 100 = $ **–16.8**

49.22. Incandescent Lamp Characteristics

The operating characteristics of an incandescent lamp are materially affected by departure from its normal working voltage. Initially, there is a rapid heating up of the lamp due to its low thermal capacity, but then soon its power intake becomes steady. If the filament resistance were not dependent on its temperature, the rate of generation of heat would have been directly proportional to the square of voltage applied across the lamp. However, because of **(i)** positive temperature coefficient of resistance and **(ii)** complex mechanism of heat transfer from filament to gas, the relations between the lamp characteristics and its voltage are mostly experimental. Some of the characteristics of gas-filled lamps are given below.

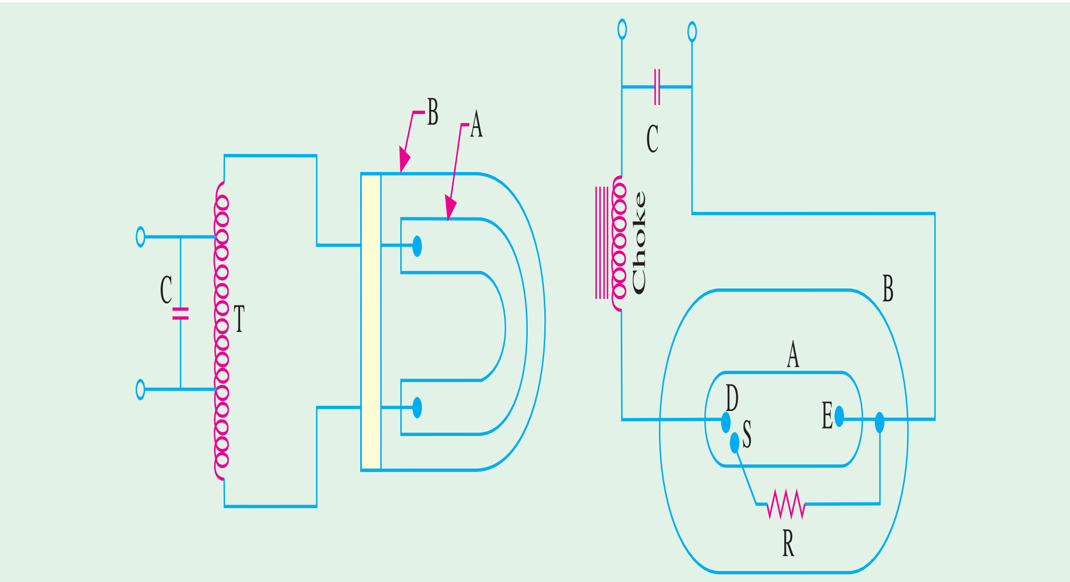


Fig. 49.47

(i) It is found that candle power or lumen output of the lamp varies with the voltage as lumen output $\propto V^{3.3}$.

(*ii*) Variation of lumen output in terms of current is given by : lumen output $\propto I^5$

(*iii*) Life of the lamp is given by : life $\propto 1/V^{13}$

(*iv*) Wattage is given by $W \propto V^{1.43}$

(*v*) Its lumen/watt is given by : lm/watt $\propto V^2$

The characteristic curves are plotted in Fig. 49.47. The life characteristic is very revealing. Even a small undervoltage considerably increases its life whereas overvoltage of as small a value as 5% shortens its life by 50%.

49.23. Clear and Inside-frosted Gas-filled Lamps

The advantages of clear glass-filled lamps are that they facilitate light control and are necessary for use in lighting units where accurate distribution is required such as in flood-lights for buildings, projectors and motor-car headlights. However, they produce hard shadows and glare from filaments. Inside-frosted gas-filled lamps have luminous output nearly 2 per cent less than clear glass lamps of the same rating, but they produce softer shadows and practically eliminate glare from filaments. Such lamps are ideal for use in industrial open fittings located in the line of sight at low mounting heights and in diffuse fittings of opal glass type in order to avoid the presence of filament striations on the surface of the glassware etc.

Another new type of incandescent lamp is the inside-silica coated lamp which, due to the fine coating of silica on the inside of its bulb, has high diffusion of light output. Hence, the light from the filament is evenly distributed over the entire bulb surface thus eliminating the noticeably-bright spot around the filament area of an inside-frosted lamp. Such lamps are less glaring, soften shadows and minimize the brightness of reflections from specular (shiny) surfaces.

49.24. Discharge Lamps

In all discharge lamps, an electric current is passed through a gas or vapour which renders it luminous. The elements most commonly used in this process of producing light by gaseous conduction are neon, mercury and sodium vapours. The colours (*i.e.* wavelength) of light produced depends on the nature of gas or vapour. For example, the neon discharge yields orange-red light of nearly 6,500 A.U. which is very popular for advertising signs and other spectacular effects. The pressure used in neon tubes is usually from 3 to 20 mm of Hg. Mercury-vapour light is always bluish green and deficient in red rays, whereas sodium vapour light is orange-yellow.

Discharge lamps are of two types. The first type consists of those lamps in which the colour of light ***is the same as produced by the discharge through the gas or vapour***. To this group belong the neon gas lamps, mercury vapour (M.V.) and sodium vapour lamps. The other type consists of vapour lamps which use the phenomenon of fluorescence. In their case, the discharge through the vapour produces ultra-violet waves which cause fluorescence in certain materials known as phosphors. The radiations from the mercury discharge (especially 2537 A° line) impinge on these phosphors which absorb them and then re-radiate them at longer wave-lengths of visible spectrum. The inside of the fluorescent lamp is coated with these phosphors for this purpose. Different phosphors have different exciting ranges of frequency and give lights of different colours as shown in table 49.2.

Table 49.2

Phosphor	*Lamp Colour*	*Exciting range A°*	*Emitted wavelenght A°*
Calcium Tangstate	Blue	2200 - 3000	4400
Zinc Silicate	Green	2200 - 2960	5250
Cadmium Borate	Pink	2200 - 3600	6150
Cadmium Silicate	Yellow-pink	2200 - 3200	5950

49.25. Sodium Vapour Lamp

Sodium vapour lamp

One type of low-pressure sodium-vapour lamp along with its circuit connection is shown in Fig. 49.48. It consists of an inner U-tube A made of a special sodium-vapour-resisting glass. It houses the two electrodes and contains sodium together with the small amount of neon-gas at a pressure of about 10 mm of mercury and one per cent of argon whose main function is to reduce the initial ionizing potential. The discharge is first started in the neon gas (which gives out redish colour). After a few minutes, the heat of discharge through the neon gas becomes sufficient to vaporise sodium and then discharge passes through the sodium vapour. In this way, the lamp starts its normal operation emitting its characteristic yellow light.

The tungsten-coated electrodes are connected across auto-transformer *T* having a relatively high leakage reactance. The open-circuit voltage of this transformer is about 450 V which is sufficient to initiate a discharge through the neon gas. The leakage reactance is used not only for starting the current but also for limiting its value to safe limit. The electric discharge or arc strikes immediately after the supply is switched on whether the lamp is hot or cold. The normal burning position of the lamp is horizontal although two smaller sizes of lamp may be burnt vertically. The lamp is surrounded by an outer glass envelope *B* which serves to reduce the loss of heat from the inner discharge tube *A*. In this way, *B* helps to maintain the necessary high temperature needed for the operation of a sodium vapour lamp irrespective of draughts. The capacitor *C* is meant for improving the power factor of the circuit.

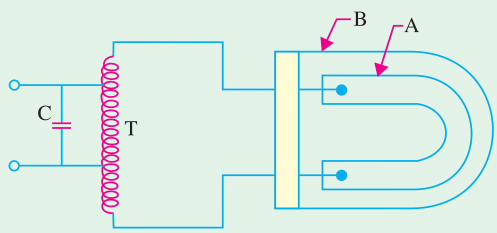


Fig. 49.48

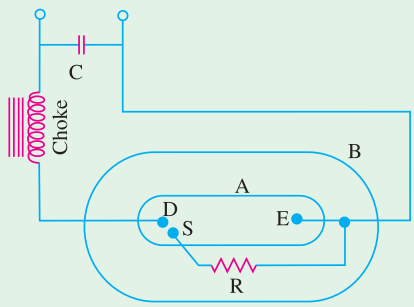


Fig. 49.49

The light emitted by such lamps consists entirely of yellow colour. Solid objects illuminated by sodium-vapour lamp, therefore, present a picture in monochrome appearing as various shades of yellow or black.

49.26. High-pressure Mercury Vapour Lamp

Like sodium-vapour lamp, this lamp is also classified as electric discharge lamp in which light is produced by gaseous conduction. Such a lamp usually consists of two bulbs — an arc-tube containing the electric discharge and an outer bulb which protects the arc-tube from changes in temperature. The inner tube or arc tube *A* is made of quartz (or hard glass) the outer bulb *B* of hard glass. As shown in Fig. 49.49, the arc tube contains a small amount of mercury and argon gas and houses three electrodes *D*, *E* and *S*. The main electrodes are *D* and *E* whereas *S* is the auxiliary starting electrode. *S* is connected through a high resistance *R* (about 50 kΩ) to the main electrode situated at the outer end of the tube. The main electrodes consist of tungsten coils with electron-emitting coating or elements of thorium metal.

When the supply is switched on, initial discharge for the few seconds is established in the argon gas between *D* and *S* and then in the argon between *D* and *E*. The heat produced due to this discharge through the gas is sufficient to vaporise mercury. Consequently, pressure inside *A* increases to about one or two atmospheres and the p.d. across *D* and *E* grows from about 20 to 150 V, the operation taking about 5-7 minutes. During this time, discharge is established through the mercury vapours which emit greenish-blue light.

The choke serves to limit the current drawn by the discharge tube *A* to a safe limit and capacitor *C* helps to improve the power factor of the circuit.

True colour rendition is not possible with mercury vapour lamps since there is complete absence of red-light from their radiations. Consequently, red objects appear black, all blues appear mercury-spectrum blue and all greens the mercury-spectrum green with the result that colour values are distorted.

Correction for colour distortion can be achieved by

1. Using incandescent lamps (which are rich in red light) in combination with the mercury lamps.
2. Using colour-corrected mercury lamps which have an inside phosphor coat to add red colour to the mercury spectrum.

Stroboscopic (Flickering) effect in mercury vapour lamps is caused by the 100 on and off arc strikes when the lamps are used on the 50-Hz supply. The effect may be minimized by

1. Using two lamps on lead-lag transformer
2. Using three lamps on separated phases of a 3-phase supply and
3. Using incandescent lamps in combination with mercury lamps.

In the last few years, there has been tremendous improvement in the construction and operation of mercury-vapour lamps, which has increased their usefulness and boosted their application for all types of industrial lighting, floodlighting and street lighting etc. As compared to an incandescent lamp, a mercury-vapour lamp is (*a*) smaller in size (*b*) has 5 to 10 times longer operating life and (*c*) has 3 times higher efficiency *i.e.* 3 times more light output for given electrical wattage input.

Typical mercury-vapour lamp applications are :

1. High-bay industrial lighting — where high level illumination is required and colour rendition is not important.
2. Flood-lighting and street-lighting
3. Photochemical applications — where ultra-violet output is useful as in chlorination, water sterilization and photocopying etc.
4. For a wide range of inspection techniques by ultra-violet activation of fluorescent and phosphorescent dyes and pigments.
5. Sun-tan lamps — for utilizing the spectrum lines in the erythemal region of ultra-violet energy for producing sun-tan.

49.27. Fluorescent Mercury-vapour Lamps

Basically, a fluorescent lamp consists of a long glass tube internally coated with a suitable fluorescent powder. The tube contains a small amount of mercury along with argon whose function is to facilitate the starting of the arc. There are two sealed-in electrodes at each end of the tube. Two basic types of electrodes are used in fluorescent lamps :

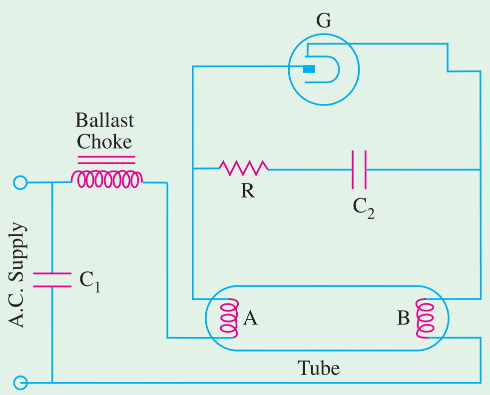


Fig. 49.50

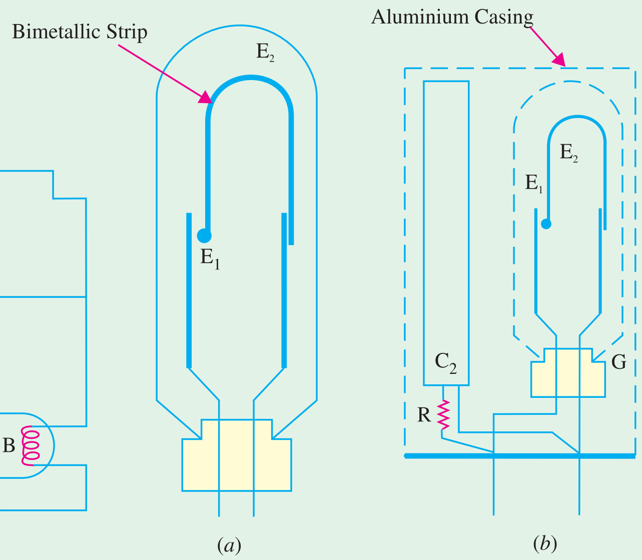


Fig. 49.51

1. The coated-coil tungsten wire type. This type is used in standard pre-heat, rapid-start, instant-start lamps etc.

2. The inside-coated metal cylinder type which operates at a lower and more even temperature than the tungsten type and is called 'cold cathode'.*

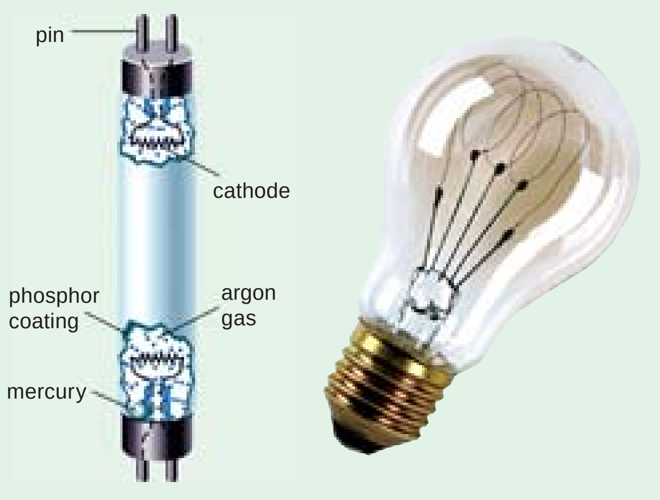
Circuits employed for the control of fluorescent lamps can be divided into two main groups ***(i)*** switch-start circuits and ***(ii)*** startless circuits requiring no starter. There are two types of starters ***(a)*** glow type — which is a voltage-operated device and ***(b)*** thermal switch — which is a current-operated device. Fig. 49.50 shows a flourescent tube fitted with a glow starter G. As shown separately in Fig. 49.51 (*a*), the glow switch consists of two electrodes enclosed in a glass bulb filled with a mixture of helium and hydrogen or argon or neon at low pressure. One electrode E_1 is fixed whereas the other E_2 is movable and is made of a U-shaped bimetallic strip. To reduce radio interference, a small capacitor C_2 is connected across the switch. The resistor R checks capacitor surges and prevents the starter electrodes or contacts from welding together. The complete starter switch along with the capacitor and resistor is contained in an aluminium casing is shown in Fig. 49.51 (*b*). Normally, the contacts are open and when supply is switched on, the glow switch receives almost full mains voltage**. The voltage is sufficient to start a glow discharge between the two electrodes E_1 and E_2 and the heat generated is sufficient to bend the bimetallic strip E_2 till it makes contact with the fixed electrode E_1, thus closing the contacts. This action completes the main circuit through the choke and the lamp electrodes A and B (Fig. 49.50). At the same time, since the glow between E_1 and E_2 has been shorted out, the bimetallic strip cools and the contacts E_1 and E_2 open. By this time, lamp electrodes A and B become heated to incandescence and the argon gas in their immediate vicinity is ionized. Due to opening of the glow switch contacts, a high inductive e.m.f. of about 1000 volts is induced in the choke. This voltage surge is sufficient to initiate a discharge in the argon gas lying between electrodes A and B. The heat thus produced is sufficient to vaporize mercury and the p.d. across the fluorescent tube falls to about 100 or 110 V which is not sufficient to restart the glow in

* A cold cathode fluorescent lamp requires higher operating voltage than the other type. Although cold cathode lamps have less efficiency, they have much longer life than other lamps.

** It is so because only the small discharge current flows and voltage drop across the choke is negligible.

G. Finally, the discharge is established through the mercury vapour which emits ultra-violet radiations. These radiations impinge on the fluorescent powder and make it emit visible light.

The function of the capacitor C_1 is to improve the power factor of the circuit. It may be noted that the function of the highly-inductive choke (also called ballast) is (*i*) to supply large potential for starting the arc or discharge and (*ii*) to limit the arc current to a safe value.



A fluorescent lamp radiates more light compared to an incandescent filament bulb, for the same amount of power consumed

49.28. Fluorescent Lamp Circuit with Thermal Switch

The circuit arrangement is shown in Fig. 49.52. The switch has a bimetallic strip close to a resistance R which produces heat. The switch is generally enclosed in hydrogen-filled glass bulb G. The two switch electrodes E_1 and E_2 are normally closed when the lamp is not in operation. When normal supply is switched on, the lamp filament electrodes A and B are connected together through the thermal switch and a large current passes through them. Consequently, they are heated to incandescence. Meanwhile heat produced in resistance R causes the bimetallic strip E_2 to break contact. The inductive surge of about 1000 V produced by the choke is sufficient to start discharge through mercury vapours as explained in Art. 49.27. The heat produced in R keeps the switch contacts E_1 and E_2 open during the time lamp is in operation.

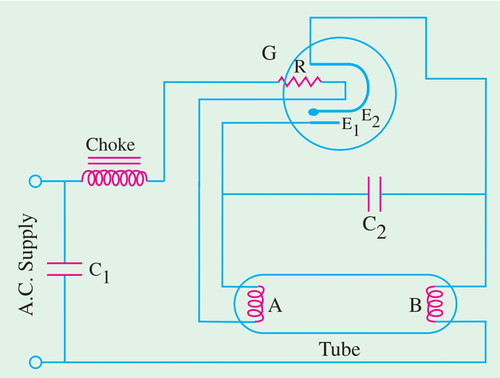


Fig. 49.52

49.29. Startless Fluorescent Lamp Circuit

Such a circuit (Fig. (49.53) which does not require the use of a starter switch is commercially known as 'instant-start' or 'quick-start'. In this case, the normal starter is replaced by a filament heating transformer whose secondaries SS heat up the lamp electrodes A and B to incandescence in a fraction of a second. This combination of pre-heating and application of full supply voltage across lamp electrodes A and B is sufficient to start ionization in the neighbourhood of the electrodes which further spreads to the whole tube. An earthed strip E is used to ensure satisfactory starting.

The advantages of startless method are

1. It is almost instantaneous starting.
2. There is no flickering and no false starts.
3. It can start and operate at low voltage of 160-180 V.
4. Its maintenance cost is lower due to the elimination of any starter-switch replacements.
5. It lengthens the life of the lamp.

49.30. Stroboscopic Effect of Fluorescent Lamps

Stroboscopic or flickering effect produced by fluorescent lamps is due to the periodic fluctuations in the light output of the lamp caused by cyclic variations of the current on a.c. circuits. This phenomenon creates multiple-image appearance of moving objects and makes the movement appear jerky. In this connection, it is worth noting that

1. This flicker effect is more pronounced at lower frequencies.
2. Frequency of such flickers is twice the supply frequency.
3. The fluorescent powder used in the tube is slightly phosphorescent, hence stroboscopic effect is reduced to some extent due to after-glow.

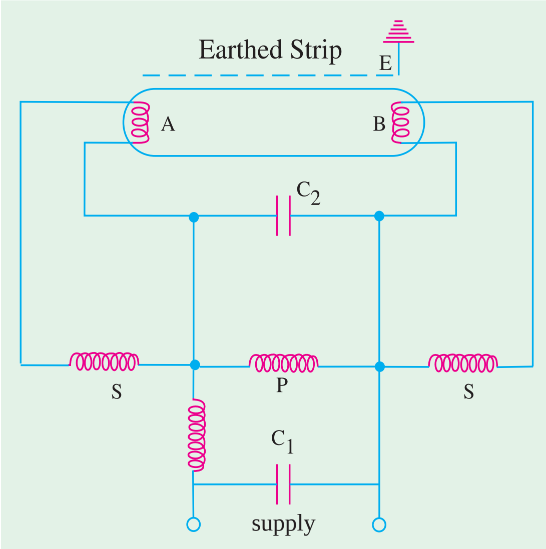


Fig. 49.53

Stroboscopic effect is very troublesome in the following cases :

1. When an operator has to move objects very quickly particularly those having polished finish. These objects would appear to move with jerky motion which over a long period would produce visual fatigue.
2. In the case of rotating machines whose frequency of rotation happens to be a multiple of flicker frequency, the machines appear to decrease in speed of rotation or be stationary. Sometimes the machines may even seem to rotate in the opposite direction.

Some of the methods employed for minimizing stroboscopic effect are given below :

1. By using three lamps on the seperate phases of a 3-phase supply. In this case, the three light waves reaching the working plane would overlap by 120° so that the resultant fluctuation will be very much less than from a single fluorescent lamp.
2. By using a ‘twin lamp’ circuit on single-phase supply as shown in Fig. 49.54, one of the chokes has a capacitor in series with it and the lamp. In this way, a phase displacement of nearly 120° is introduced between the branch currents and also between the two light waves thereby reducing the resultant fluctuation.
3. By operating the lamps from a high frequency supply. Obviously, stroboscopic effect will entirely disappear on d.c. supply.

49.31. Comparison of Different Light Sources

1. Incandescent Lamps. They have instantaneous start and become momentarily off when the supply goes off. The colour of their light is very near the natural light. Their initial cost of installation is minimum but their running cost is maximum. They work equally well both on d.c. and a.c. supply and frequent switching does not affect their life of operation. Change of supply voltage affects their efficiency, output and life in a very significant

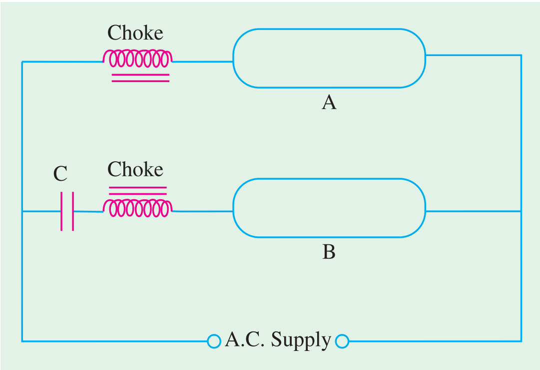


Fig. 49.54

way. They have an average working life of 1000 hours and luminous efficiency of 12 lm/W. Since their light has no stroboscopic effects, the incandescent lamps are suitable for domestic, industrial, street lighting and floodlights etc. They are available in a wide range of voltage ratings and, hence are used in automobiles, trains, emergency lights, aeroplanes and signals for railways etc.

2. Flourescent Lamps. They have a reaction time of one second or a little more at the start. They go off and restart when the supply is restored. The colour of their light varies with the phosphor coating. Their initial cost of installation is maximum but running cost is minimum. Since stroboscopic effect is present, they are suitable for semi-direct lighting, domestic, industrial, commercial, roads and halls etc. Change in voltage affects their starting although light output does not change as remarkably as in the case of incandescent lamps. Colour of their light changes with the different phosphor coating on the inner side of the tube. Frequent switching affects their life period. They have quite high utility but their voltage rating is limited. Hence, their use is confined to mains voltage or complicated inverter circuits which convert 12 V d.c. into high volt d.c.. They have an average working life of 4000 hours and a luminous efficiency of 40 lm/W.

3. Mercury Vapour Lamps. They take 5 to 6 minutes for starting. They go off and cannot be restarted after the recovery of the voltage till the pressure falls to normal. They suffer from high colour distortion. Their initial cost of installation is high but lesser than that of flourescent lamps. Their running cost is much less than incandescent lamps but higher than flourescent tubes for the same levels of illumination. Stroboscopic effect is present in their light. They are suitable for open space like yards, parks and highway lighting etc. Change in voltage effects their starting time and colour of radiations emitted by them. Switching does not affect their life period. They have very limited utility that too on mains voltage. They are suitable for vertical position of working. They have an average working life of 3000 hours and an efficiency of 40 lm/W.

4. Sodium Vapour Lamps. They have a starting time of 5 to 6 minutes. They go off and cannot be restarted after the recovery of the voltage till its value falls to the normal value. The colour of their light is yellowish and produces colour distortion.Their initial cost of installation is maximum although their running cost is less than for filament lamps but more than for flourescent lamps. They have stroboscopic effect and are suitable for use in open spaces, highways and street lighting etc. Change in voltage affects their starting time and colour of their radiations. They work on a.c. voltage and frequent switching affects their life. They are not suitable for local lighting. The colour of their light cannot be changed. They are very suitable for street lighting purposes. Their position of working is horizontal. They have a working life of about 3000 hours and efficiency of 60-70 lm/W.

Tutorial Problem No. 49.3

1. Explain $\cos^3 \phi$ law. *(J.N. University, Hyderabad, November 2003)*
2. A lamp of 500 candle power is placed at the centre of a room,(20 m × 10 m × 5 m.) Calculate the illumination in each corner of the floor and a point in the middle of a 10 m wall at a height of 2 m from floor. *(J.N. University, Hyderabad, November 2003)*
3. Enumerate the various factors, which have to be considered while designing any lighting scheme. *(J.N. University, Hyderabad, November 2003)*
4. Prove that 1 candle/sq. feet = ($\pi fl - L$.) *(J.N. University, Hyderabad, November 2003)*
5. A lamp giving 300 c.p. in all directions below the horizontal is suspended 2 metres above the centre of a square table of 1 metre side. Calculate the maximum and minimum illumination on the surface of the table. *(J.N. University, Hyderabad, November 2003)*
6. Explain the various types of lighting schemes with relevant diagrams. *(J.N. University, Hyderabad, November 2003)*
7. Discuss inverse square law? Corire law of Illustration. *(J.N. University, Hyderabad, November 2003)*

8. A lamp fitted with 120 degrees angled cone reflector illuminates circular area of 200 meters in diameter.The illumination of the disc increases uniformly from 0.5 metre-candle at the edge to 2 meter-candle at the centre.
   Determine :
   (*i*) the total light received (*ii*) Average illumination of the disc (*iii*) Average c.p. of the source.
   (J.N. University, Hyderabad, November 2003)
9. Discuss the flood lighting with suitable diagrams. ***(J.N. University, Hyderabad, November 2003)***
10. Explain the measurement techniques used for luminous intensity.
    (J.N. University, Hyderabad, November 2003)
11. Write short notes on :
    (*i*) Bunsen photometer head (*ii*) Lummer-Brodherm photometer head (*iii*) Flicker photometer head.
    (J.N. University, Hyderabad, November 2003)
12. What do you understand by polar curves as applicable to light source? Explain.
    (J.N. University, Hyderabad, November 2003)
13. Mean spherical Candlepower. ***(J.N. University, Hyderabad, April 2003)***
14. Explain how you will measure the candlepower of a source of light.
    (J.N. University, Hyderabad, April 2003)
15. Explain the Rosseau's construction for calculating M.S.C.P. of a lamp.
    (J.N. University, Hyderabad, April 2003)
16. What do you mean by International Luminosity curve? Explain.
    (J.N. University, Hyderabad, April 2003)
17. Explain in detail the primary standard of luminous intensity with relevant diagram.
    (J.N. University, Hyderabad, April 2003)
18. Explain with sketches the constructional features of a filament lamp.
    (J.N. University, Hyderabad, April 2003)
19. Explain how the standard lamps canbe calibrated w.r.t. primary and secondary standards.
    (J.N. University, Hyderabad, April 2003)
20. Briefly explain the various laboratory standards used in Illumination.
    (J.N. University, Hyderabad, April 2003)
21. Write short notes on :
    (*a*) High pressure mercury vapour lamp (*i*) M.A. Type (*ii*) M.T. Type
    (*b*) Mercury fluorescent lamp.
    (J.N. University, Hyderabad, April 2003)
22. Explain with connection diagram the operation of the low pressure fluorescent lamp and state its advantage. ***(J.N. University, Hyderabad, April 2003)***
23. Explain clearly the following :
    Illumination, Luminous efficiency, MSCP, MHCP and solid angle.
    (J.N. University, Hyderabad, December 2002/January 2003)
24. A small light source with uniform intensity is mounted at a height of 10 meters above a horizontal surface. Two points A and B both lie on the surface with point A directly beneath the source. How far is B from A if the illumination at B is only 1/15th of that at A?
    (J.N. University, Hyderabad, December 2002/January 2003)
25. Discuss the : (*i*) Specular reflection principle (*ii*) Diffusion principle of street lighting.
    (J.N. University, Hyderabad, December 2002/January 2003)
26. Determine the height at which a light source having uniform spherical distribution should be placed over a floor in order that the intensity of horizontal illumination at a given distance from its vertical line may be greatest. ***(J.N. University, Hyderabad, December 2002/January 2003)***
27. What is a Glare? ***(J.N. University, Hyderabad, December 2002/January 2003)***
28. With the help of a neat diagram, explain the principle of operation of fluorescent lamp.
    (J.N. University, Hyderabad, December 2002/January 2003)

29. A machine shop 30 m long and 15 m wide is to have a general illumination of 150 lux on the work plane provided by lamps mounted 5 m above it. Assuming a coefficient of utilization of 0.55, determine suitable number and position of light. Assume any data if required.
(J.N. University, Hyderabad, December 2002/January 2003)

30. Laws of illumination. *(J.N. University, Hyderabad, December 2002/January 2003)*

31. Working principle of sodium vapour lamp.
(J.N. University, Hyderabad, December 2002/January 2003)

32. Explain the principle of operation of sodium vapour lamp and its advantages.
(J.N. University, Hyderabad, December 2002/January 2003)

33. A corridor is lighted by lamps spaced 9.15 m apart and suspended at a height of 4.75 m above the centre line of the floor. If each lamp gives 100 candle power in all directions, find the maximum and minimum illumination on the floor along the centre line. Assume and data if required.
(J.N. University, Hyderabad, December 2002/January 2003)

34. Discuss the laws of illumination and its limitations in practice.
(J.N. University, Hyderabad, December 2002/January 2003)

35. State the functions of starter and choke in a fluorescent lamp.
(J.N. University, Hyderabad, December 2002/January 2003)

36. Mercury Vapour Lamp. *(J.N. University, Hyderabad, December 2002/January 2003)*

37. Describe briefly (*i*) conduit system (*ii*) C.T.S. system of writing.
(Bangalore University, January/February 2003)

38. With a neat circuit diagram, explain the two way control of a filament lamp.
(Belgaum Karnataka University, February 2002)

39. With a neat sketch explain the working of a sodium vapour lamp.
(Belgaum Karnataka University, February 2002)

40. Mention the different types of wiring. With relevant circuit diagrams and switching tables, explain two-way and three-way control of lamps.
(Belgaum Karnataka University, January/February 2003)

41. With a neat sketch explain the working of a fluorescent lamp.
(Belgaum Karnataka University, January/February 2003)

OBJECTIVE TESTS – 49

1. Candela is the unit of
(*a*) flux
(*b*) luminous intensity
(*c*) illumination
(*d*) luminance.

2. The unit of illuminance is
(*a*) lumen
(*b*) cd/m^2
(*c*) lux
(*d*) steradian.

3. The illumination at various points on a horizontal surface illuminated by the same source varies as
(*a*) $\cos^3\theta$
(*b*) $\cos\theta$
(*c*) $1/r^2$
(*d*) $\cos^2\theta$.

4. The M.S.C.P. of a lamp which gives out a total luminous flux of 400 π lumen is .... candela.
(*a*) 200
(*b*) 100
(*c*) 50
(*d*) 400.

5. A perfect diffuser surface is one that
(*a*) diffuses all the incident light
(*b*) absorbs all the incident light
(*c*) transmits all the incident light
(*d*) scatters light uniformly in all directions.

6. The direct lighting scheme is most efficient but is liable to cause
(*a*) monotony
(*b*) glare
(*c*) hard shadows
(*d*) both (*b*) and (*c*).

7. Total flux required in any lighting scheme depends inversely on
(*a*) illumination

(*b*) surface area
(*c*) utilization factor
(*d*) space/height ratio.

8. Floodlighting is NOT used for .... purposes.
(*a*) reading
(*b*) aesthetic
(*c*) advertising
(*d*) industrial.

9. Which of the following lamp has minimum initial cost of installation but maximum running cost ?
(*a*) incandescent
(*b*) fluorescent
(*c*) mercury vapour
(*d*) sodium vapour.

10. An incandescent lamp can be used
(*a*) in any position
(*b*) on both ac and dc supply
(*c*) for street lighting
(*d*) all of the above.

11. The average working life of a fluorescent lamp is about....................... hours.
(*a*) 1000
(*b*) 4000
(*c*) 3000
(*d*) 5000.

12. The luminous efficiency of a sodium vapour lamp is about .... lumen/watt.
(*a*) 10
(*b*) 30
(*c*) 50
(*d*) 70

13. Which of the following statements is correct?
(*a*) Light is a form of heat energy
(*b*) Light is a form of electrical energy
(*c*) Light consists of shooting particles
(*d*) Light consists of electromagnetic waves

14. Luminous efficiency of a fluorescent tube is
(*a*) 10 lumens/watt
(*b*) 20 lumens/watt
(*c*) 40 lumens/watt
(*c*) 60 lumens/watt

15. Candela is the unit of which of the following?
(*a*) Wavelength
(*b*) Luminous intensity
(*c*) Luminous flux
(*d*) Frequency

16. Colour of light depends upon
(*a*) frequency
(*b*) wave length
(*c*) both (a) and (b)
(*d*) speed of light

17. Illumination of one lumen per sq. metre is called ....
(*a*) lumen metre
(*b*) lux
(*c*) foot candle
(*d*) candela

18. A solid angle is expressed in terms of ....
(*a*) radians/metre
(*b*) radians
(*c*) steradians
(*d*) degrees

19. The unit of luminous flux is ....
(*a*) watt/m^2
(*b*) lumen
(*c*) lumen/m^2
(*d*) watt

20. Filament lamps operate normally at a power factor of
(*a*) 0.5 lagging
(*b*) 0.8 lagging
(*c*) unity
(*d*) 0.8 leading

21. The filament of a GLS lamp is made of
(*a*) tungsten
(*b*) copper
(*c*) carbon
(*d*) aluminium

22. Find diameter tungsten wires are made by
(*a*) turning
(*b*) swaging
(*c*) compressing
(*d*) wire drawing

23. What percentage of the input energy is radiated by filament lamps?
(*a*) 2 to 5 percent
(*b*) 10 to 15 percent
(*c*) 25 to 30 percent
(*d*) 40 to 50 percent

24. Which of the following lamps is the cheapest for the same wattage?
(*a*) Fluorescent tube
(*b*) Mercury vapour lamp
(*c*) GLS lamp
(*d*) Sodium vapour lamp

25. Which of the following is not the standard rating of GLS lamps?
(*a*) 100 W
(*b*) 75 W
(*c*) 40 W
(*d*) 15 W

26. In houses the illumination is in the range of
(*a*) 2–5 lumens/watt
(*b*) 10–20 lumens/watt

(c) 35–45 lumens/watt
(d) 60–65 lumens/watt

27. "The illumination is directly proportional to the cosine of the angle made by the normal to the illuminated surface with the direction of the incident flux". Above statement is associated with
(a) Lambert's cosine law
(b) Planck's law
(c) Bunsen's law of the illumination
(d) Macbeth's law of illumination

28. The colour of sodium vapour discharge lamp is
(a) red
(b) pink
(c) yellow
(d) bluish green

29. Carbon arc lamps are commonly used in
(a) photography
(b) cinema projectors
(c) domestic lighting
(d) street lighting

30. Desired illumination level on the working plane depends upon
(a) age group of observers
(b) whether the object is stationary or moving
(c) Size of the object to be seen and its distance from the observer
(d) whether the object is to be seen for longer duration or shorter duration of time
(e) all above factors

31. On which of the following factors dies the depreciation or maintenance factor depend?
(a) Lamp cleaning schedule
(b) Ageing of the lamp
(c) Type of work carried out at the premises
(d) All of the above factors

32. In lighting installating using filament lamps 1% voltage drop results into
(a) no loss of light
(b) 1.5 percent loss in the light output
(c) 3.5 percent loss in the light output
(d) 15 percent loss in the light output

33. For the same lumen output, the running cost of the fluorescent lamp is
(a) equal to that of filament lamp
(b) less than that of filament lamp
(c) more than that of filament lamp
(d) any of the above

34. For the same power output
(a) high voltage rated lamps will be more sturdy
(b) low voltage rated lamps will be more sturdy
(c) both low and high voltage rated lamps will be equally sturdy

35. The cost of a fluorescent lamp is more than that of incandescent lamp because of which of the following factors?
(a) More labour is required in its manufacturing
(b) Number of components used is more
(c) Quantity of glass used is more
(d) All of the above factors

36. Filament lamp at starting will take current
(a) less than its full running current
(b) equal to its full running current
(c) more than its full running current

37. A reflector is provided to
(a) protect the lamp
(b) provide better illumination
(c) avoid glare
(d) do all of the above

38. The purpose of coating the fluorescent tube from inside with white power is
(a) to improve its life
(b) to improve the appearance
(c) to change the colour of light emitted to white
(d) to increase the light radiations due to secondary emissions

39. .... will need lowest level of illumination.
(a) Audiotoriums
(b) Railway platform
(c) Displays
(d) Fine engravings

40. Due to moonlight, illumination is nearly
(a) 3000 lumens/m^2
(b) 300 lumens/m^2
(c) 30 lumens/m^2
(d) 0.3 lumen/m^2

41. Which of the following instruments is used for the comparison of candle powers of different sources?
(a) Radiometer
(b) Bunsen meter
(c) Photometer
(d) Candle meter

42. .... photometer is used for comparing the lights of different colours?
(a) Grease spot
(b) Bunsen
(c) Lummer brodhum
(d) Guilds flicker

43. In the fluorescent tube circuit the function of choke is primarily to
(a) reduce the flicker

(*b*) minimise the starting surge
(*c*) initiate the arc and stabilize it
(*d*) reduce the starting current

44. ...... cannot sustain much voltage fluctuations.
(*a*) Sodium vapour lamp
(*b*) Mercury vapour lamp
(*c*) Incandescent lamp
(*d*) Fluorescent lamp

45. The function of capacitor across the supply to the fluorescent tube is primarily to
(*a*) stabilize the arc
(*b*) reduce the starting current
(*c*) improve the supply power factor
(*d*) reduce the noise

46. ...... does not have separate choke
(*a*) Sodium vapour lamp
(*b*) Fluorescent lamp
(*c*) Mercury vapour lamp
(*d*) All of the above

47. In sodium vapour lamp the function of the leak transformer is
(*a*) to stabilize the arc
(*b*) to reduce the supply voltage
(*c*) both (*a*) and (*b*)
(*d*) none of the above

48. Most affected parameter of a filament lamp due to voltage change is
(*a*) wattage
(*b*) life
(*c*) luminous efficiency
(*d*) light output

49. In electric discharge lamps for stabilizing the arc
(*a*) a reactive choke is connected in series with the supply
(*b*) a condenser is connected in series to the supply
(*c*) a condenser is connected in parallel to the supply
(*d*) a variable resistor is connected in the circuit

50. For precision work the illumination level required is of the order of
(*a*) 500-1000 lumens/m^2
(*b*) 200-2000 lumens/m^2
(*c*) 50-100 lumens/m^2
(*d*) 10-25 lumens/m^2

51. ...... is a cold cathode lamp.
(*a*) Fluorescent lamp
(*b*) Neon lamp
(*c*) Mercury vapour lamp
(*d*) Sodium vapour lamp

52. In case of ...... least illumination level is required.
(*a*) skilled bench work
(*b*) drawing offices
(*c*) hospital wards
(*d*) find machine work

53. For normal reading the illumination level required is around
(*a*) 20-40 lumens/m^2
(*b*) 60-100 lumens/m^2
(*c*) 200-300 lumens/m^2
(*d*) 400-500 lumens/m^2

54. In electric discharge lamps light is produced by
(*a*) cathode ray emission
(*b*) ionisation in a gas or vapour
(*c*) heating effect of current
(*d*) magnetic effect of current

55. A substance which change its electrical resistance when illuminated by light is called ....
(*a*) photoelectric
(*b*) photovoltaic
(*c*) phatoconductive
(*d*) none of the above

56. In case of ...... power factor is the highest.
(*a*) GLS lamps
(*b*) mercury arc lamps
(*c*) tube lights
(*d*) sodium vapour lamps

57. A mercury vapour lamp gives ..... light.
(*a*) white
(*b*) pink
(*c*) yellow
(*d*) greenish blue

58. Sometimes the wheels of rotating machinery, under the influence of fluorescent lamps appear to be stationary. This is due to the
(*a*) low power factor
(*b*) stroboscopic effect
(*c*) fluctuations
(*d*) luminescence effect

59. Which of the following bulbs operates on least power?
(*a*) GLS bulb
(*b*) Torch bulb
(*c*) Neon bulb
(*d*) Night bulb

60. The flicker effect of fluorescent lamps is more pronounced at
(*a*) lower frequencies
(*b*) higher frequencies
(*c*) lower voltages
(*d*) higher voltages

61. Which of the following application does not need ultraviolet lamps?
(*a*) Car lighting
(*b*) Medical purposes
(*c*) Blue print machine
(*d*) Aircraft cockpit dashboard lighting

62. Which gas can be filled in GLS lamps?
(*a*) Oxygen
(*b*) Carbon dioxide
(*c*) Xenon
(*d*) Any inert gas

ANSWERS

1. (*b*)	**2.** (*c*)	**3.** (*a*)	**4.** (*b*)	**5.** (*d*)	**6.** (*d*)	**7.** (*c*)	**8.** (*a*)	**9.** (*a*)	**10.** (*d*)
11. (*b*)	**12.** (*d*)	**13.** (*d*)	**14.** (*d*)	**15.** (*b*)	**16.** (*c*)	**17.** (*b*)	**18.** (*c*)	**19.** (*b*)	**20.** (*c*)
21. (*a*)	**22.** (*d*)	**23.** (*b*)	**24.** (*c*)	**25.** (*b*)	**26.** (*d*)	**27.** (*a*)	**28.** (*c*)	**29.** (*b*)	**30.** (*e*)
31. (*d*)	**32.** (*c*)	**33.** (*b*)	**34.** (*b*)	**35.** (*d*)	**36.** (*c*)	**37.** (*d*)	**38.** (*d*)	**39.** (*b*)	**40.** (*d*)
41. (*c*)	**42.** (*d*)	**43.** (*c*)	**44.** (*c*)	**45.** (*c*)	**46.** (*a*)	**47.** (*c*)	**48.** (*b*)	**49.** (*a*)	**50.** (*a*)
51. (*b*)	**52.** (*c*)	**53.** (*b*)	**54.** (*b*)	**55.** (*c*)	**56.** (*a*)	**57.** (*d*)	**58.** (*b*)	**59.** (*b*)	**60.** (*a*)
61. (*a*)	**62.** (*d*)								

GO To FIRST

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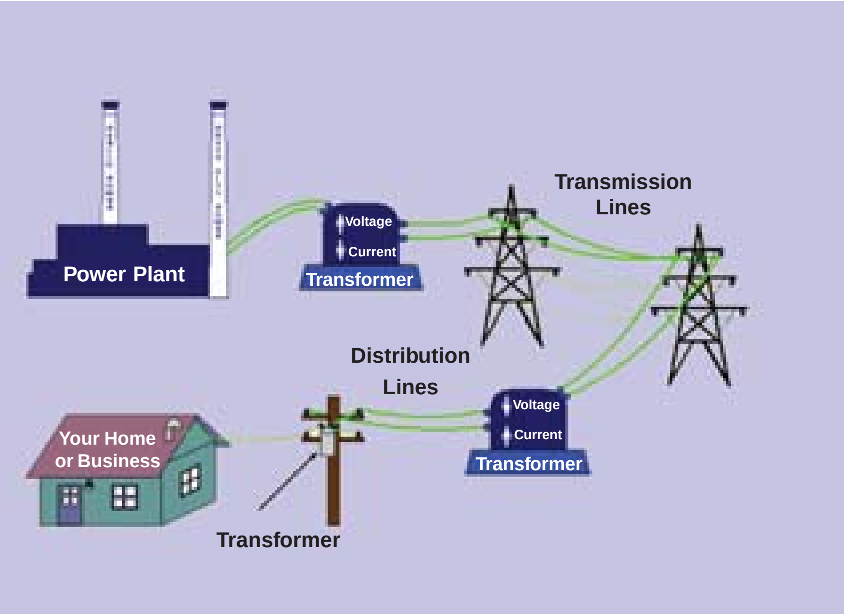
CHAPTER 50

TARIFFS AND ECONOMIC CONSIDERATIONS

Learning Objectives

- Economic Motive
- Depreciation
- Indian Currency
- Factors Influencing Cost and Tariffs of Electric Supply
- Demand
- Average Demand
- Maximum Demand
- Demand Factor
- Diversity of Demand
- Diversity Factor
- Load Factor
- Plant Factor or Capacity Factor
- Utilization Factor (or Plant use Factor)
- Connected Load Factor
- Tariffs
- Flat Rate
- Sliding Scale
- Two-part Tariff
- Kelvin's Law
- Effect of Cable Insulation
- Note on Power Factor
- Disadvantages of Low Power Factor
- Economics of Power Factor
- Economical Limit of Power Factor Correction



For the successful running of an electricity production, transmission and distribution system, it is necessary to properly account for the various direct and indirect costs involved, before fixing the final kWh charges for the consumers

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50.1. Economic Motive

In all engineering projects with the exception of the construction of works of art or memorial buildings, the question of cost is of first importance. In fact, in most cases the cost decides whether a project will be undertaken or not although political and other considerations may intervene sometimes. However, the design and construction of an electric power system is undertaken for the purpose of producing electric power to be sold at a profit. Hence, every effort is made to produce the power as cheaply as possible. The problem of calculating the cost of any scheme is often difficult because the cost varies considerably with time, tariffs and even with convention. In general, the cost of producing electric power can be roughly devided into the following two portions :

(*a*) Fixed Cost. These do not vary with the operation of the plant *i.e.* these are independent of the number of units of electric energy produced and mainly consist of :

1. Interest on capital investment,

2. Allowance for depreciation (*i.e.* wearing out of the depreciable parts of the plant augmented by obsolescence, buildings, the transmission and distribution system etc.),

3. Taxes and insurance, **4.** most of the salaries and wages, **5.** small portion of the fuel cost.

(*b*) Running or Operating Costs. These vary with the operation of the plant *i.e.* these are proportional to the number of units of electric energy generated and are mostly made up of :

1. most of the fuel cost, **2.** small portion of salaries and wages, **3.** repair and maintenance.

50.2. Depreciation

It is obvious that from the very day the construction of a generating plant is completed, deterioration starts and due to wear and tear from use and the age and physical decay from lapse of time, there results a reduction in the value of the plant — a loss of some part of the capital investment in the perishable property. The rate of wear and disintegration is dependent upon ***(i)*** conditions under which the plant or apparatus is working, ***(ii)*** how it is protected from elements and ***(iii)*** how promptly the required repairs are carried out.

Hence, as the property decreases from its original cost when installed, to its final scrap or salvage value at the end of its useful life, it is essential that the owner will have in hand at any given time as much money as represents the shrinkage in value and at the time of actual retirement of the plant, he must certainly have in hand the full sum of the depreciable part of the property. By adding this amount to the net salvage value of the plant, the owner can rebuild the same type of property as he did in the first instance or he can build some other property of an equivalent earning capacity.

The useful life of the apparatus ends when its repair becomes so frequent and expensive that it is found cheaper to retire the equipment and replace it by a new one.

It may be pointed out here that in addition to depreciation from wear and tear mentioned above, there can also be depreciation of the apparatus due to the inadequacy from obsolescence, both sentimental and economic, from the requirements of the regulating authorities and from accidental damages and if any of these factors become operative, they may force the actual retirement of the apparatus much before the end of its normal useful life and so shorten the period during which its depreciation expenses can be collected. These factors will necessitate increased depreciation rate and the consequent build up of the depreciation reserve as to be adequate for the actual retirement.

Some of the important methods of providing for depreciation are :

1. Straight-line method,
2. Diminishing-value method,
3. Retirement-expense method,
4. Sinking-fund method.

In the straight-line method, provision is made for setting aside each year an equal proportional part of the depreciable cost based on the useful life of the property. Suppose a machine costs

Rs. 45,000 and its useful life is estimated as ten years with a scrap value of Rs. 5,000, then the annual depreciation value will be 1/10 of Rs. 40,000 *i.e.* Rs. 4000. This method is extremely simple and easy to apply when the only causes for retirement of the machine are the wear and tear or the slow action of elements. But it is extremely difficult to estimate when obsolescence or accidental damage may occur to the machine. This method ignores the amount of interest earned on the amount set aside yearly.

In the diminishing value method, provision is made for setting aside each year a fixed rate, first applied to the original cost and then to the diminishing value; such rate being based upon the estimated useful life of the apparatus. This method leads to heaviest charges for depreciation in early years when maintenance charges are lowest and so evens out the total expense on the apparatus for depreciation plus the maintenance over its total useful life. This method has the serious disadvantage of imposing an extremely heavy burden on the early years of a new plant which has as yet to develop its load and build up its income as it goes along.

The retirement expense method which is not based on the estimated life of the property, aims at creating an adequate reserve to take care of retirement before such retirements actually occurs. Because of many objections raised against this method, it is no longer used now.

In the sinking-fund method, provision is made for setting aside each year such a sum as, invested at certain interest rate compounded annually, will equal the amount of depreciable property at the end of its useful life. As compared to straight-line method, it requires smaller annual amounts and also the amounts for annuity are uniform. This method would be discussed in detail in this chapter.

Suppose P is the capital outlay required for an installation and r p.a. is the interest per unit (6% is equivalent to $r = 0.06$). The installation should obviously provide rP as annual interest which is added to its annual running cost. Were the installation to last forever, then this would have been the only charge to be made. But as the useful life of the installation has a definite value, it is necessary to provide a sinking fund to produce sufficient amount at the end of the estimated useful life to replace the installation by a new one. Let the cost of replacement be denoted by Q. This Q will be equal to P if the used installation has zero scrap value, less than P if it has positive scrap value and greater than P if it has a negative scrap value. If the useful life is n years, then the problem is to find the annual charge q to provide a sinking fund which will make available an amount Q at the end of n years. Since amount q will earn an annual interest rq, hence its value after one year becomes $q + rq = q(1 + r)$. This sum will earn an interest of $r \times q(1 + r)$ and hence its value at the end of two years will become $q(1 + r) + qr(1 + r)$ or $q(1 + r)^2$. Similarly, its value at the end of three years is $q(1 + r)^3$. *i.e.* its value is multiplied by $(1 + r)$ every year so that the first payment becomes worth $q(1 + r)^n$ at the end of n years. The second payment to the sinking-fund is made at the beginning of the second year, hence its value at the end of the useful life of the installation becomes $q(1 + r)^{n-1}$ because this amount earns interest only for $(n - 1)$ years. The total sum available at the end of n years is therefore

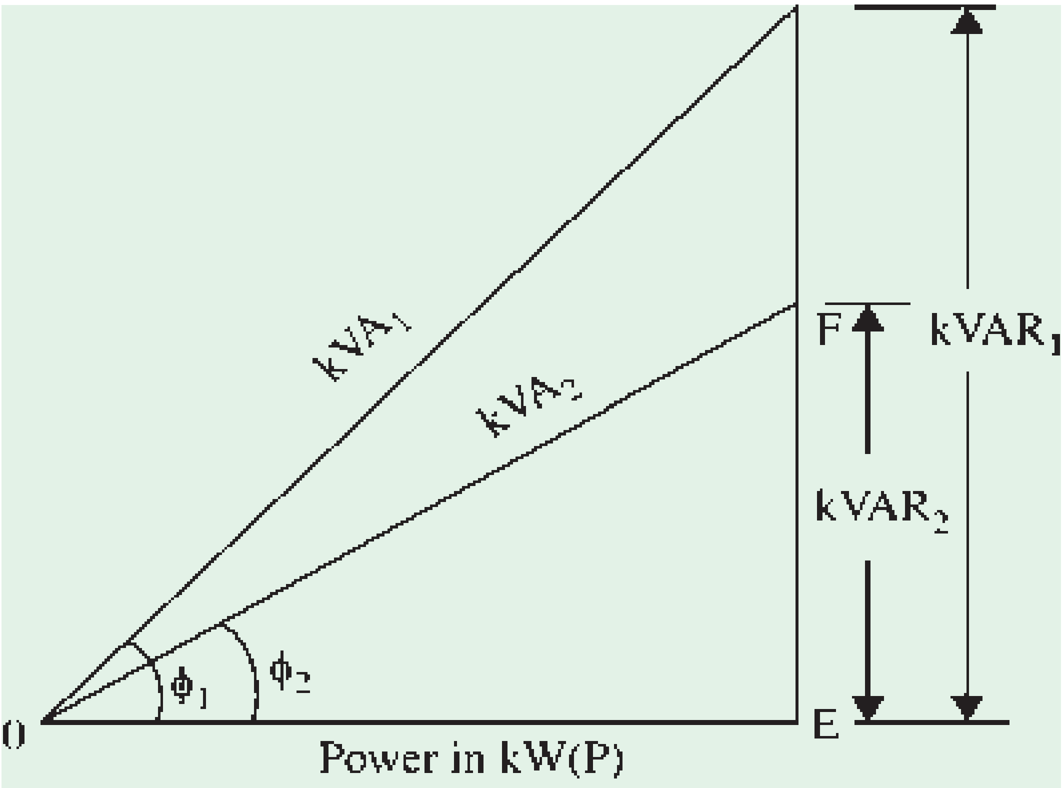
$$= q(1+r)^n + q(1+r)^{n-1} + \ldots\ldots\ldots\ldots q(1+r)^2 + q(1+r)$$

$$= q\,\frac{(1+r)^{n+1} - (1+r)}{(1+r) - 1} = q\,\frac{1+r}{r}\,[(1+r)^n - 1]$$

This sum must, obviously, be equal to the cost of renewal Q.

$$\therefore \qquad Q = q\,[(1+r)^n - 1] \text{ or } q = Q\,\frac{r}{1+r} \div [(1+r)^n - 1]$$

Hence, the total annual charge on the installation is $(rP + q)$ *i.e.* the plant should bring in so much money every year.



An overview of a combined cycle power plant

50.3. Indian Currency

The basic unit of Indian currency is rupee (Re). Its plural form is rupees (Rs.) One rupee contain 100 paisa. Higher multiples of rupees in common use are :

1 lakh (or lac) = Rs. 100,000 = Rs. 10^5 = Rs. 0.1 million

1 crore = 100 lakh = Rs. 10^7 = Rs. 10 million

Example 50.1. *Find the total annual charge on an installation costing Rs. 500,000 to buy and install, the estimated life being 30 years and negligible scrap value. Interest is 4% compounded annually.*

Solution. Since scrap value is negligible, $Q = P$. Now Q = Rs. 500,000; $r = 0.04$, $n = 30$ years.

$$\therefore \qquad q = 500,000 \times \frac{0.04}{1.04} \div [1.04^{30} - 1] = \frac{500,000 \times 0.04}{1.04 \times 2.236} = 8,600$$

Hence, the total annual charge on the installation is

$$= rP + q = (0.04 \times 500,000) + 8,600 = \textbf{Rs. 28,600}$$

Example 50.2. *A power plant having initial cost of Rs. 2.5 lakhs has an estimated salvage value of Rs. 30,000 at the end of its useful life of 20 years. What will be the annual deposit necessary if it is calculated by :*

(i) straight-line depreciation method. (ii) sinking-fund method with compound interest at 7%.

(Electrical Engineering-III, Poona Univ.)

Solution. Here, $Q = P$ – scrap value = Rs. 250,000 – Rs. 30,000 = Rs. 220,000

$$r = 0.07,\ n = 20$$

(*i*) Total depreciation of 20 years = Rs. 220,000

$\therefore$ annual depreciation = Rs. 220,000/20 = Rs. 11,300.

$\therefore$ annual deposit $= rP + q = 0.07 \times 250{,}000 + 11{,}000 =$ **Rs. 28,500**

(*ii*)
$$q = Q\frac{r}{r+1} \div [(1+r)^n - 1]$$

$$= \text{Rs. } 220{,}000\ [1.0720 - 1] = \text{Rs. } 5015.$$

Annual deposit $= 0.07 \times 250{,}000 + 5015 =$ **Rs. 22,515**

Example 50.3. *A plant initially costing Rs. 5 lakhs has an estimated salvage value of Rs. 1 lakh at the end of its useful life of 20 years. What will be its valuation half-way through its life* **(*a*)** *on the basis of straight-line depreciation and* **(*b*)** *on the sinking-fund basis at 8% compounded annually?*

Solution. (*a*) In this method, depreciation is directly proportional to time.

Total depreciation in 20 years = Rs. (5 – 1) = Rs. 4 lakhs

$\therefore$ depreciation in 10 years = Rs. 4/2 = Rs. 2 lakhs

$\therefore$ its value after 10 years = (5 – 2) = **Rs. 3 lakhs.**

(*b*) Now, $Q = 5 - 1 =$ Rs. 4 lakhs ; $r = 0.08$, $n = 20$

The annual charge is
$$q = Q\frac{r}{r+1} \div [(1+r)^n - 1]$$

$$\therefore \quad q = 4 \times 10^5 \times \frac{0.08}{1.08}[1.08^{20} - 1] = \text{Rs. } 8095$$

At the end of 10 years, the amount deposited in the sinking fund would become

$$= q\frac{1+r}{r}[(1+r)^n - 1] = 8095 \times \frac{1.08}{0.08} \times (1.08^{10} - 1) = \text{Rs. } 126{,}647$$

$\therefore$ value at the end of 10 years = Rs. 500,000 – Rs. 126,647

= Rs. 373,353 = **Rs. 3.73353 lakhs.**

50.4. Factors Influencing Costs and Tariffs of Electric Supply

In the succeeding paragraphs we will discuss some of the factors which determine the cost of

Automation of electricity production, transmission and distribution helps in the effective cost management generating electric energy and hence the rates or tariffs of charging for this energy. The cost is composed of ***(i) standing charges which are independent of the output*** and ***(ii) running or operating charges which are proportional to the output.*** The size or capacity of the generating plant and hence the necessary capital investment is determined by the maximum demand imposed on the generating plant.

50.5. Demand

By '***demand***' of a system is meant its load requirement (usually in kW or kVA) averaged over a suitable and specified *interval* of time of short duration.

It should be noted that since '***demand***' means the load averaged over an ***interval*** of time, there is no such thing as ***instantaneous*** demand.

50.6 Average Demand

By ***average*** demand of an installation is meant its average power requirement during some specified period of time of considerable duration such as a day or month or year giving us daily or monthly or yearly average power respectively.

Obviously, the average power demand of an installation during a specific period can be obtained by dividing the energy consumption of the installation in kWh by the number of hours in the period.

In this way, we get the arithmetical average.

$$\text{Average power} = \frac{\text{kWh consumed in the period}}{\text{hours in the period}}$$

50.7. Maximum Demand

The maximum demand of an installation is defined as the greatest of all the demands which have occurred during a given period.

It is measured, according to specifications, over a prescribed time interval during a certain period such as a day, a month or a year.

It should be clearly understood that it is not the greatest instantaneous demand but the greatest average power demand occuring during any of the relatively short intervals of 1-minute, 15-minute or 30 minute duration within that period.

In Fig. 50.1 is shown the graph of an imaginary load extending over a period of 5 hours. The maximum demand on 30 min. interval basis occurs during the interval *AB i.e.* from 8-30 p.m to 9-00 p.m. Its value as calculated in Fig. 50.1 is 288 kW. A close inspection of the figure shows that average load is greater during the 30 min. interval *AB* than it is during any other 30-min interval during this period of 5 hours. The average load over 30-min. interval *AB* is obtained first by scaling kW instantaneous demands at five equidistant points between ordinates *AC* and *BD* and then by taking arithmetic average of these values as shown. Hence, 30 min. maximum demand from the above load graph is 288 kW.

It may be noted that the above method of averaging can be made to yield more accurate results by ***(i)*** considering a large number of ordinates and ***(ii)*** by scaling the ordinates more precisely.

It may also be noted that if the maximum demand were to be based on a 15 min. interval, then it will occur during the 15-min. interval *MN* and its value will be 342 kW as shown in Fig. 50.1. It is seen that not only has the position of maximum demand changed but its value has also changed. The 30-min. maximum demand has lesser value than 15-min. max. demand. In the present case, 1-min. max. demand will have still greater value and will occur somewhere near point *M*.

From the above discussion, it should be clear that the unqualified term "maximum demand" is indefinite and has no specific meaning. For example, a statement that "maximum demand is 150 kW" carries no specific meaning. To render any statement of maximum demand meaningful, it is necessary

(i) to indicate the *period* of load duration under consideration and ***(ii)*** to specify the ***time interval*** used *i.e.* 15-min. or 30-min. etc. and also ***(iii)*** the method used for averaging the demand during that interval.

Now, let us see why it is the *average maximum demand* over a definite *interval* of time that is of interest rather than the *instantaneous maximum demand*.

Maximum demand determinations are mostly used for estimating the capacity (and hence cost) of the generator and other electrical apparatus required for serving a certain specific load.

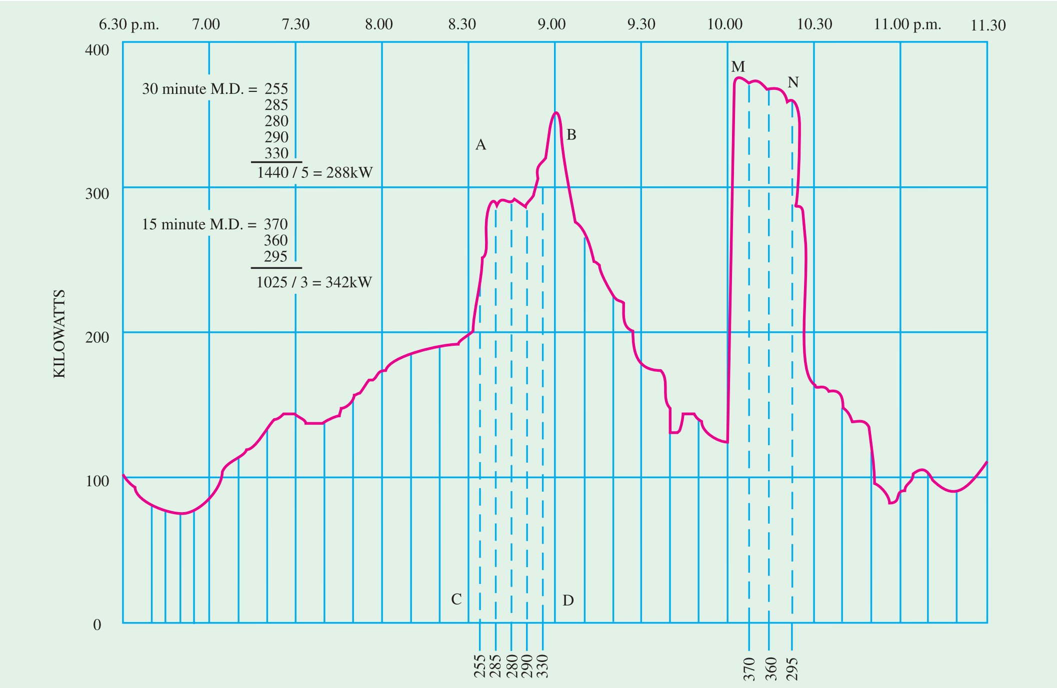


Fig. 50.1

The one main reason why maximam demand values are important is because of the direct bearing they have in establishing the capacity of the generating equipment or indirectly, the initial investment required for serving the consumers. The amount of this investment will have a further effect on fixing of rates for electric service. Since all electric machines have ample overload capacity *i.e.* they are capable of taking 100% or more overloads for short periods without any permanent adverse effects, it is not logical or economically desirable to base the continuous capacity requirements of generators on instantaneous maximum loads which will be imposed on them only momentarily or for very short periods.

Consider the graph of the power load (Fig. 50.2) to be impressed on a certain generator. Let it be required to find the rating of a generator capable of supplying this load. It is seen that there are peak loads of short durations at point *A*, *B*, *C* and *D* of values 250, 330, 230 and 260 kW. However, during the interval *EF* a demand of 210 kW persists for more than half an hour. Hence, in this particular case, the capacity of the generator required, as based on 30-min. maximum demand, should be 210 kW, it being of course, assumed that 4-hour load conditions graphed in Fig. 50.2 are typical of the conditions which exist during any similar period of generator's operation.

In the end, it may be remarked that the exact time interval for maximum demand determinations, over which the greatest demand is averaged varies not only with the characteristics of the load but with the policy of the firm measuring the load. However, 15-min. interval is now most generally used, peak load of shorter durations being considered as temporary overloads.

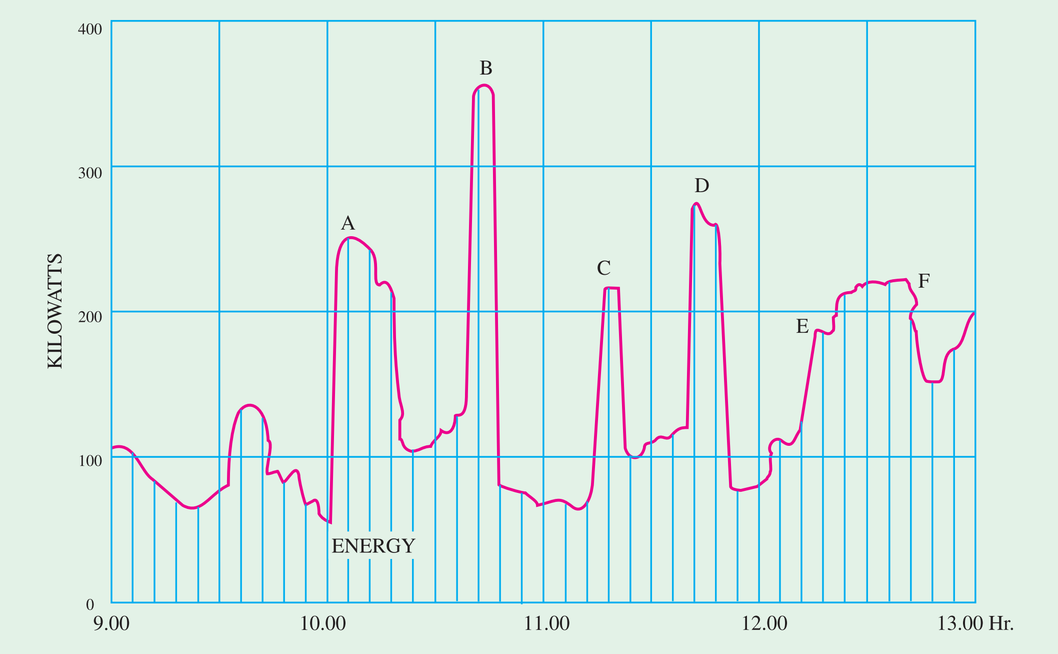


Fig. 50.2

50.8. Demand Factor

Demand factors are used for estimating the proportion of the total connected load which will come on the power plant at one time. It is defined as the ***ratio of actual maximum demand made by the load to the rating of the connected load.***

$$\text{Demand factor} = \frac{\text{maximum demand}}{\text{connected load}}$$

The idea of a demand factor was introduced because of the fact that normally the kW or kVA maximum demand of a group of electrical devices or 'receivers' is always less than the sum of the kW or kVa ratings or capacities of these receivers. There are two reasons for the existence of this condition **(*i*)** the electrical apparatus is usually selected of capacity somewhat greater than that actually required in order to provide some reserve or overload capacity and **(*ii*)** in a group of electrical devices it very rarely happens that all devices will, at the same time, impose their maximum demands which each can impose *i.e.* rarely will all 'receivers' be running full-load simultaneously.

The demand factor of an installation can be determined if **(*i*)** maximum demand and **(*ii*)** connected load are known.

Maximum demand can be determined as discussed in Art. 50.7 whereas connected load can be calculated by adding together the name-plate ratings of all the electrical devices in the installation. The value of demand factor is always less than unity.

Demand factors are generally used for determining the capacity and hence cost of the power equipment required to serve a given load. And because of their influence on the required investment, they become important factors in computing rate schedules.

As an example, suppose a residence has the following connected load : three 60-W lamps; ten 40-W lamps; four 100-W lamps and five 10-W lamps. Let us assume that the demand meter indicates a 30-min. maximum demand of 650 W. The demand factor can be found as follows :

Connected load $= (3 \times 60) + (10 \times 40) + (4 \times 100) + (5 \times 10) = 1{,}030$ W

30-min. max.demand $= 650$ W

Hence, the demand factor of this lighting installation is given as

$$= \frac{\text{max. demand}}{\text{connected load}} = \frac{650}{1.030} = \mathbf{0.631 \text{ or } 63.1\%}$$

Demand factors of lighting installations are usually fairly constant because lighting loads are not subject to such sudden and pronounced variations as like power loads.

50.9. Diversity of Demand

In central-station parlance, diversity of demand implies that maximum demands of various consumers belonging to different classes and the various circuit elements in a distribution system are not coincident. In other words, the maximum demands of various consumers occur at different times during the day and not simultaneously. It will be shown later that from the economic angle, it is extremely fortunate that there exists a diversity or non-simultaneity of maximum demand of various consumers which results in lower costs of electric energy.

For example, residence lighting load is maximum in the evening whereas manufacturing establishments require their maximum power during daytime hours. Similarly, certain commercial establishments like department stores usually use more power in day-time than in the evening whereas some other stores like drug stores etc. use more power in the evening.

The economic significance of the concept of diversity of demand can only be appreciated if one considers the increase in the capacity of the generating and distributing plant (and hence the corresponding increase in investment) that would be necessary, if the maximum demands of all the consumers occurred simultaneously. It is of great concern to the engineer because he has to take it into consideration while planning his generating and distributing plant. Also diversity is an important element in fixing the rates of electric service. If it were not for the fact that the coincident maximum demand imposed on a certain station is much less than the sum of maximum demands of all the consumers fed by that station, the investment required for providing the electric service would have been far in excess of that required at present. Because of the necessity of increase in investment, that cost of electric supply would also have been increased accordingly.

50.10. Diversity Factor

The non-coincidence of the maximum demands of various consumers is taken into consideration in the so-called diversity factor which is defined as the ratio of the sum of the individual maximum demands of the different elements of a load during a specified period to the simultaneous (or coincident) maximum demand of all these elements of load during the same peirod.

$$\text{Diversity factor*} = \frac{\text{maximum demand}}{\text{connected load}}$$

Its value is usually much greater than unity. It is clear that if all the loads in a group impose their maximum demands simultaneously, then diversity factor is equal to unity. High value of diversity factor means that more consumers can be supplied for a given station maximum demand and so lower prices can be offered to consumers. Usually domestic load gives higher value of diversity factor than industrial load. As shown in Fig. 50.3, suppose that the maximum demands of six elements of a load as observed from their maximum demand meters M_1 and M_2 etc. are 620 W, 504 W, 435 W, 380 W, 160 W and 595 W respectively.

* Sometimes, the diversity factor is given by certain authors as the reciprocal of the value so obtained.

Also, suppose that the (coincident) maximum demand of the whole group as observed by the maximum demand meter *MT* is only 900 W. It is so because the maximum demands of all load elements did not occur simultaneously.

Sum of individual maximum demands

$$= 620 + 504 + 435 + 380 + 160 + 595 = 2694 \text{ W} = 2.694 \text{ kW}$$

$\therefore$ diversity factor $= 2.694/0.9 = \mathbf{2.99}$

It may be noted here that because of the diversity of demand, the maximum demand on a transformer is less than the sum of the maximum demands of the consumers supplied by that transformer. Further, the maximum demand imposed on a feeder is less than the sum of the maximum demands of transformers connected to that feeder. Similarly, the maximum demand imposed on the generating station is less than the sum of the maximum demands of all the feeders suplied station is less than the sum of the maximum demands of all the feeders supplied from the station. The effective demand of a consumer on a generator is given as follows :

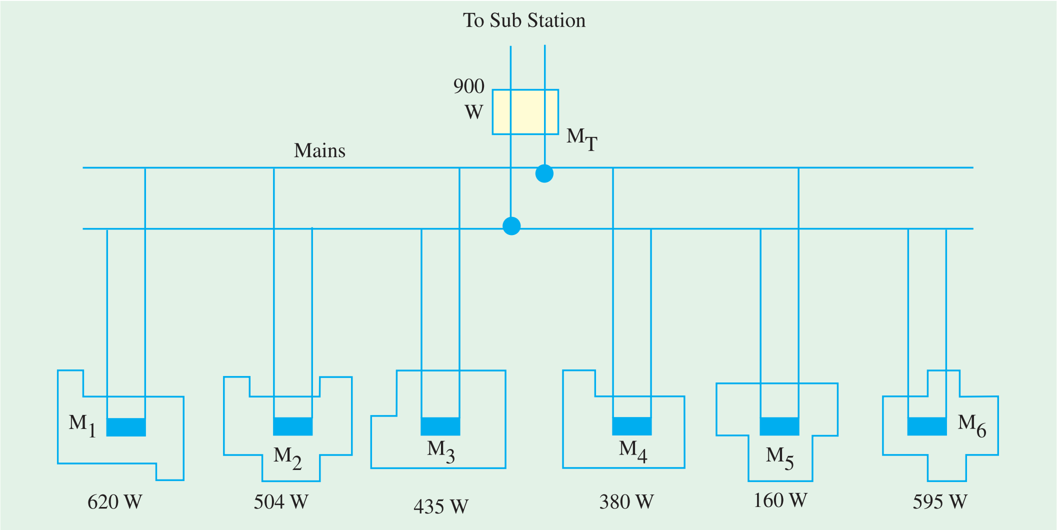


Fig. 50.3

Multiply this connected load by demand factor and then divide the product by diversity factor for consumer to generator.

As an example, let us find the total diversity factor for a residence lighting system whose component diversity factors are : between consumers 2.6 ; between transformers 1.32 ; between feeders 1.13 and between sub-stations 1.1. The total diversity factor between the consumers and the generating equipment would be the product of these component factors *i.e.*

$$= 2.6 \times 1.32 \times 1.13 \times 1.1 = 4.266$$

The factor may now be used in determining the effective demand of consumers on the generator.

It may be proved that generating equipment can be economized by grouping on one supply source different elements of load having high diversity factor. In fact, the percentage of the generating equipment which can be eliminated is equal to 100 percent minus the reciprocal of diversity factor expressed as a percentage. Suppose four loads of maximum demand 120, 360, 200 and 520 kVA respectively are to be supplied. If each of these loads were supplied by a separate transformer, then aggregate transformer capacity required would be = 120 + 360 + 200 + 520 = 1200 kVA. Suppose these loads had a diversity factor of 2.5 among themselves, then (coincident) maximum demand of the whole group would be 1200/2.5 = 480 kVA.

In other words, a single 480 kVA transformer can serve the combined load. The saving = 1200 – 480 = 720 kVA which expressed as a percentage is 720 × 100/1200 = 60%. Now, reciprocal of diversity factor = 1/2.5 = 0.4 or 40 %. The percentage saving in the required apparatus is also = 100 – 40 = 60% which proves the statement made above.

50.11. Load Factor

It is defined ***as the ratio of the average power to the maximum demand***.

It is necessary that in each case the time interval over which the maximum demand is based and the ***period**** over which the power is averaged must be definitely specified.

If, for example, the maximum demand is based on a 30-min. interval and the power is averged over a month, then it is known as 'half-hour monthly' load factor.

Load factors are usually expressed as percentages. The average power may be either generated or consumed depending on whether the load factor is required for generating equipment or receiving equipment.

When applied to a generating station, annual load factor is

$$= \frac{\text{No. of units actually supplied/year}}{\text{Max. possible No. of units that can be supplied}}$$

It may be noted that *maximum* in this definition means the value of the maximum peak load and *not the maximum* kW *installed capacity of the plant equipment of the station.*

$$\therefore \quad \text{annual load factor} = \frac{\text{No. of units actually supplied/year}}{\text{Max. possible demand} \times 8760}$$

$$\text{Monthly load factor} = \frac{\text{No. of units actually supplied/month}}{\text{Max. possible demand} \times 24 \times 30}$$

When applied to a consuming equipment

$$\text{annual load factor} = \frac{\text{No. of units consumed/year}}{\text{Max. demand} \times 8760}$$

$$\text{monthly load factor} = \frac{\text{No. of units consumed/month}}{\text{Max. demand} \times 24 \times 30}$$

$$\text{Daily load factor} = \frac{\text{No. of units consumed/day}}{\text{Max. demand} \times 24}$$

$$\text{In general, load factor} = \frac{\text{Average power}}{\text{Max. demand}} \text{ per year or per month or per day}$$

The value of maximum demand can be found by using a maximum demand meter set for 30-min. or 15-min. interval as already explained in Art. 50.7. The average power can also be found either by graphic method explained below or by using a planimeter.

In the graphic method, momentary powers are scaled or read from the load-graph at the end of a number of suitable and equal time intervals over the entire time comprehended by the graph. Then these are added up. Average power is obtained by dividing this sum by the number of periods into which the total time was apportioned.

* If not specified, it is assumed to be one year of 24 × 365 = 8,760 hours.

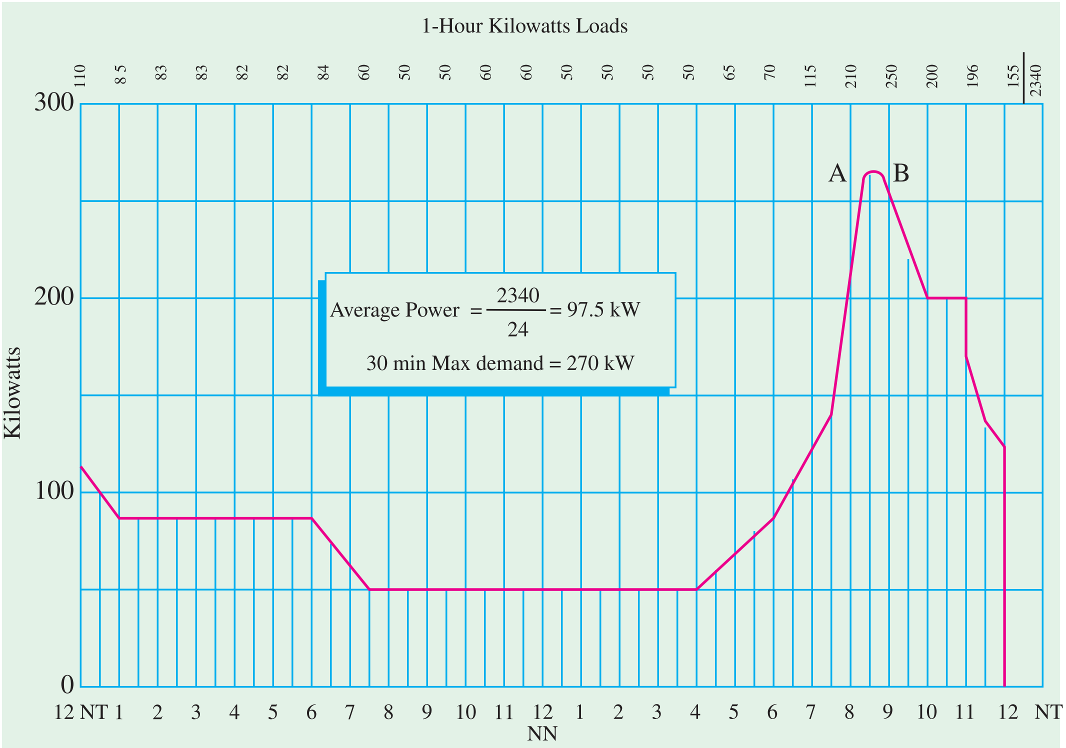


Fig. 50.4

The number of time intervals into which the entire time is apportioned is determined by the contour of the graph and the degree of accuracy required.

The general, the accuracy will increase with the increase in the number of the intervals. For a graph having a smooth contour and comprehending 24 hours, sufficient accuracy can be obtained by taking 1 hour intervals. However, in the case of a graph which has extremely irregular contours and comprehends short time intervals, reasonable accuracy can only be obtained if 15-min. or even 1 min. intervals are used.

As an example, let us find the load factor of a generating equipment whose load graph (imaginary one) is shown in Fig. 50.4. For calculating average power over a period of 24 hours, let us take in view of the regularity of the curve, a time interval of 1 hour as shown. The average power is 97.5 kW. Now, the 30-min. maximum demand is 270 kW and occurs during 30-min. interval of *A B*. Obviously load factor = 97.5 × 100/270 = 36.1%.

50.12. Significance of Load Factor

Load factor is, in fact, an index to the proportion of the whole time a generator plant or system is being worked to its full capacity. The generating equipment has to be selected on the basis of maximum power demand that is likely to be imposed on it. However, because of general nature of things, it seldom happens that a generating equipment has imposed on it during all the 8,760 hrs of a year the maximum load which it can handle. But whether the equipment is being worked to its full capacity or not, there are certain fixed charges (like interest, depreciation, taxes, insurance, part of staff salaries etc.) which are adding up continuously. In other words, the equipment is costing money to its owner whether working or idle. The equipment earns a net profit only during those hours when it is fully loaded and the more it is fully loaded, the more is the profit to the owner. Hence, from the standpoint of economics, it is desirable to keep the equipment loaded for as much time as possible i.e. it is economical to obtain high load factors.

If the load factor is poor *i.e.* kWh of electric energy produced is small, then charge per kWh would obviously be high. But if load factor is high *i.e.* the number of kWh generated is large, then cost of production and hence charge per kWh are reduced because now the standing charges are distributed over a larger number of units of energy.

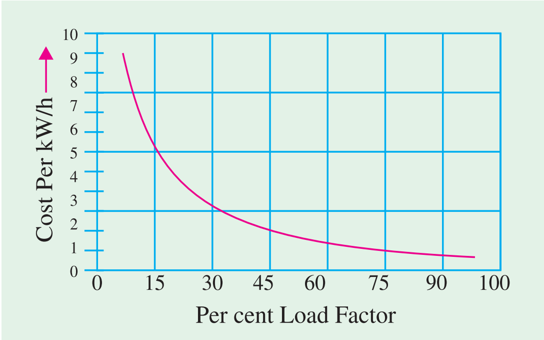


Fig. 50.5

The fact that fixed charges per kWh increase with decreasing load factor and vice versa is brought out in Ex. 50.15 and is graphically shown in Fig. 50.5.

It may be remarked here that increase of diversity in demand increases the load factor almost in direct proportion.

Load factor of a generating plant may be improved by seeking and accepting off-peak loads at reduced rates and by combining lighting, industrial and inter-urban railway loads.

Example 50.4. *A consumer has the following connected load : 10 lamps of 60 W each and two heaters of 1000 W each. His maximum demand is 1500 W. On the average, he uses 8 lamps for 5 hours a day and each heater for 3 hours a day. Find his total load, montly energy consumption and load factor.* **(Power Systems-I, AMIE, Sec. B, 1993)**

Solution. Total connected load = $10 \times 60 + 2 \times 1000$ = **2600 W**.

Daily energy consumption is = $(8 \times 60 \times 5) + (2 \times 1000 \times 3)$ = 8400 Wh = **8.4 kWh**

Monthly energy consumption = 8.4×30 = **252 kWh**

$$\text{Monthly load factor} = \frac{252}{1500 \times 10^{-3} \times 24 \times 30} = \mathbf{0.233 \text{ or } 23.3\ \%}$$

Example 50.5. *The load survey of a small town gives the following categories of expected loads.*

	Type	*Load in kW*	*% D.F.*	*Group D.F.*
1.	*Residential lighting*	*1000*	*60*	*3*
2.	*Commercial lighting*	*300*	*75*	*1.5*
3.	*Street lighting*	*50*	*100*	*1.0*
4.	*Domestic power*	*300*	*50*	*1.5*
5.	*Industrial power*	*1800*	*55*	*1.2*

What should be the kVA capacity of the S/S assuming a station p.f. of 0.8 lagging ?

Solution. (*i*) Residential lighting. Total max. demand = 1000×0.6 = **600 kW**

Max. demand of the group = 600/3 = **200 kW**

(*ii*) Commercial lighting Total max. demand = 300×0.75 = **225 kW**

Max. demand of the group = 225/1.5 = **150 kW**

(*iii*) Street lighting Total max. demand = **50 kW**

Max. demand of the group = 50/1 = **50 kW**

(*iv*) Domestic power Total max. demand = 300×0.5 = **150 kW**

Max. demand of the group = 150/1.5 = **100 kW** (*v*) **Industrial power** Total max. demand = 1800×0.55 = **990 kW**

Max. demand of the group = 990/1.2 = **825 kW**

Total max. demand at the station = 200 + 150 + 50 + 100 + 825 = **1325 kW.**

Capacity of the sub-station required at a p.f. of 0.8 lagging = 1325 / 0.8 = **1656 kVA**

Example 50.6. *A consumer has the following load-schedule for a day :*

From midnight (12 p.m.) to 6 a.m. = 200 W ; From 6 a.m. to 12 noon = 3000 W

From 12 noon to 1 p.m. = 100 W ; From 1 p.m. to 4 p.m. = 4000 W

From 4 p.m. to 9 p.m. = 2000 W ; From 9 p.m. to mid-night (12 p.m.) = 1000 W

Find the load factor.

If the tariff is 50 paisa per kW of max. demand plus 35 paisa per kWh, find the daily bill the consumer has to pay. **(Electrical Engineering-III, Poona Univ.)**

Solution. Energy consumed per day *i.e.* in 24 hours

$= (200 \times 6) + (3000 \times 6) + (100 \times 1) + (4000 \times 3) + (2000 \times 5) + (1000 \times 3) = 44{,}300$ Wh

Average power = 44,300/24 = 1846 W = 1.846 kW

$$\text{Daily load factor} = \frac{\text{average power}}{\text{max. power demand}} = \frac{1846}{4000} = 0.461 \text{ or } \mathbf{46.1\%}$$

Since max. demand = 4 kW M.D. charge = $4 \times 1/2$ = Rs. 2/-

Energy consumed = 44.3 kWh Energy charge = Rs. $44.3 \times 35/100$ = Rs. 15.5/-

$\therefore$ daily bill of the consumer = Rs. 2 + Rs. 15.5 = **Rs. 17.5**

Example 50.7. *A generating station has a connected load of 43,000 kW and a maximum demand of 20,000 kW, the units generated being 61,500,000 for the year. Calculate the load factor and demand factor for this case.*

Solution.
$$\text{Demand factor} = \frac{\text{maximum demand}}{\text{connected load}} = 0.465 \text{ or } \mathbf{46.5\%}$$

Average power = 61,500,000/8,760 = 7020 W ($\because$ 1 year = 8760 hr)

$$\therefore \quad \text{Load factor} = \frac{\text{average power}}{\text{max. power demand}} = \frac{7020}{20{,}000} = 0.351 \text{ or } \mathbf{35.1\%}$$

Example 50.8. *A 100 MW power station delivers 100 MW for 2 hours, 50 MW for 6 hours and is shut down for the rest of each day. It is also shut down for maintenance for 45 days each year. Calculate its annual load factor.* **(Generation and Utilization, Kerala Univ.)**

Solution. The station operates for (365 – 45) = 320 days in a year. Hence, number of MWh supplied in one year = $(100 \times 2 \times 320) + (50 \times 6 \times 320)$ = 160,000 MWh

Max. No. of MWh which can be supplied per year with a max. demand of 100 MW is

$= 100 \times (320 \times 24) = 768{,}000$ MWh

$$\therefore \quad \text{load factor} = \frac{160{,}000}{760{,}000} \times 100 = \mathbf{20.8\%}$$

Example 50.9. *Differentiate between fixed and running charges in the operation of a power company.*

Calculate the cost per kWh delivered from the generating station whose

(*i*) *capital cost = Rs. 10^6,*
(*ii*) *annual cost of fuel = Rs. 10^5,*
(*iii*) *wages and taxes = Rs. 5×10^5,*
(*iv*) *maximum demand laod = 10,000 kW,*
(*v*) *rate of interest and depreciation = 10%*
(*vi*) *annual load factor = 50%.*

Total number of hours in a year is 8,760. **(Electrical Technology-I, Bombay Univ.)**

Solution. Average power demand = max. load × load factor = 10,000 × 0.5 = 5,000 kW

Units supplied/year = 5,000 × 8,760 = 438 × 10^5 kWh

Annual cost of the fuel plus wages and taxes = Rs. 6 × 10^5

Interest and depreciation charges/year = 10% of Rs. 106 = Rs. 10^5

Total annual charges = Rs. 7 × 10^5 ; Cost / kWh = Rs. 7 × 10^5/438 × 10^5 = **1.6 paisa**

Example 50.10. *A new colony of 200 houses is being established, with each house having an average connected load of 20 kW. The business centre of the colony will have a total connected load of 200 kW. Find the peak demand of the city sub-station given the following data.*

	Demand factor	*Group D.F.*	*Peak D.F.*
Residential load	*50%*	*3.2*	*1.5*
Business load	*60%*	*1.4*	*1.2*

Solution. The three demand factors are defined as under :

$$\text{Demand factor} = \frac{\text{max. demand}}{\text{connected load}}$$

$$\text{group D.F.} = \frac{\text{sum of individual max. demands}}{\text{actual max. demand of the group}}$$

$$\text{Peak D.F.} = \frac{\text{max. demand of consumer group}}{\text{demand of consumer group at the time of system peak demand}}$$

Max. demand of each house = 2 × 0.5 = 1.0 kW

Max. demand of residential consumer = 1 × 200/3.2 = 62.5 kW

Demand of the residential consumer at the time of the system peak = 62.5 / 1.5 = 41.7 kW

Max. demand of commercial consumer = 200 × 0.6 = 120 kW

Max. demand of commercial group = 120/1.4 = 85.7 kW

Commercial demand at the time of system peak = 85.7 / 1.2 = 71.4 kW

Total demand of the residential and commercial consumers at the time of system peak

= 41.7 + 71.4 = **113 kW**

Example 50.11. *In Fig. 50.6 is shown the distribution network from main sub-station. There are four feeders connected to each load centre sub-station. The connected loads of different feeders and their maximum demands are as follows :*

Feeder No.	*Connected load, kW*	*Maximum Demand, kW*
1.	*150*	*125*
2.	*150*	*125*
3.	*500*	*350*
4.	*750*	*600*

If the actual demand on each load centre is 1000 kW, what is the diversity factor on the feeders? If load centres B, C and D are similar to A and the diversity factor between different load centres is 1.1, calculate the maximum demand of the main sub-station. What would be the kVA capacity of the transformer required at the main sub-station if the overall p.f. at the main sub-station is 0.8 ?

Solution. Diversity factor of the feeders

$$= \frac{\text{total of max. demand of different feeders}}{\text{simultaneous max. demand}} = \frac{125 + 125 + 350 + 600}{1000} = 1.2$$

Total max. demand of all the 4 load centre sub-stations = 4 × 1000 = **4000 kW**

Diversity factor of load centres = 1.1

Simultaneous max. demand on the main sub-station = 4000/1.1 = 3636 kW

The kVA capacity of the transformer to be used at the sub-station = 3636/0.8 = **4545**

Example 50.12. *If a generating station had a maximum load for the year of 18,000 kW and a load factor of 30.5% and the maximum loads on the sub-stations were 7,500, 5,000, 3,400, 4,600 and 2,800 kW, calculate the units generated for the year and the diversity factor.*

Solution. Load factor $= \dfrac{\text{average power}}{\text{maximum power demand}}$ $\therefore\ 0.305 = \dfrac{\text{average power}}{18{,}000}$

$\therefore$ average power = 18,000 × 0.305 kW

kWh generated per year = 8,000 × 0.305 × 8.760 = **48.09 × 106 kWh**

Sum of individual maximum demands = 7,500 + 5,000 + 3,400 + 4,600 + 2,800 = 23,300

$\therefore$ diversity factor=23,300/18,000 = **1.3 (approx.)**

Example 50.13. *A power station is supplying four regions of load whose peak loads are 10MW, 5 MW, 8 MW and 7 MW. the diversity factor of the load at the station is 1.5 and the average annual load factor is 60%. Calculate the maximum demand on the station and the annual energy supplied from the station. Suggest the installed capacity and the number of units taking all aspects into account.* **(A.M.I.E. Sec. B, Winter 1990)**

Solution. Diversity factor $= \dfrac{\text{sum of individual max. demands}}{\text{max. demands of the whole load}}$

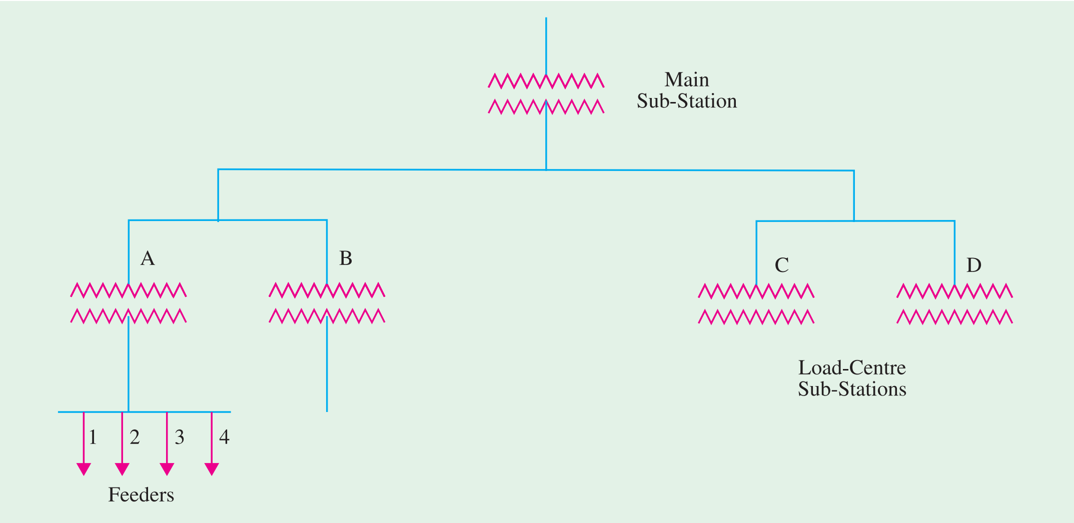


Fig. 50.6

$\therefore$ Max. demand of the whole load imposed on the station = (10 + 5 + 8 + 7)/1.5 = **20 MW**

Now, annual load factor $= \dfrac{\text{No. of units supplied/year}}{\text{Max. demand} \times 8760}$

$\therefore$ No. of units supplied / year = $0.6 \times 20 \times 10^3 \times 8760$ = **105.12×10^6 kWh**

Provision for future growth in load may be made by making installed capacity 50% more than the maximum demand of the whole load. Hence, installed capacity is 20 × 1.5 = 30 MW. Four generators, two of **10 MW** each and other two of **5 MW** each may be installed.

Example 50.14. *The capital cost of 30 MW generating station is Rs. 15 × 10^6. The annual expenses incurred on account of fuel, taxes, salaries and maintenance amount to Rs. 1.25 × 10^6. The station operates at an annual load factor of 35%. Determine the generating cost per unit delivered, assuming rate of interest 5% and rate of depreciation 6%.* **(Electrical Power-I, Bombay Univ.)**

Solution. Average power = max. power × load factor = $30 \times 10^6 \times 0.35$ = 10,500 kW

Units produced/year = 10,500 × 8,760 = 91.98×10^6 kWh; Annual expenses = Rs. 1.25×10^6

Depreciation plus interest = 11% of capital cost = 11% of Rs. 15×10^6 = Rs. 1.65×10^6

Total expenses/year = Rs. 1.25×10^6 + Rs. 1.65×10^6 = Rs. 2.9×10^6

∴ cost/kWh = Rs. $2.9 \times 10^6 / 91.98 \times 10^6$ = **3.15 paisa / kWh.**

Example 50.15. *A generating plant has a maximum capacity of 100 kW and costs Rs.300,000. The fixed charges are 12% consisting of 5% interest, 5% depreciation and 2% taxes etc. Find the fixed charges per kWh generated if load factor is **(i)** 100% and **(ii)** 25%.*

Solution. Annual fixed charges = Rs. 300,000 × 12/100 = Rs. 36,000

With a load factor of 100%, number of kWh generated per year = 100 × 1 × 8,760

= 876,000 kWh.

Similarly, units generated with a load factor of 25% = 100 × 0.25 × 8,760 = 219,000 kWh.

(i) Fixed charge / kWh = 36,000 × 100/876,000 = **4.1 paisa**

(ii) Fixed charge / kWh = 36,000 × 100/219,000 = **16.4 paisa**

As seen, the charge has increased four-fold. In fact, charge varies inversely as the load factor.

Example 50.16. *The annual working cost of a thermal station is represented by the formula Rs. (a + b kW + c kWh) where a, b and c are constants for that particular station, kW is the total installed capacity and kWh is the energy produced per annum.*

*Determine the values of a, b and c for a 100 MW station having annual load factor of 55% and for which **(i)** capital cost of buildings and equipment is Rs. 90 million, **(ii)** the annual cost of fuel, oil, taxation and wages and salaries of operating staff is Rs. 1,20,000, **(iii)** interest and depreciation on buildings and equipment are 10% p.a., **(iv)** annual cost of orginasation, interest on cost of site etc. is Rs. 80,000.*

Solution. In the given formula, *a* represents the fixed cost, *b* semi-fixed cost and *c* the running cost. Here, *a* = **Rs. 80,000**

An overview of a thermal power plant

Now, $b \times$ kW minimum demand = semi-fixed cost

or, $b \times 100 \times 10^3 = 90 \times 10^6 \quad \therefore\ b = \mathbf{900}$

Total units generated per annum = kW max. demand $\times$ load factor $\times$ 8760

$= 100 \times 10^3 \times 0.55 \times 8760 = 438 \times 10^6$ kWh

Since running cost is Rs. 1,20,000

$\therefore \quad c \times 438 \times 10^6 = 1{,}20{,}000$ or $c = \mathbf{0.00027}$

Example 50.17. *In a steam generating station, the relation between the water evaporated W kg and coal consumed C kg and power in kW generated per 8-hour shift is as follows :*

$$W = 28{,}000 + 5.4 \text{ kWh}; \; C = 6000 + 0.9 \text{ kWh}$$

What would be the limiting value of the water evaporated per kg of coal consumed as the station output increases ? Also, calculate the amount of coal required per hour to keep the station running at no-load.

Solution. For an 8-hour shift, Wt. of water evaporated per kg of coal consumed is

$$\frac{W}{C} = \frac{28{,}000 + 5.4 \text{ kWh}}{6{,}000 + 0.9 \text{ kWh}}$$

As the station output increases, the ratio W/C approaches the value 5.4 / 0.9 = 6

Hence, weight of water evaporated per kg of coal approaches a limiting value of **6 kg** as the station output increases.

Since at no-load, there is no generation of output power, kWh = 0. Substituting this value of kWh in the above ratio we get,

Coal consumption per 8-hour shift = 6000 kg

$\therefore$ coal consumption per hour on no-load $= \dfrac{6000}{8} = \mathbf{725\ kg.}$

Example 50.18. *Estimate the generating cost per kWh delivered from a generating station from the following data :*

Plant capacity = 50 MW ; annual load factor = 40%; capital cost = Rs. 3.60 crores; annual cost of wages, taxation etc. = Rs. 4 lakhs; cost of fuel, lubrication, maintenance etc. = 2.0 paise per kWh generated, interest 5% per annum, depreciation 5% per annum of initial value.

(Electrical Technology, M.S. Univ. Baroda)

Solution. Average power over a year = maximum power $\times$ load factor

$= 50 \times 10^6 \times 0.4 = 2 \times 10^7$ W $= 2 \times 10^4$ kW

Units produced/year $= 20{,}000 \times 8{,}760 = 1{,}752 \times 10^5$ kWh

Depreciation plus interest = 10% of initial investment $= 0.1 \times 3.6 \times 10^7$

$= \mathbf{Rs.\ 3.6 \times 10^6}$

Annual wages and taxation etc. = Rs. 4 lakhs = Rs. 0.4×10^6

Total cost/year = Rs. $(3.6 + 0.4) \times 10^6$ = Rs. 4×10^6

Cost / kWh $= 4 \times 10^6 \times 100/1{,}752 \times 10^5 =$ **2.28 paisa**

Adding the cost of fuel, lubrication and maintenance etc., we get

Cost per kWh delivered = 2.0 + 2.28 = **4.28 paisa.**

Example 50.19. *The following data relate to a 1000 kW thermal station :*

Cost of Plant	= *Rs. 1,200 per kW*
Interest, insurance and taxes	= *5% p.a.*
Depreciation	= *5% p.a.*

Cost of primary distribution system = *Rs. 4,00,000*
Interest, insurance, taxes and depreciaton = *5% p.a.*
Cost of coal including transportation = *Rs. 40 per tonne*
Operating cost = *Rs. 4,00,000 p.a.*
Plant maintenance cost : *fixed* = *Rs. 20,000 p.a.*
variable = *Rs. 30,000 p.a.*
Installed plant capacity = *10,000 kW*
Maximum demand = *9,000 kW*
Annual load factor = *60%*
Consumption of coal = *25,300 tonne*

Find the cost of power generation per kilowatt per year, the cost per kilowatt-hour generated and the total cost of generation per kilowatt-hour. Transmission/primary distribution is chargeable to generation. **(Power Systems-I, AMIE, Sec. B, 1993)**

Solution. Cost of the plant = Rs. 1200 per kW

Fixed cost per annum is as under :

(*i*) on account of capital cost = $(1200 \times 10{,}000) \times 0.1 + 400 \times 103 \times 0.05$
= **Rs. 1.22×10^6**

(*ii*) part of maintenance cost = Rs. 20,000 = Rs. 0.02×10^6

∴ total fixed cost = $1.22 \times 10^6 + 0.02 \times 10^6$ = **Rs. 1.24×10^6**

Running or variable cost per annum is as under : (*i*) operation cost = Rs. 4,00,000, (*ii*) part of maintenance cost = Rs. 30,000,

(*iii*) fuel cost = Rs. $25{,}300 \times 40$ = Rs. 10,12,000

Total cost = 4,00,000 + 30,000 + 10,12,000 = Rs. 1.442×10^6

$$\text{Load factor} = \frac{\text{average demand}}{\text{maximum demand}} \quad \text{or} \quad 0.6 = \frac{\text{average demand}}{9{,}000 \times 8{,}760}$$

∴ average demand = **47,305 MWh**

Total cost per annum = $1.24 \times 10^6 + 1.442 \times 10^6$ = Rs. 2.682×10^6

Cost per kWh generated = $2.682 \times 10^6 / 47{,}305 \times 10^3$ = Rs. 0.0567 = **5.7 paisa.**

Since total installed capacity is 10,000 kW, the cost per kW per year

= $2.682 \times 10^6 / 10{,}000$ = **Rs. 268.2**

Example 50.20. *A consumer has an annual consumption of 176,400 kWh. The charge is Rs. 120 per kW of maximum demand plus 4 paisa per kWh.*

(i) *Find the annual bill and the overall cost per kWh if the load factor is 36%.*

(ii) *What is the overall cost per kWh, if the consumption were reduced 25% with the same load factor ?*

(iii) *What is the overall cost per kWh, if the load factor is 27% with the same consumption as in (i)* **(Utili. of Elect. Power, AMIE Sec. B)**

Solution. (*i*) Since load factor is 0.36 and there are 8760 hrs in a year,

Annual max. demand = $176{,}400/0.36 \times 8760$ = **55.94 kW**

The annual bill will be based on maximum annual demand charges plus the annual energy consumption charge.

∴ annual bill = Rs. $(55.94 \times 120 + 176{,}400 \times 0.04)$ = **Rs. 13,768**

Overall cost/kWh = Rs. 13,768 / 176,400 = **7.8 paisa.**

(ii) In this case, the annual consumption is reduced to 176,400 × 0.75 = 132,300 kWh but the load factor remains the same.

Annual max. demand = 132,300/0.36 × 8760 = 41.95 kW

∴ annual bill = Rs. (41.95 × 120 + 132,300 × 0.04) = **Rs. 10,326**

Overall cost/kWh = 10,326/132,300 = **7.8 paisa**

It will be seen that the annual max. demand charge is reduced but the overall cost per kWh remains the same.

(iii) Since, load factor has decreased to 0.27,

Annual max. demand = 176,400/0.27 × 8760 = 74.58 kW

Annual bill = Rs. (74.58 × 120 + 176,400 × 0.04) = Rs. 16,006

Overall cost/kWh = Rs. 16,006/176,400 = **9.1 paisa**

Here, it will be seen that due to decrease in load factor, the annual bill as well as cost per kWh have increased.

50.13. Plant Factor or Capacity Factor

This factor relates specifically to a generating plant unlike load factor which may relate either to generating or receiving equipment for the whole station.

It is defined as the ***ratio of the average load to the rated capacity of the power plant i.e. the aggregate rating of the generators***. It is preferable to use continuous rating while calculating the aggregate.

$$\therefore \quad \text{plant factor} = \frac{\text{average load}}{\text{rated capacity of plant}} = \frac{\text{average demand on station}}{\text{max. installed capacity of the station}}$$

It may be of interest to note that if the maximum load corresponds exactly to the plant ratings, then load factor and plant factor will be identical.

50.14. Utilization Factor (or Plant Use Factor)

It is given by the ratio of the kWh generated to the product of the capcity of the plant and the number of hours the plant has been actually used.

$$\text{Utilization factor} = \frac{\text{station output in kWh}}{\text{plant capacity} \times \text{hours of use}}$$

If there are three units in a plant of ratings kW_1, kW_2 and kW_3 and their operation hours are h_1, h_2 and h_3 respectively, then

$$\text{Utilization factor} = \frac{\text{station output in kWH}}{(kW_1 \times h_1) + (kW_2 \times h_2) + (kW_3 \times h_3)}$$

50.15. Connected Load Factor

The factor relates only to the receiving equipment and is defined as the ratio of the average power input to the connected load.

To render the above value specific, it is essential*

(i) to define the period during which average is taken and

(ii) to state the basis on which the connected load is computed.

* Wherever feasible, it should be stated on continuous-rating basis. Lighting connected load is taken equal to the sum of the wattages of all lamps in the installation whereas motor connected load is equal to the sum of the name-plate outputs of all motors (and not their input ratings).

$$\text{Connected-load factor} = \frac{\text{average power input}}{\text{connected load}}$$

It can be proved that the connected-load factor of a ***receiving*** equipment is equal to the product of its demand factor and its load factor.

$$\text{Connected-load factor} = \frac{\text{average power input}}{\text{connected load}}$$

$$= \frac{\text{Average power}}{\text{Max. demand}} \times \frac{\text{max demand}}{\text{connected load}} = \text{load factor} \times \text{demand factor}$$

50.16. Load Curves of a Generating Station

The total power requirement of a generating station can be estimated provided variation of load with time is known. Following curves help to acquire this knowledge.

(*i*) Load Curve (or Chronological Curve)

It represents the load in its proper time sequence. As shown in Fig. 50.7 (*a*), this curve is obtained by plotting the station load (in kW) along Y-axis and the time when it occurs along X-axis. Usually, such curves are plotted for one day *i.e.* for 24 hours by taking average load (kW) on hourly basis. The area under the curve represents the total energy consumed by the load in one day. Following information can be obtained from the load curve :

(*a*) maximum load imposed on the station, **(*b*)** size of the generating unit required and **(*c*)** daily operating schedule of the station.

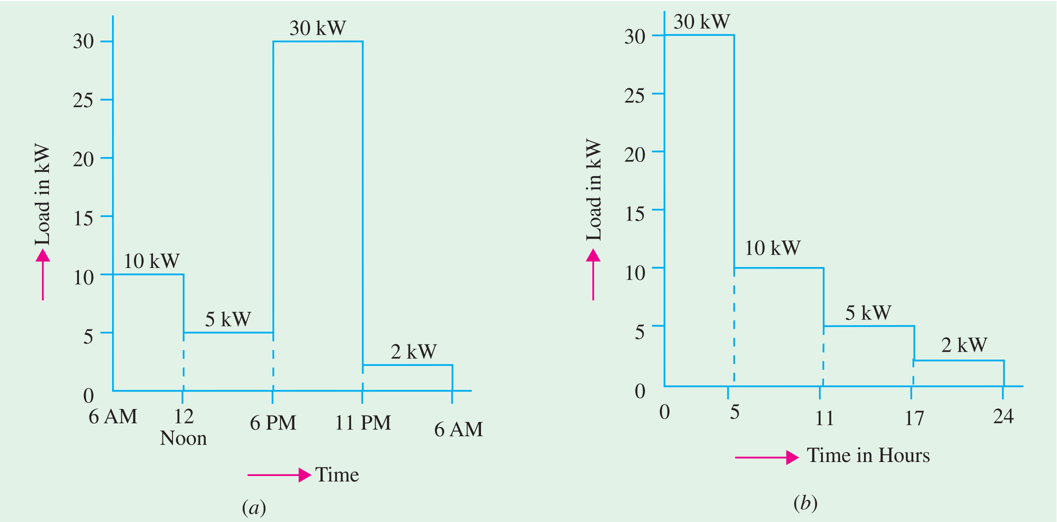


Fig. 50.7

(*ii*) Load Duration Curve

It represents the same data (*i.e.* load *vs* time) but the ordinates are rearranged in magnitude sequence (not time sequence). Here, greatest load is plotted on the left, lesser load towards the right and the least load on the extreme right. In other words, loads are plotted in descending order. As seen from Fig. 50.7 (*a*) maximum load on the station is 30 kW which lasts for 5 hours from 6 p.m. to 11 p.m. It is plotted first in Fig. 50.7 (*b*). The next lower load is 10 kW from 6 a.m. to 12 noon *i.e.* for 6 hours. It has been plotted next to the highest load. The other lesser loads are plotted afterwards. The areas under the load curve and load duration curve are equal and each represents the total units consumed during a day of 24 hours.

(*iii*) Load Energy Curve (or Integrated Load Duration Curve)

It represents the relation between a particular load on the station and the total number of kWhs produced ***at or below this load***. The load in kW is taken along the ordinate (*Y*-axis) and kWh generated ***upto this load*** along the abscissa (*X*-axis) as shown in Fig. 50.8. This curve is derived from the load duration curve. For example, for a load of 2 kW, the number of units generated is $2 \times 24 = 48$ kWh. It corresponds to point *A* on the curve. For a load of 5 kW, the units generated are $= 5 \times 17 + 2 \times 7 = 99$ kWh. It corresponds to point *B*. For a load of 10 kW, the units generated are $= 10 \times 11 + 5 \times 6 + 2 \times 7 = 154$ kWh (point *C*). Finally, for a load of 30 kW, the number of units generated is $= 30 \times 5 + 10 \times 6 + 5 \times 6 + 2 \times 7 = 254$ kWh (point *D*.)

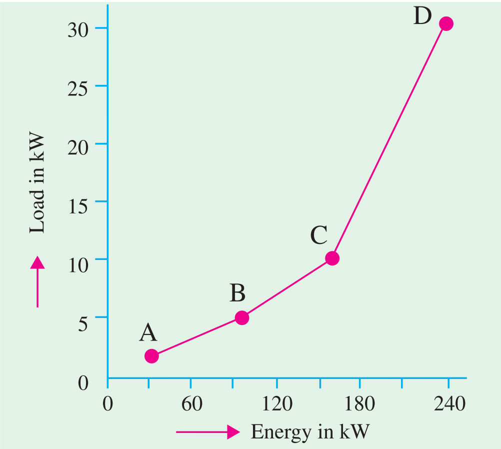


Fig. 50.8

Example 50.21. *A power station has a load cycle as under :*

60 MW for 6 hr; 200 MW for 8 hr ; 160 MW for 4 hr ; 100 MW for 6 hr

If the power station is equipped with 4 sets of 75 MW each, calculate the load factor and the capacity factor from the above data. Calculate the daily fuel requirement if the calorific value of the oil used were 10,000 kcal/kg and the average heat rate of the station were 2,860 kcal/kWh.

(Electric Power Systems-III, Gujarat Univ.)

Solution. Daily load factor $= \dfrac{\text{units actually supplied in a day}}{\text{max. demand} \times 24}$

Now, MWh supplied per day $= (260 \times 6) + (200 \times 8) + (160 \times 4) + (100 \times 6) = 4{,}400$

$\therefore$ station daily load factor $= \dfrac{4{,}440}{260 \times 24} = 0.704$ or **70.4%**

Capacity factor $= \dfrac{\text{average demand on station}}{\text{installed capacity of the station}}$

No. of MWh supplied/day $= 4{,}400$ $\quad\therefore\quad$ average power/day $= 4{,}400/24$ MW

Total installed capacity of the station $= 75 \times 4 = 300$ MW

$\therefore$ capacity factor $= \dfrac{4{,}400/24}{300} = 0.611$ or **61.1%**

Energy supplied/day $= 4{,}400 \text{ MWh} = 44 \times 10^5$ kWh

Heat required/day $= 44 \times 10^5 \times 2{,}860$ kcal

Amount of fuel required/day $= 44 \times 2{,}860 \times 10^5 / 10^5$ kg $=$ **125 tonne.**

Example 50.22. *A generating station has two 50 MW units each running for 8,500 hours in a year and one 30 MW unit running for 1,250 hours in one year. The station output is 650×10^6 kWh per year. Calculate **(i)** station load factor, **(ii)** the utilization factor.*

Solution. (*i*) $\text{kW}_1 \times h_1 = 50 \times 10^3 \times 8{,}500 = \mathbf{425 \times 10^6\ kWh}$

(*ii*) $\text{kW}_2 \times h_2 = 50 \times 10^3 \times 8{,}500 = \mathbf{425 \times 10^6\ kWh}$

(*iii*) $\text{kW}_3 \times h_3 = 30 \times 10^3 \times 1{,}250 = \mathbf{37.5 \times 10^6\ kWh}$

$\therefore$ $\Sigma\,(\text{kW}) \times h = (2 \times 425 + 37.5) \times 10^6 = \mathbf{887.5 \times 10^6\ kWh}$

Total installed capacity of the station $= 2 \times 50 + 30 = 130 \times 10^3$ kW

(*i*) Assuming that maximum demand equals installed capacity of the station,

$$\text{annual load factor} = \frac{\text{units generated/year}}{\text{max. demand} \times 8{,}760} = \frac{650 \times 10^6}{130 \times 10^3 \times 8760} = \mathbf{0.636 \text{ or } 63.6\%}$$

Note. In view of the above assumption, this also represents the plant or capacity factor.

(ii) $$\text{utilization factor} = \frac{\text{station output in kWh}}{\Sigma(\text{kW}) \times h} = \frac{650 \times 10^6}{887.5 \times 10^6} = \mathbf{0.732 \text{ or } 73.2\%}$$

Example 50.23. *The yearly duration curve of a certain plant may be considered as a straight line from 40,000 kW to 8,000 kW. To meet this load, three turbo-generators, two rated at 20,000 kW each and one at 10,000 kW are installed. Determine **(a)** the installed capacity, **(b)** plant factor, **(c)** maximum demand, **(d)** load factor and **(e)** utilization factor.* **(Ranchi Univ.)**

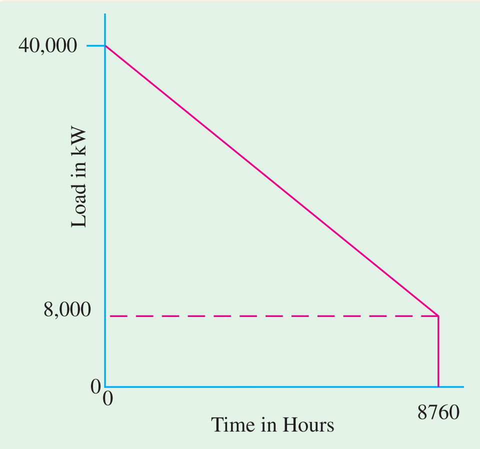


Fig. 50.9

Solution. *(a)* installed capacity

= 20 + 20 + 10 = **50 MW**

(b) Average demand on plant = (40,000 + 8,000)/2

= 24,000 kW

plant factor = av. demand / installed capacity

= 24,000/50,000 = **0.48% or 48%**

(c) Max. demand, obviously, is **40,000 kW**

(d) From load duration curve, total energy generated/year

$= 24{,}000 \times 8760$ kWh $= 21 \times 10^7$ kWh.

Load factor $= 21 \times 10^7 / 40{,}000 \times 8{,}760 =$ **0.6 or 60%**

(e) $$\text{u.f.} = \frac{\text{max. demand}}{\text{plant capacity}} \times 100 = \frac{40{,}000}{50{,}000} \times 100 = \mathbf{80\%}$$

Example 50.24. *The load duration curve of a system is as shown in Fig. 50.10. The system is supplied by three stations; a steam station, a run-of-river station and a reservoir hydro-electric station. The ratios of number of units supplied by the three stations are as below :*

Steam	:	*Run of river*	:	*Reservoir*
7	:	*4*	:	*1*

The run-of-river station is capable of generating power continuosuly and works as a peak load station. Estimate the maximum demand on each station and also the load factor of each station.

(Ranchi Univ.)

Solution. Here 100% time will be taken as 8760 hours.

Total units generated = area under the curve

$$= \frac{1}{2}(160 + 80) \times 10^3 \times 8760 = 1051.2 \times 10^6 \text{ kWh}$$

From the given ratio, the number of units supplied by each station can be calculated

Units Generated

Run-of-river-station $= 1051.2 \times 10^6 \times 4/12 = 350.4 \times 10^6$ kWh

Steam station $= 1051.2 \times 10^6 \times 7/12 = 613.2 \times 10^6$ kWh

Reservoir HE station $= 1051.2 \times 10^6 \times 1/12 = 87.6 \times 10^6$ kWh

Max. demand of *ROR* station $= 350.4 \times 10^6 / 8760 =$ **40 MW**

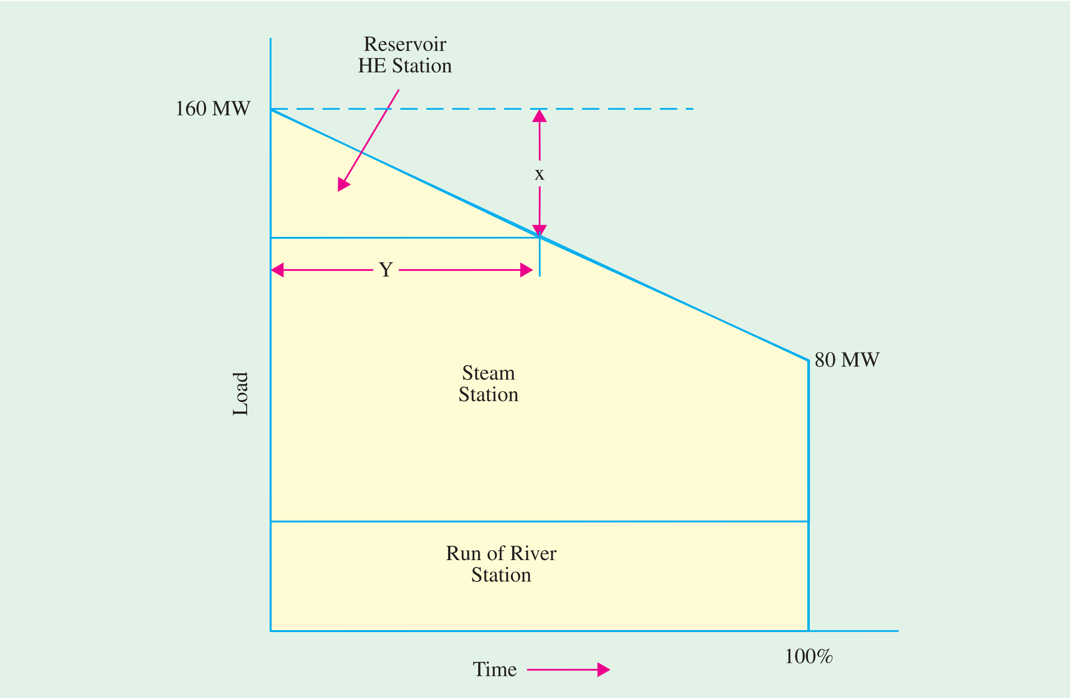


Fig. 50.10

Let x MW be the maximum demand of the reservoir plant. As shown in Fig. 50.10. let it operate for y hours.

Obviously, $y = \dfrac{x}{80} \times 8760$

Area under the curve for reservoir station = $\dfrac{1}{2} xy \times 10^3$ kWh

$$= \frac{1}{2} x \times \frac{x}{80} \times 8760 \times 10^3 = 54,750\, x^2$$

$\therefore$ $54,750\, x^2 = 87.6 \times 10^6$; x = **40 MW**

$\therefore$ max. demand of steam station= 160 – (40 + 40) = **80 MW.**

Load factor

Since ROR station works continuously as a base load station, its load factor is **100%.**

Reservoir station $= 87.6 \times 10^6 / 40 \times 10^3 \times 8760$ = **0.25 or 25%**

Steam station $= 613.2 \times 10^6 / 80 \times 10^3 \times 8760$ = **0.875 or 87.5%.**

Example 47.25. *A load having a maximum value of 150 MW can be supplied by either a hydro-electric plant or a steam power plant. The costs are as follows :*

Capital cost of steam plant = Rs. 700 per kW installed

Capital cost of hydro-electric plant = Rs. 1,600 per kW installed

Operating cost of steam plant = Rs. 0.03 per kWh

Operating cost of hydro-electric plant = Rs. 0.006 per kWh

Interest on capital cost 8 per cent. Calculate the minimum load factor above which the hydro-electric plant will be more economical.

Solution. Let x be the total number of units generated per annum.

Steam Plant

Capital cost $= \text{Rs. } 700 \times 150 \times 10^3 = \textbf{Rs. } \mathbf{10.5 \times 10^7}$

Interest charges $= 8\%$ of Rs. $10.5 \times 10^7 = \textbf{Rs. } \mathbf{8.4 \times 10^6}$

$\therefore$ fixed cost/unit $= \text{Rs. } 8.4 \times 10^6 / x$; operating cost / unit = **Re. 0.03**

$\therefore$ total cost/unit generated $= \textbf{Rs. } \mathbf{(8.4 \times 10^4 / x + 0.03)}$

Hydro Plant

Capital cost $= \text{Rs. } 1600 \times 150 \times 10^3 = \textbf{Rs. } \mathbf{24 \times 10^7}$

Interest charges $= 8\%$ of Rs. $24 \times 10^7 = \textbf{Rs. } \mathbf{19.2 \times 10^6}$

Total cost/unit $= \textbf{Rs. } \mathbf{(19.2 \times 10^6 / x + 0.006)}$

The two overall costs will be equal when

$$(8.4 \times 10^6 / x) + 0.03 = (19.2 \times 10^6 / x) + 0.006 \; ; \; x = 45 \times 10^7 \text{ kWh}$$

Obviously, if units generated are more than 45×10^7 kWh, hydro-electric station will be cheaper.

Load factor $= 45 \times 10^7 / 150 \times 10^3 \times 8760 =$ **0.342 or 34.2%**

This represents the minimum load factor beyond which hydro-electric station would be economical.

Example 50.26. *A power system having maximum demand of 100 MW has a load 30% and is to be supplied by either of the following schemes :*

(a) *a steam station in conjunction with a hydro-electric station, the latter supplying* 100×10^6 *units per annum with a max. output of 40 MW,*

(b) *a steam station capable of supply the whole load,*

(c) *a hydro station capable of supplying the whole load,*

Compare the overall cost per unit generated assuming the following data :

	Steam	*Hydro*
Capital cost / kW	*Rs. 1,250*	*Rs. 2,500*
Interest and depreciation on the capital cost	*12%*	*10%*
Operating cost/kWh	*5 paise*	*1.5 paise*
Transmission cost/kWh	*Negligible*	*0.2 paise*

Show how overall cost would be affected in case (ii) and (iii) above if the system load factor were improved to 90 per cent. **(Elect. Power System-III, Gujarat Univ.)**

Solution. Average power $= 100 \times 0.3 = 30 \text{ MW} = 3 \times 10^4$ kW

Units generated in one year $= 3 \times 10^4 \times 8{,}760 = 262.8 \times 10^6$ kWh

(*a*) Steam Station in Conjunction with Hydro Station

Units supplied by hydro-station = $\mathbf{100 \times 10^6}$ **kWh**

Units supplied by steam station $= (262.8 - 100) \times 10^6 = \mathbf{162.8 \times 10^6}$ **kWh**

Since, maximum output of hydro-station is 40 MW, the balance (100 – 40) = 60 MW is supplied by the steam station.

(*i*) Steam Station

Capital cost $= \text{Rs. } 60 \times 10^3 \times 1{,}250 = \textbf{Rs. } \mathbf{75 \times 10^6}$

Annual interest and depreciation - Rs. $0.12 \times 75 \times 10^6 = \textbf{Rs. } \mathbf{9 \times 10^6}$

Operating cost $= \text{Rs. } 0.5 \times 162.8 \times 10^6 / 100 = \textbf{Rs. } \mathbf{8.14 \times 10^6}$

Transmission cost = negligible

Total annual cost $= \text{Rs. } (9 + 8.14) \times 10^6 = \textbf{Rs. } \mathbf{17.14 \times 10^6}$

(*ii*) Hydro Station

Capital cost $= \text{Rs. } 40 \times 10^3 \times 2{,}500 = \text{Rs. } 100 \times 10^6$

Annual interest and depreciation $= \text{Rs. } 0.1 \times 100 \times 10^6 = \text{Rs. } 10 \times 10^6$

Operating cost $= \text{Rs. } 0.15 \times 100 \times 10^6/100 = \text{Rs. } 1.5 \times 10^6$

Transmission cost $= \text{Rs. } .02 \times 100 \times 10^6 / 100 = \text{Rs. } 0.2 \times 10^6$

Total annual cost $= \text{Rs. } (10 + 1.5 + 1.2) \times 10^6 = \text{Rs. } 11.7 \times 10^6$

Combined annual charge for steam and hydro stations

$= \text{Rs. } (17.14 + 11.7) \times 10^6 = \text{Rs. } 28.84 \times 10^6$

$\therefore$ Overall cost/kWh $= \text{Rs. } \dfrac{28.84 \times 10^6}{262.8 \times 10^6} =$ **10.97 paise**

(*b*) Steam Station Alone

Capital cost $= \text{Rs. } 1{,}250 \times 100 \times 10^3 = \text{Rs. } 125 \times 10^6$

Annual interest and depreciation $= \text{Rs. } 0.12 \times 125 \times 10^6 = \text{Rs. } 15 \times 10^6$

$\therefore$ fixed charge / unit $= \text{Rs. } 15 \times 10^6/262.8 \times 10^6 = 5.71$ paise

Operating cost/unit = 5 paisa ; Transmission cost / unit = 0

$\therefore$ overall cost per unit $= (5.71 + 5) =$ **10.71 paise**

(*c*) Hydro Station Alone

Annual interest and depreciation on capital cost

$= \text{Rs. } 0.1\,(2{,}500 \times 100 \times 10^3) = \text{Rs. } 25 \times 10^6$

$\therefore$ fixed charge / unit $= \text{Rs. } 25 \times 10^6 / 262.8 \times 10^6 = 9.51$ paise

Operating cost/unit = 1.5 paisa ; Transmission cost/unit = 0.2 paise

$\therefore$ overall cost/unit $= (9.51 + 1.5 + 0.2) =$ **11.21 paise**

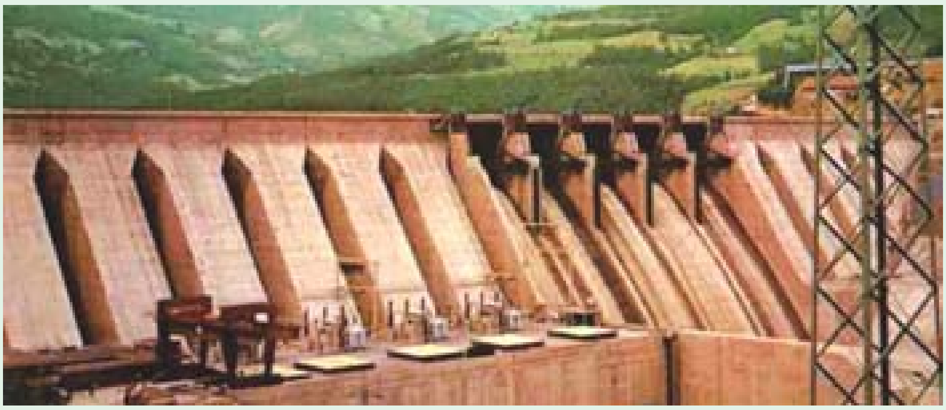
(*d*) (*i*) Steam Station

Since number of units generated will increase three-fold, fixed charge per unit will decrease to one-third of its previous value *i.e.* to 5.71/3 = 1.9 paisa. Since other charges are unaffected by change in load factor.

$\therefore$ Overall cost/unit $= (1.9 + 5) =$ **6.9 paise**

(*ii*) Hydro Station

For same reasons, fixed cost per unit becomes 9.51/3 = 3.17 paise Overall cost/unit = (3.17 + 1.5 + 0.2) = **4.87 paise**



An overview of a hydroelectric plant

Example 50.27. *The capital cost of a hydro-power station of 50,000 kW capacity is Rs. 1,200 per kW. The annual charge on investment including depreciation etc. is 10%. A royality of Rs. 1 per kW per year and Rs. 0.01 per kWh generated is to be paid for using the river water for generation of power. The maximum demand is 40,000 kW and the yearly load factor is 80%. Salaries, maintenance charges and supplies etc. total Rs. 6,50,000. If 20% of this expense is also chargeable as fixed charges, determine the generation cost in the form of A per kW plus B per kWh.*

(A.M.I.E. Sec. B,)

Solution. Capital cost of station $= \text{Rs. } 1200 \times 50{,}000 = \text{Rs. } 6 \times 10^7$

Annual charge on investment including depreciaiton

$= 10\%$ of Rs. $6 \times 10^7 = \text{Rs. } 6 \times 10^6$

Total running charges $= 80\%$ of Rs. 6,50,000 = Rs. 5,20,000

Fixed charges $= 20\%$ of Rs. 6,50,000 = Rs. 1,30,000

Total annual fixed charges $= \text{Rs. } 6 \times 10^6 + \text{Rs. } 0.13 \times 10^6 = \text{Rs. } 6.13 \times 10^6$

Cost per M.D. kW due to fixed charges $= \text{Rs. } 6.13 \times 10^6 / 40{,}000 = \text{Rs. } 153.25$

Cost per M.D. kW due to royalty = Rs. 1

Total cost per M.D. kW = Rs. 154.25

Total No. of units generated per annum $= 40{,}000 \times 0.8 \times 8760 = 28 \times 10^7$ kWh

Cost per unit due to running charges $= \text{Rs. } 5{,}20{,}000 / 28 \times 10^7 = 0.18$ p

Royalty cost/unit = 1 p

$\therefore$ total cost/unit = 1.18 p

$\therefore$ generation cost = Rs. 154.25 kWh + 1.18 p kWh

$= \textbf{Rs. (154.25 kW + 1.18} \times \textbf{10}^{-2}\textbf{ kWh)}$

Example 50.28. *The capital costs of steam and water power stations are Rs. 1,200 and Rs. 2,100 per kW of the installed capacity. The corresponding running costs are 5 paise and 3.2 paise per kWh respectively.*

The reserve capacity in the case of the steam station is to be 25% and that for the water power station is to be 33.33% of the installed capacity.

At what load factor would the overall cost per kWh be the same in both cases ? Assume interest and depreciation charges on the capital to be 9% for the thermal and 7.5% for the hydro-electric station. What would be the cost of generating 500 million kWh at this load factor ?

Solution. Let x be the maximum demand in kWh and y the load factor.

Total No. of units produced $= xy \times 8760$ kWh

Steam Station

installed capacity $= 1.25\,x$ (including reserve capacity)

capital cost $= \text{Rs. } 1200 \times 1.25\,x = \text{Rs. } 1500\,x$

annual interest and depreciation $= 9\%$ of Rs. $1500\,x = \text{Rs. } 135\,x$

annual running cost $= \text{Rs. } 8760\,xy \times 5/100 = \text{Rs. } 438\,xy$

Total annual cost $= \text{Rs. } (135\,x + 438\,xy)$

$\therefore$ total cost / unit $= \textbf{Rs. } \boldsymbol{(135\,x + 438\,xy / 8760\,xy}$

Hydro-electric Station

installed capacity $= 1.33\,x$ (including reserve capacity)

capital cost $= \text{Rs. } 1.33\,x \times 2100 = \text{Rs. } 2800\,x$

annual interest and depreciation $= 7.5\%$ of Rs. $2800\,x = \text{Rs. } 210\,x$

annual running cost $= \text{Rs. } 8760xy \times 3.2/100 = \text{Rs. } 280\,xy$

$\therefore$ total annual cost $=$ Rs. $(210x + 280\,xy)$

$\therefore$ total cost/unit $=$ Rs. $(210x + 280\,xy)/8760\,xy$

For the two costs to be the same, we have

$$\frac{135\,x + 438\,xy}{8760\,xy} = \frac{210\,x + 280\,xy}{8760\,xy}, \quad y = \mathbf{0.475 \text{ or } 47.5\%}$$

Cost of 500 × 10⁶ Units

Now, maximum demand $= 500 \times 10^6 / 8760 \times 0.475 = 12 \times 10^4$ kW

$\therefore \qquad x = 12 \times 10^4,\ y = 0.475$

$\therefore$ generating cost $=$ Rs. $(136 \times 12 \times 10^4 + 438 \times 12 \times 10^4 \times 0.475) =$ **41,166,000**

Example 50.29. *In a particular area, both steam and hydro-stations are equally possible. It has been estimated that capital cost and the running costs of these two types will be as follows:*

Capital cost/kW	*Running cost/kWh*	*Interest*
Hydro : Rs. 2,200	*1 Paise*	*5%*
Steam : Rs. 1,200	*5 Paise*	*5%*

If expected average load factor is only 10%, which is economical to operate : steam or hydro? If the load factor is 50%, would there be any change in the choice ? If so, indicate with calculation.

(Electric Power-II Punjab Univ. 1991)

Solution. Let x be the capacity of power station in kW.

Case I. Load factor = 10%

Total units generated/annum $= x \times 0.1 \times 8760 = 876\,x$ kWh

(a) Hydro Station

capital cost $=$ Rs. $2200\,x$

annual fixed charges $=$ 5% of Rs. $2200\,x =$ Rs. $110\,x$

annual running charges $=$ Rs. $876x \times 1/100 =$ Rs. $8.76\,x$

total annual charges $=$ Rs. $(110 + 8.76)x$

total cost/unit $=$ Rs. $(110 + 8.76)\,x / 876\,x =$ **13.5 p**

(b) Steam Station

capital cost $=$ Rs. $1200\,x$

annual fixed charges $=$ 5% of Rs. $1200\,x =$ Rs. $60\,x$

annual running charges $=$ Rs. $876\,x \times 5/100 =$ Rs. $43.8\,x$

total annual charges $=$ Rs. $(60 + 43.8)x$

overall cost/unit $=$ Rs. $103.8\,x/8.76\,x =$ **11.85 p**

Obviously, steam station is more economical to operate.

Case II. Load factor = 50%

Total units generated/annum $= x \times 8760 \times 0.5 = 4380\,x$

(a) Hydro Station

If we proceed as above, we find that total cost/unit = **3.5 p**

(b) Steam Station

total cost/unit = **6.35 p**

Obviously, in this case, hydro-station is more economical.

Example 50.30. *The annual working cost of a thermal station can be represented by a formula Rs. (a + b kW + c kWh) where a, b and c are constants for a particular station, kW is the total installed capactiy and kWh the energy produced per annum. Explain the significance of the constants a, b and c and the factors on which their values depend.*

Determine the values of a, b and c for a 60 MW station operating with annual load factor of 40% for which :

(i) *capital cost of buildings and equipment is Rs.* 5×10^5

(ii) *the annual cost of fuel, oil, taxation and wages and salaries of operating staff is Rs.90,000*

(iii) *the interest and depreciation on buildings and equipment are 10% per annum*

(iv) *annual cost of organisation and interest on cost of site etc. is Rs. 50,000.*

Solution. Here, *a* represents fixed charge due to the annual cost of the organisation, interest on the capital investment on land or site etc.

The constant *b* represents semi-fixed cost. The constant *b* is such that when multiplied by the maximum kW demand on the station, it equals the annual interest and depreciation on the capital cost of the buildings equipment and the salary of the charge engineer.

Constant *c* repesents running cost and its value is such that when multiplied by the annual total kWh output of the station, it equals the annual cost of the fuel, oil, taxation, wages and salaries of the operating staff.


An overview of a thermal power plant near Tokyo, Japan

(i) Here, a = **50,000**

(ii) $b \times$ max. kW demand = annual interest on the capital cost of the buildings and equipment etc. $= 0.1 \times 5 \times 10^5$

$\therefore \quad b \times 60 \times 10^3 = 0.5 \times 10^5$ or b = **0.834**

(iii) annual average power $= 0.5 \times 60 \times 10^3 = 30 \times 10^3$ kW

Units produced annually $= 30 \times 10^3 \times 8{,}760 = 262.8 \times 10^6$ kWh

$\therefore \quad c \times 262.8 \times 10^6 = 90{,}000$; c = **0.0034**

50.17. Tariffs

The size and cost of installations in a generating station is determined by the maximum demand made by the different consumers on the station. Each consumer expects his maximum demand to be met at any time of the day or night. For example, he may close down his workshop or house for a month or so but on his return he expects to be able to switch on his light, motor and other equipment without any previous warning to the supply company. Since electric energy, unlike gas or water cannot be stored, but must be produced as and when required, hence the generating equipment has to be held in 'readiness' to meet every consumer's full requirement at all hours of the day.

This virtually amounts to allocating a certain portion of the generating plant and the associated distribution system to each consumer for his individual use. Hence, it is only fair that a consumer should pay the fixed charges on that portion of the plant that can be assumed to have been exclusively allocated to him plus the charges proportional to the units actually used by him.

Hence, any method of charging or tariff, in the fairness to the supply company, should take into account the two costs of producing the electric energy **(*i*)** fixed or standing cost proportional to the maximum demand and **(*ii*)** running cost proportional to the energy used. Such two-part tariffs are in common use. Some of the different ways of rate making are described below :

50.18. Flat Rate

This was the earliest type of tariff though it is not much used these days because, strictly speaking, it is not based on the considerations discussed above. In this system, charge is made at a simple flat rate per unit. But the lighting loads and power loads are metered separately and charged at different rates. Since the lighting load has a poor load factor *i.e*. the number of units sold is small in relation to the installed capacity of the generating plant, the fixed cost per kWh generated is high and this is taken into account by making the price per unit comparatively high. But since the power load is more predictable and has a high load factor, the cost per kWh generated is much lower which results in low rate per unit.

50.19. Sliding Scale

In this type of tariff, the fixed costs are collected by charging the first block of units at a higher rate and then reducing the rates, usually in many steps, for units in excess of this quantity.

50.20. Two-part Tariff

This tariff is based on the principles laid down in Art. 50.16. It consists of two parts **(*i*)** a fixed charge proportional to the maximum demand (but independent of the units used) and **(*ii*)** a low running charge proportional to the actual number of units used.

The maximum demand during a specified period, usually a quarter, is measured by a maximum demand indicator. The maximum demand indicator is usually a watt-hour meter which returns to zero automatically at the end of every half hour but is fitted with a tell-tale pointer which is left behind at the maximum reduing reached during the quarter under consideration.

This type of tariff is expressed by a first degree equation like Rs. $A \times$ kW + $B \times$ kWh where Rs. A is the charge per annum per kW of maximum demand and B is the price per kWh.

Sometimes, the customer is penalized for his poor load power factor by basing the fixed charges on kVA instead of per kW of maximum demand.

Example 50.31. *Compute the cost of electrical energy and average cost for consuming 375 kWh under 'block rate tariff' as under :*

First 50 kWh at 60 paisa per kWh ; next 50 kWh at 50 paisa per kWh; next 50 kWh at 40 paisa per kWh; next 50 kWh at 30 paisa per kWh.

Excess over 200 kWh at 25 paisa per kWh. **(Utilisation of Elect. Power, AMIE Sec. B)**

Solution. Energy charge for the first 50 kWh is = Rs. 0.6×50 = Rs. 30

Energy charge for the next 50 kWh at 50 paisa / kWh = Rs. 0.5×50 = Rs. 25

Energy charge for the next 50 kWh at 40 paisa / kWh = Rs. 0.4×50 = Rs. 20

Energy charge for the next 50 kWh at 30 paisa / kWh = Rs. 0.3×50 = Rs. 15

Energy charge for the rest (375 – 200) *i.e.* 175 kWh =Rs. 0.25×175 = Rs. 43.75

Total cost of energy for 375 kWh = Rs. (30 + 25 + 20 + 15 + 43.75) = **Rs. 133.75**

Average cost of electrical energy/kWh = Rs. 133.75 / 375 = **36 paise.**

Example 50.32. *The output of a generating station is 390×10^6 units per annum and installed capacity is 80,000 kW. If the annual fixed charges are Rs. 18 per kW of isntalled plant and running charges are 5 paisa per kWh, what is the cost per unit at the generating station ?*

(Eletrical Technology, Bombay Univ.)

Solution. Annual fixed charges = Rs. $18 \times 80{,}000$ = Rs. 1.44×10^6

Fixed charges/kWh = Rs. $1.44 \times 10^6/390 \times 10^6 = 0.37$ paisa

Running charges/kWh = 5 paisa

Hence, cost at the generating station is = $5 + 0.37$ = **5.37 paise/kWh.**

Example 50.33. *A power station has an installed capacity of 20 MW. The capital cost of station is Rs. 800 per kW. The fixed costs are 30% of the cost of investment. On full-load at 100% load factor, the variable costs of the station per year are 1.5 times the fixed cost. Assume no reserve capacity and variable cost to be proportional to the energy produced, find the cost of generation per kWh at load factors of 100% and 20%. Comment on the results.* **(Ranchi University)**

Solution. Capital cost of the station = Rs. $800 \times 20{,}000$ = Rs. 16×10^6

(*a*) At 100% load factor

Fixed cost = Rs. $16 \times 10^6 \times 13/100$ = Rs. 2.08×10^6

Variable cost = Rs. $1.5 \times 2.08 \times 10^6$ = Rs. 3.12×10^6

Total operating cost per annum = Rs. $(2.08 + 3.12) \times 10^6$ = Rs. 5.2×10^6

Total No. of units generated = kW max. demand $\times$ LF $\times$ 8760 = $20{,}000 \times 1 \times 8760$

= 175.2×10^6 kWh

Cost of generation per kWh = $5.2 \times 10^6 \times 100/175.2 \times 10^6$ = **3.25 paise.**

(*b*) At 20% load factor :

Fixed cost = Rs. 2.08×10^6 as before

Total units generated per annum = $20{,}000 \times 0.2 \times 8760 = 35.04 \times 10^6$ kWh

Since variable cost is proportional generated, the variable = Rs. $3.12 \times 10^6 \times 35.04 \times 10^6 / 175.2 \times 10^6$ to kWh

cost at load of 20% is = Rs. 0.624×10^6

Total operating cost = Rs. $(2.08 + 0.624) \times 10^6 = 2.704 \times 10^6$

Cost of generation per kWh = $2.704 \times 10^6 \times 100/35.04 \times 10^6$ = **7.7 paise**

It is obvious from the above calculations that as the station load factor is reduced, the cost of electric generation is increased.

Example 50.34. *The annual output of a generating sub-station is 525.6×10^6 kWh and the average load factor is 60%. If annual fixed charges are Rs. 20 per kW installed plant and the annual running charges are 1 paisa per kWh, what would be the cost per kWh at the bus bars ?*

Solution. Average power supplied per annum = $525.6 \times 10^6 / 8760 = 60{,}000$ kW

Max. demand $= \dfrac{\text{average power}}{\text{load factor}} = \dfrac{60{,}000}{0.6} = 100{,}000$ kW*

Annual fixed charges = Rs. $20 \times 100{,}000$ = Rs. 2×10^6

Fixed charges / kWh $= \dfrac{2 \times 10^6}{525.6 \times 10^6} = \dfrac{2 \times 100}{525.6} = 0.38$ paisa

Annual running charges per kWh = 1 paisa. Hence, cost per kWh at the bus-bars = **1.38 paisa.**

* It has been assumed that the installed capacity is equal to the maximum demand of 100,000 kW.

Example 50.35. *A certain factory working 24 hours a day is charged at Rs. 10 per kVA of max. demand plus 5 paisa per kV ARh. The meters record for a month of 30 days; 135,200 kWh, 180,020 kV ARh and maximum demand 310 kW. Calculate*

(i) *M.D. charges,* ***(ii)*** *monthly bill,* ***(iii)*** *load factor,* ***(iv)*** *average power factor.*

(Electric Engineering-II, Bangalore Univ.)

Solution. Average deamnd $= 135{,}200/24 \times 30 = 188$ kW

Average reactive power $= 180{,}020/24 \times 30 = 250$ kVAR

Now, $\tan\phi = $ kVAR/kW $= 250/88$; $\phi = 53°$; $\cos\phi = 0.6$;

M.D. kVA $= 310/0.6 = 517$

(*i*) M.D. charge $=$ Rs. $10 \times 517 =$ **Rs. 5170**

(*ii*) reactive energy charges $=$ Rs. $5 \times 180{,}020/100 =$ Rs. 9001

$\therefore$ monthly bill $=$ **Rs. 14,171**

(*iii*) monthly load factor $= 188/310 = 0.606$ or 60.6% (*iv*) average p.f. $=$ **0.6**

Example 50.36. *The cost data of a power supply company is as follows :*

Station maximum demand = 50 MW ; station load factor = 60% ; Reserve capacity = 20% ; capital cost = Rs. 2,000 per kW; interest and depreciation = 12% ; salaries (annual) = Rs. 5×10^5; *fuel cost (annual) = Rs.* 5×10^6 ; *maintenance and repairs (annual) Rs.* 2×10^5 ; *losses in distribution = 8% ; load diversity factor = 1.7.*

Calculate the average cost per unit and the two-part tariff, assuming 80 per cent of salaries and repair and maintenace cost to be fixed. **(Electrical Power-I, Bombay Univ.)**

Solution. Station capacity $= 50 + 20\%$ of $50 = 60$ MW

Average power $= 60 \times 0.6 = 36$ MW $= 36 \times 10^3$ kW

Capital investment $=$ Rs. $60 \times 10^3 \times 2000 =$ Rs. 12×10^7

Interest + depreciation $=$ Rs. $0.12 \times 12 \times 10^7 =$ Rs. 14.4×10^6

Total cost both fixed and running

$=$ Rs. $14.4 \times 10^6 +$ Rs. $5 \times 10^5 +$ Rs. $5 \times 10^6 +$ Rs. $2 \times 10^5 =$ Rs. 201×10^5

No. of units generated annually $= 8760 \times 36 \times 10^3$

$\therefore$ overall cost/unit $=$ Rs. $\dfrac{201 \times 10^5}{8760 \times 36 \times 10^3} =$ **6.37 paise**

Fixed charges

Annual interest and depreciation $=$ Rs. 14.4×10^6

80% of salaries $=$ Rs. $0.8 \times 5 \times 10^5 =$ Rs. 4×10^5

80% of repair and maintenance cost $=$ Rs. $0.8 \times 2 \times 10^5 =$ Rs. 1.6×10^5

Total fixed charges $=$ Rs. 149.6×10^5

Aggregate maximum demand of all consumers

$=$ Max. demand on generating station $\times$ diversity factor

$= 60 \times 10^3 \times 1.7 = 10^2 \times 10^3$ kW

$\therefore$ annual cost/kW of maximum demand $=$ Rs. $149.6 \times 10^5/10^2 \times 10^3 =$ **Rs. 149.6**

Running Charges

Cost of fuel $=$ Rs. 50×10^5 ; 20% of salaries $=$ Rs. 1×10^5

20% of maintenance $= 0.4 \times 10^5$;

Total $=$ Rs. 51.4×10^5

Considering distribution loss of 8%, cost per unit delivered to the consumer is

$= \text{Rs. } 51.4 \times 105/0.92 \times 8760 \times 36 \times 10^3 =$ **1.77 paise**

Hence, two-part tariff is : **Rs. 149.6 per kW max.** demand plus **1.77 paisa per kWh** consumed.

Example 50.37. *A certain electric supply undertaking having a maximum demand of 110 MW generates 400 × 10^6 kWh per year. The supply undertaking supplies power to consumers having an aggregate demand of 170 MW. The annual expenses including capital charges are :*

Fuel $= \text{Rs. } 5 \times 10^6$

Fixed expenses connected with generation $= \text{Rs. } 7 \times 10^6$

Transmission and distribution expenses $= \text{Rs. } 8 \times 10^6$

Determine a two-part tariff for the consumers on the basis of actual cost.

Assume 90% of the fuel cost as variable charges and transmission and distrbution losses as 15% of energy generated. **(Electrical Power-I. Bombay Univ.)**

Solution. Total fixed charges per annum are as under :

Fixed charges for generation $= \text{Rs. } 7 \times 10^6$

Transmission and distribution expenses = Rs. 8 $\times 10^6$

10% of annual fuel cost $= \text{Rs. } 0.5 \times 10^6$ Total $= \text{Rs. } 15.5 \times 10^6$

This cost has to be spread over the aggregate maximum demand of all consumers *i.e.* 170 MW.

∴ cost per kW of maximum deamnd = Rs. $15.5 \times 10^6 / 170 \times 10^3$ = **Rs. 91.2**

Running charges = 90% of fuel cost = Rs. 4.5×10^6

These charges have to be spread over the number of kWh acutally delivered.

No. of units delivered = 85% of No. of units generated

$= 0.85 \times 400 \times 10^6 = 340 \times 10^6$

∴ running cost / kWh $= \text{Rs. } 4.5 \times 10^6/340 \times 10^6 =$ **1.32 paise**

Hence, the two-part tariff for the consumer would be **Rs. 91.2 per kW** of maximum demand and **1.32 paisa per kWh** consumed.

Example 50.38. *A customer is offered power at Rs. 80 per annum per kVA of maximum demand plus 8 paisa per unit metered. He proposes to install a motor to carry his estimated maximum demand of 300 b.h.p. (223.8 kW). The motor available has a power factor of 0.85 at full-load. How many units will he require at 20% load factor and what willl be his annual bill ?*

(Electric Power II Punjab Univ. 1992)

Solution. Assuming a motor efficiency of 90%, the full-load power intake of motor = 223.8/0.9 = 746/3 kW. This represents the max. demand.

Now, load factor $= \dfrac{\text{average}}{\text{max.demand}}$

∴ average power = max. demand × load factor = (746/3) × 0.2 = **149.2/3 kW**

∴ annual consumption = 149.2 × 8,760/3 = 435,700 kWh

Max. kVA of demand = 300 × 746/0.9 × 0.85 × 1000 = 292.5

∴ cost per kVA of maximum demand = 292.5 × 80 = Rs. 23,400

Cost of units consumed/annum = 435,700 × 8/100 = Rs. 34,856

∴ annual bill = Rs. 23,400 + Rs. 34,856 = **Rs. 58,256**

Example 50.39. *How two-part tariff is modified for penalising low p.f. consumers ?*

An industry consumes 4 million kWh/year with a maximum demand of 1000 kW at 0.8 p.f. What is its load factor ?

(a) *Calculate the annual energy charges if tariff in force is as under :*
Max. demand charge = Rs. 5 per kVA per month. Energy charges = Rs. 0.35 per kWh
(b) *Also calculate reduction in this bill if the maximum demand is reduced to 900 kW at 0.9 p.f. lagging.* **(Electrotechnics, Gujarat Univ.)**

Solution. Load factor $= 4 \times 10^6/1000 \times 8760 = 0.4566$ or 45.66%

(*a*) Max. kVA demand $= 1000/0.8 = 1250$

Annual M.D. charge/kVA $= 5 \times 12 =$ Rs. 60

M.D. charge for 1250 kVA = Rs. 60×1250 = Rs. 75×10^3

Annual energy charge = Rs. $4 \times 10^6 \times 0.35$ = Rs. 1400×10^3

Annual energy bill = **Rs. 1475×10^3**

(*b*) Since load factor remains the same, annual average power $= 900 \times 0.4566$ kW

Units consumed annually $= 8760 \times 900 \times 0.4566 = 3.6 \times 10^6$

Max. kVA demand $= 900/0.9 = 1000$, M.D. charge for 1000 kVA = Rs. 60×1000 = Rs. 60×10^3

Energy charge = Rs. $3.6 \times 10^6 \times 0.35$ = Rs. 1260×10^3; Annual bill = Rs.$1{,}320 \times 10^3$

Annual saving = Rs. $(1475 - 1320) \times 10^3$ = **Rs. 155×10^3**

Example 50.40. *A supply is offered on the basis of fixed charges of Rs. 30 per annum plus 3 paise per unit or alternatively, at the rate of 6 paisa per unit for the first 400 units per annum and 5 paise per unit for all the additional units. Find the number of units taken per annum for which the cost under these two tariffs becomes the same.* **(Electrical Technology-I, Bombay Univ.)**

Solution. Let x kWh be the annual consumption of the consumer for which the two tariffs are equally advantageous.

Cost according to the first tariff = Rs. $30 + 3x / 100$

Charges according to the alternative tariff are

= Rs. $6 \times 400/100$ + Rs. $(x - 400) \times 5/100$ = Rs. $[24 + (x - 400)/20]$

Since charges in both cases are to be equal

$$\therefore \quad 30 + \frac{3x}{100} = 24 + \frac{(x-400)}{20}$$

or $x =$ **1300 kWh.**

Example. 50.41. *If power is charged for at the rate of Rs. 75 per kVA of maximum demand and 4 paisa per unit, what is the cost per unit at 25% yearly load factor* **(a)** *for unity power factor demand and* **(b)** *for 0.7 power factor demand.*

Solution. **(*a*) At 25% load factor and unity power factor**

Maximum demand charge *per unit* $= \dfrac{75 \times 100}{8760 \times 0.25} = 3.43$ paise

Energy charge per unit = 4 paisa ; Cost *per unit* = 4 + 3.43 = **7.43 paise**

(*b*) At 25% load factor and 0.7 power factor

Maximum demand charge *per unit* $= \dfrac{75 \times 100}{0.7 \times 0.25 \times 8760} =$ **4.9 paise**

Energy charge per unit = 4 paise ; Cost per unit = 4 + 4.9 = **8.9 paise**

Exmaple 50.42. *Explain different methods of tariff. A tariff in force is Rs. 50 per kVA of max. demand per year plus 10 p per kWh. A consumer has a max. demand of 10 kW with a load factor of 60% and p.f. 0.8 lag.*

(i) *Calculate saving in his annual bill if he improves p.f. to 0.9 lag.*

(ii) *Show the effect of improving load factor to 80% with the same max. demand and p.f. 0.8 lag on the total cost per kWh.* **(Electrical Engineering-III, Poona Univ.)**

Solution. *(i)* Max. kVA demand at 0.8 p.f. = 10/0.8 = 12.5

M.D. charges = Rs. 50 × 12.5 = Rs. 625

Max. kVA demand at 0.9 p.f. = 10/0.9 ; M.D. charge = Rs. 50 × 10/0.9 = Rs. 555.55

Since energy consumed remains constant, saving is due only to reduction in M.D. charges.

∴ saving = Rs. (625 – 555.55) = **Rs. 69.45**

(ii) With 60% load factor, average power = 10 × 0.6 = 6 kW. The number of units consumed annually = 6 × 8760 = 52,560 kWh. Also, annual M.D. charge = Rs. 50 × 12.5 = Rs. 625

M.D. charge/unit consumed = 625 × 100/52,560 = 1.19 paise

Total cost per unit = 10 + 1.19 = **11.19 paise**

With a laod p.f. of 80% average power = 10 × 0.8 = 8 kW

Number of units consumed annually = 8 × 8,760 = 70,080 kWh. The M.D. charge as before = Rs. 625.

∴ M.D. charge/unit consumed = 625 × 100/70,080 = 0.89 paise

∴ total cost/unit consumed = 10 + 0.89 = **10.89 paise**

Obviously, with improvement in load factor, total cost per unit is reduced.

Example 50.43. *A consumer has a maximum demand (M.D.) of 20 kW at 0.8 p.f. lagging and an annual load factor of 60%. There are two alternative tariffs (i) Rs. 200 per kVA of M.D. plus 3p per kWh consumed and (ii) Rs. 50 per kVA of M.D. plus 7p per kWh consumed. Determine which of the tariffs will be economical for him.* **(Electrical Engineering, Banaras Hindu Univ.)**

Solution. An M.D. of 20 kW at 0.8 p.f. = 20/0.8 = 25 kVA

Average power = M.D. × load factor = 20 × 0.6 = 12 kW

Energy consumed/year = 12 × 8760 = 105120 kWh

(a) M.D. charge = Rs. (200 × 25) = Rs. 5000

Energy charge = Rs. 3 × 105, 120/100 = Rs. 3153.60

Annual charges = Rs. 5000 + Rs. 3153.60 = **Rs. 8153.60**

(b) M.D. charge = Rs. (50 × 25) = Rs. 1250

$$\text{Energy charge} = \text{Rs. } \frac{7 \times 105120}{100} = \text{Rs. } 7358.40$$

Annual charge = Rs. 1250 + Rs. 7358.40 = **Rs. 8608.40**

Obviously, tariff *(a)* is economical.

Example 50.44. *Determine the load factor at which the cost of supplying a unit of electricity from a Diesel station and from a steam station is the same if the respective annual fixed and running charges are as follows.*

Station	*Fixed charges*	*Running charges*
Diesel	*Rs. 300 per kW*	*25 paise/kWh*
Steam	*Rs. 1200 per kW*	*6.25 paise/kWh*

(Electrical Power-I, Gujarat Univ.)

Solution. *(i)* **Diesel Station**

Suppose that the energy supplied in one year is one unit *i.e.* one kWh.

∴ annual average power = 1 kWh/8760 *h* = 1/8760 kW

Now, annual load factor (L) $= \dfrac{\text{annual average power}}{\text{annual max. demand}}$

$\therefore$ max. demand $= \dfrac{\text{average power}}{L} = \dfrac{1}{8760\,L}$ kW

$\therefore$ fixed charge = Rs. 300 × 1/8760 L = 30,000 / 8760 L paise

Running charge = 1 × 25 = 25 paise

Hence, fixed and running charges per kWh supplied by Diesel station = $\left(\dfrac{30{,}000}{8760\,L}\right)$paise

***(ii)* Steam Station**

In the same way, fixed charge = Rs. 1200/8760 L = 120,000/8760 L paise

Running charge = 6.25 paise

$\therefore$ fixed plus running charges for supplying one kWh of energy are

= (120,000/8760 L) + 6.25 paise.

Since the two charges have to be the same

$$\therefore \quad \frac{120{,}000}{8760\,L} + 6.25 = \frac{30{,}000}{8760\,L} + 25\ ;\ L = \mathbf{0.55 \text{ or } 55\%.}$$

Example 50.45. *A factory has a maximum load of 300 kW at 0.72 p.f. with an annual consumption of 40,000 units, the tariff is Rs. 4.50 per kVA of maximum demand plus 2 paisa/unit. Find out the average price per unit. What will be the annual saving if the power factor be improved to units ?*

(Electrical Technology-I, Bombay Univ.)

Sulotion. kVA load at 0.72 p.f. = 300/0.72 = 1250/3

$\therefore$ max. kVA demand charge = Rs. 4.5 × 1250/3 = Rs. 1875

$\therefore$ M.D. charge per unit = Rs. 1875 / 40,000 = 4.69 paise

$\therefore$ total charge per unit = 2 + 4.69 = **6.69 paise**

Max. kVA demand at unity p.f. = 300

M.D. charge per unit = Rs. 300 × 4.5 = Rs. 1350

Annual saving = Rs. 1875 – Rs. 1350 = **Rs. 525.**

Example 50.46. *There is a choice of two lamps, one costs Rs. 1.2 and takes 100 W and the other costs Rs. 5.0 and takes 30 W ; each gives the same candle power and has the same useful life of 1000 hours. Which will prove more economical with electrical energy at Rs. 60 per annum per kW of maximum demand plus 3 paise per unit ? At what load factor would they be equally advantageous?*

Solution. *(i)* First Lamp

Initial cost of the first lamp per hour = Rs. 120/1000 = 0.12 paisa

Max. demand / hr. = 100/1000 = 0.1 kW

Max. demand charge/hr. = (60 × 100) × 0.1/8760 = 0.069 paisa

Energy charge/hr. = 3 × 0.1 = 0.3 paisa

$\therefore$ Total cost/hr. = 0.12 + 0.069 + 0.3 = **0.489 paisa**

***(ii)* Second Lamp**

Initial cost/hr. = 500/1000 = 0.5 paisa ; Max. demand /hr ; = 30/1000 = 0.03 kW

Max. demand charge/hr. = (60 × 100) × 0.03 / 8760 = 0.02 paisa

Energy charge/hr. = 3 × 0.03 = 0.09 paisa $\therefore$ total cost/hr. = 0.5 + 0.02 + 0.09 = **0.61 paisa**

Hence, the first lamp would be more economical.

It would be seen that the only charge which will vary with load factor is the max. demand charge. Moreover the maximum demand charge varies inversely as the load factor. Let x be the load factor at which both lamps are equally advantageous. Then

$$0.12 + \frac{0.069}{x} + 0.3 = 0.05 + \frac{0.02}{x} + 0.09 \quad \therefore \quad x = \mathbf{0.29}$$

Hence, both lamps would be equally advantageous at **29%** load factor.

Example 50.47. *The following data refers to a public undertaking which supplies electric energy to its consumers at a fixed tariff of 11.37 paise per unit.*

Total installed capacity = 344 MVA ; Total capital investment = Rs. 22.4 crores;

Annual recurring expenses = Rs. 9.4 crores ; Interest charge = 6% ; depreciation charge = 5%

Estimate the annual load factor at which the system should operate so that there is neither profit nor loss to the undertaking. Assume distribution losses at 7.84% and the average system p.f. at 0.86.

(Electrical Power-I, Bombay Univ.)

Solution. Yearly load factor (L) $= \dfrac{\text{No. of kWh supplied in a year}}{\text{Max. No. of kWh which can be supplied}}$ *

$$= \frac{\text{kWh supplied/year}}{\text{Max. output in kW} \times 8760}$$

$\therefore$ kWh supplied/year $= \text{Max. output in kW} \times 8760 \times L$

$$= (344{,}000 \times 0.86) \times 8760 \times L = L \times 25.92 \times 10^8$$

Considering distribution losses of 7.84%, the units actually supplied

$$= 92.16 \text{ per cent of } (L \times 25.92 \times 10^8) = L \times 23.89 \times 10^8$$

Amount collected @ of 11.37 paise / kWh = Rs. $L \times 23.89 \times 10^8 \times 11.37 / 100$

If there is to be no profit or gain, then this amount must just equal the fixed and running charges.

Annual interest and depreciation on capital investment

$$= 11\% \text{ of Rs. } 22.4 \times 10^7 = \text{Rs. } 2.46 \times 10^7$$

Total annual expenses $= \text{Rs. } 2.46 \times 10^7 + \text{Rs. } 9.4 \times 10^7 = \text{Rs. } 11.86 \times 10^7$

$\therefore \quad L \times 23.89 \times 10^8 \times 11.37 / 100 = 11.86 \times 10^7$; L = **0.437 or 4.37%**

Example 50.48. *An area has a M.D. of 250 MW and a load factor of 45%. Calculate the overall cost per unit generated by* ***(i)*** *steam power station with 30 per cent reserve generating capacity and* ***(ii)*** *nuclear station with no reserve capacity.*

Steam station : Capital cost per kW = Rs. 1000 ; interest and depreciation on capital costs = 15% ; operating cost per unit = 5 paise.

Nucelar station : capital cost per kW = Rs. 2000 ; interest and depreciation on capital cost = 12% ; operating cost per unit = 2 paise.

For which load factor will the overall cost in the two cases become equal ?

(Electrical Power-I, Bombay Univ.)

Solution. (*i*) Steam Station

Taking into consideration the reserve generating capacity, the installed capacity of the steam station $= 250 + (30\% \text{ of } 250) = 325 \text{ MW} = 325 \times 10^3 \text{ kW}$

average power (annual) $= \text{M.D.} \times \text{load factor} = 325 \times 10^3 \times 0.45 = 146{,}250 \text{ kW}$

$\therefore$ No. of units produced/year $= 8760 \times 146{,}250 = 128 \times 10^7$

* In this case max. demand has been taken as equal to the installed capacity.

Capital investment $= \text{Rs. } 325 \times 10^3 \times 1000 = \text{Rs. } 325 \times 10^6$

Annual interest and depreciation $= \text{Rs. } 325 \times 10^6 \times 0.15 = \text{Rs. } 48.75 \times 10^6$

These fixed charges have to be spread over the total number of units produced by the station.

$\therefore$ fixed charges / unit $= \text{Rs. } 48.75 \times 10^6 / 128 \times 10^7 = 3.8$ paise

$\therefore$ overall cost per unit generated = 3.8 + 5 = **8.8 paise**

(*ii*) Nuclear Station

Since there is no reserve capacity, installed capacity of the station equals the maximum demand of 250 MW = 25×10^4 kW.

Average power $= 0.45 \times 25 \times 10^4 = 112{,}500$ kW

Units produced annually $= 8760 \times 112{,}500 = 98.3 \times 10^7$

Capital investment $= \text{Rs. } 2000 \times 250 \times 10^3 = \text{Rs. } 5 \times 10^8$

Annual interest and depreciation $= \text{Rs. } 0.12 \times 5 \times 10^8 = \text{Rs. } 6 \times 10^7$

$\therefore$ fixed charges / unit $= \text{Rs. } 6 \times 10^7 / 98.3 \times 10^7 = 6.1$ paise

Overall cost per unit generated = 6.1 + 2.2 = **8.3 paise**

Load Factor

Suppose *L* is the load factor at which the overall cost per unit generated is the same.

$$\text{Cost / unit for steam station} = \left(\frac{48.75 \times 10^8}{325 \times 10^3 \times 8760\, L} + 5\right) = \left(\frac{1.712}{L} + 5\right) \text{paise.}$$

Similarly, overall cost/unit for nuclear station is

$$= \left(\frac{6 \times 10^7 \times 100}{250 \times 10^3 \times 8760\, L} + 2\right) = \left(\frac{2.74}{L} + 2\right) \text{paise}$$

Equating the two, we get $[(1.712/L) + 5] = [2.74/L) + 2]$ $\therefore$ $L =$ **0.34 or 34%**

Example 50.49. *The maximum demand of a customer is 25 amperes at 220 volt and his total energy consumption is 9750 kWh. If the energy is charged at the rate of 20 paise per kWh for 500 hours' use of the maximum demand plus 5 paise power unit for all additional units, estimate his annual bill and the equivalent flat rate.*

Solution. Charge at the max. demand rate.

Max. demand $= 25 \times 220/1{,}000 = 5.5$ kW

Units consumed at maximum demand rate $= 5.5 \times 500 = 2{,}750$ kWh

Max. demand charge $= 2{,}750 \times 20/100 =$ **Rs. 550**

Energy Charge

Units to be charged at lower rate = 9,750 – 2,750 = 7,000

$$\text{Charge} = \frac{7{,}000 \times 5}{100} = \text{Rs. } 350$$

$\therefore$ annual bill = Rs. 550 + 350 = **Rs. 900**

Equivalent flat rate $= 900 \times 100/9{,}750 =$ **9.2 paise**

Example 50.50. *A workshop having a number of induction motors has a maximum demand of 750 kW with a power factor of 0.75 and a load factor of 35%. If the tariff is Rs. 75 per annum per kVA of maximum demand plus 3 paise per unit, estimate what expenditure would it pay to incur to raise the power factor of 0.9.*

Solution. Max. kVA demand at 0.75 p.f. = 750/0.75 = 1000

M.D. charge $= 75 \times 1000$ = Rs. 75,000 ; Max. kVA demand at 0.9 p.f. = 750/0.9

M.D. charge = $(7{,}500/9) \times 75$ = Rs. 62,500

Difference in one year = 75,000 – 62,500 = **Rs. 12,500**

If the annual interest on money borrowed for purchasing the p.f. improvement plant is assumed 10%, then an expenditure of Rs. 125,000 is justified.

Example 50.51. *The owner of a new factory is comparing a private oil-engine generating station with public supply. Calculate the average price per unit his supply would cost him in each case, using the following data :*

Max. demand, 600 kW; load factor, 30% ; supply tariff, Rs. 70 per kW of maximum demand plus 3 paise per unit; capital cost of plant required for public supply, Rs. 10^5; capital cost of plant required for private generating station, Rs. 4×10^5 ; cost of fuel, Rs. 80 per tonne ; consumption of fuel oil; 0.3 kg per unit generated. Other work costs for private plant are as follows : lubricating oil, stores and water = 0.35 paise per unit generated ; wages 1.1 paise ; repairs and maintenance 0.3 paise per unit.

Solution. Public Supply Charges

Running charge/unit = 3.0 paise ; Average power consumption = 600×0.3 = 180 kW

Annual energy consumption = $180 \times 8{,}760$ kWh

Fixed annual charges = Rs. 70×600 = Rs. 42,000

Let us assume a capital charge rate of 10%. Hence, further annual amount to be charged from the customer is 0.1×10^5 = Rs. 10,000

$\therefore$ total fixed charges/annum = Rs. 42,000 + Rs. 10,000 = Rs. 52,000

$\therefore$ fixed charge per unit generated = $\dfrac{52{,}000 \times 100}{180 \times 8760}$ = 3.3 paise

$\therefore$ fixed plus running charges per unit generated = 3.0 + 3.3 = **6.3 paise.**

Private Supply Charges

Annual capital charges = Rs. $4 \times 10^5 \times 0.1$=Rs. 40,000

No. of units generated annually = $180 \times 8{,}760$ kWh

$\therefore$ fixed charges per unit generated = $\dfrac{4{,}000 \times 100}{180 \times 8760}$ = 2.54 paise

Cost of oil per unit generated = $\dfrac{80 \times 100 \times 0.3}{1{,}000}$ = 2.4 paise

Running charges per unit generated are :

Lubricating oil, stores, water = 0.35 paise; Wages = 1.1 paise

Repairs and maintenance = 0.3 paisa; Total = 0.35 + 1.1 + 0.3 = 1.75 paise

Total running charges = 2.4 + 1.75 = **4.15 paise**

Fixed plus running charges per unit generated = 2.54 + 4.15 = **6.69 paise**

Example 50.52. *Calculate the minimum two-part tariff to be charged to the consumers of a supply undertaking from the following data :*

Generating cost per kWh; 3.6 paise ; Generating cost per kW of maximum demand, Rs. 50

Total energy generated per year ; $4{,}380 \times 10^4$ kWh

Load factor at the generating station, 50%

Annual charges for distribution Rs. 125,000

Diversity factor for the distribution network, 1.25

Total loss between station and consumer, 10%.

Solution. Average generating power $= 4{,}380 \times 10^4/8760 = 5{,}000$ kW

$\therefore$ maximum load on generator $= 5{,}000/\text{load factor} = 5{,}000 / 0.5 = 10{,}000$ kW

$\therefore$ annual fixed charges $=$ Rs. $10{,}000 \times 50$ = Rs. 500,000

Fixed charges = Rs. 125,000 Total fixed charges = Rs. 625,000

Since diversity factor is known, consumer's max. demand $= 10{,}000 \times 1.25 = 12{,}500$ kW

Hence, Rs. 625,000 have to be equally distributed over the 12,500 kW maximum demand.

$\therefore$ cost per kW max, demand = 625,000/12,500 = Rs. 50

$\therefore$ monthly kW maximum charges is = Rs. 50/12 = Rs. 4.17

Since losses are 10%, consumer gets 0.9 kWh for every kWh generated at the station. Hence cost per kWh to the consumer is 3.6/0.9 = 4 paise.

Therefore, minimum charges are **Rs. 4.17 per kW** of max. demand per month and **4 paise per kWh** consumed.

Example 50.53. *Two systems of tariffs are available for a factory working 8 hours a day for 300 working days in a year.*

(a) High-voltage supply at 5 paise per unit plus Rs. 4.50 per month per kVA of maximum demand.

(b) Low-voltage supply at Rs. 5 per month per kVA of maximum demand plus 5.5 paise per unit.

The factory has an average load of 200 kW at 0.8 power factor and a maximum demand of 250 kW at the same p.f.

The high-voltage equipment costs Rs. 50 per kVA and losses can be taken as 4 per cent. Interest and depreciation charges are 12 per cent. Calculate the difference in the annual cost between the two systems.

Solution. First, let us find the annual cost according to rate (*a*).

(*a*) Capacity of h.v. switchgear = (250/0.8) × (100/96) kVA

Annual interest on capital investment and depreciation

$$= \left(\frac{250}{0.5} \times \frac{100}{96}\right) \times 50 \times 0.12 = \text{Rs. } 1953$$

Annual charge due to kVA max. demand is

$$= \frac{250}{0.5} \times \frac{100}{96} \times 12 \times 4.5 = \text{Rs. } 17{,}580$$

Annual charge due to kWh consumption

$$= 200 \times \frac{100}{96} \times \frac{5}{100} \times (8 \times 300) = \text{Rs. } 25{,}000$$

$\therefore$ total charges = Rs. (1953 + 17,580 + 25,000) = **Rs. 44,533**

(*b*) The total annual cost due to rate **(*b*)** would be as under.

Annual charge due to kVA max. demand is $= 250 \times 5 \times 12/0.8 =$ Rs. 18,750

Annual charge due to kWh consumption is $= 200 \times \frac{5.5}{100} \times 2{,}400 =$ Rs. 26,400

$\therefore$ total charges = Rs. 18,750 + Rs. 26,400 = Rs. 45,150

Hence, high-voltage supply is cheaper by 45,150 – 44,533 = **Rs. 617.**

Example 50.54. *Estimate what the consumption must be in order to justify the following maximum demand tariff in preference to the flat rate if the maximum demand is 6 kW.*

On Maximum Demand Tariff. A max. demand rate of 37 paise per unit for the first 200 hr. at the maximum demand rate plus 3 paisa for all units in excess.

Flat-rate tariff, 20 paise per unit.

Solution. Let x = the number of units to be consumed (within a specific period)

On Max. Demand Tariff

Units consumed at max. demand rate $= 6 \times 200 = 1200$ kWh

Units in excess of the max. demand units $= (x - 1200)$

Cost of max. demand units $= 1200 \times 37 = 44{,}400$ paise

Cost of excess units $= 3\,(x - 1200) = (3\,x - 3600)$ paise

$\therefore$ total cost on this tariff $= 44{,}400 + 3\,x - 3600 = 3\,x + 40{,}800$ paise

Flat-rate Tariff

Total cost of units on this rate $= 20\,x$ paise

$\therefore \quad 20\,x = 3\,x + 40{,}800$

or $\quad 17x = 40{,}800 \qquad \therefore \quad x =$ **2,400 units**

Tutorial Problem No. 50.1

1. A plant costing Rs. 650,000 has a useful life of 15 years. Find the amount which should be saved annually to replace the equipment at the end of that time.

(*i*) by the straight line method and (*ii*) by the sinking fund method if the annual rate of compound interest is 15%.

Assume that the salvage value of equipment is Rs. 5000.

[(i) Rs. 4,000 (ii) Rs. 1,261] (*Elect. Generation, Punjab Univ.*)

2. A plant costs Rs. 7.56×10^5 and it is estimated that after 25 years, it will have to be replaced by a new one. At that instant, its salavage value will be Rs. 1.56×10^5. Calculate.

(*i*) the annual deposit to be made in order to replace the plant after 25 years and

(*ii*) value of the plant after 10 years on the 'reducing balance depreciation method'.

[(i) 0.0612 (ii) Rs. 4.02×10^5] (*Util. of Elect. Power, AMIE Sec. B,*)

3. From the following data, estimated the generating cost per unit delivered at the station.

Capacity of the generating plant = 10 MW; annual load factor = 0.4 ; capital cost = Rs. 5 million; annual cost of fuel, oil, wages, taxes and salaries = Rs. 2×10^5; Rate of interest = 5%; rate of depreciation = 5% of initial value. **[2 paisa/kWh]**

4. From the following data, find the cost of generation per unit delivered from the station.

Capacity of the plant installed = 100 MW; annual load factor = 35% ; capital cost of power plant = Rs. 1.25 crores ; Annual cost of fuel, oil, salaries and taxation = Rs. 0.15 crore; Interest and depreciation on capital = 12%.

If the annual load factor of the station is raised to 40%, find the percentage saving in cost per unit.

[1 paisa/kWh ; 12.2%]

5. The capital costs of steam and water power station are Rs. 1200 and Rs. 2100 per kW of the installed capacity. The corresponding running costs are 5 paise and 3.2 paise per kWh respectively.

The reserve-capacity in the case of the steam station is to be 25% and that for the water power station is to be 33.33% of the installed capacity.

At what load factor will the overall cost per kWh be the same in both cases ? Assume interest and depreciation charges on the capital to be 9% for the thermal and 7.5% for the hydroelectric station. What would be cost of generating 500 million kWh at this load factor ? **[47.5% ; Rs. 4.12 crores]**

6. In a particular area, both steam and hydro stations are equally possible. It has been estimated that capital costs and the running costs of these two types will be as follows :

	Capital cost /kW	*running cost/kW*	*interest*
Hydro	Rs. 2,200	1 paisa	5%
Steam	Rs. 1,200	5 paise	5%

If expected average load factor is only 10% which is economical to operate, steam or hydro ? If the load factor is 50%, would there be any changes in the choice ? If so, indicate with calculations.

[10% load factor ; hydro ; 13.5 paise / kWh ; Steam = 11.85 paise/ kWh50% load factor ; hydro ; 3.5 paise/kWh ; Steam = 6.35 paise/kWh]

7. A consumer takes a steady load of 200 kW at a power factor of 0.8 lagging for 10 hours per day and 300 days per annum. Estimate his annual payment under each of the following tariffs :

(*i*) 10 paisa per kWh plus Rs. 100 per annum per kVA

(*ii*) 10 paisa per kWh plus Rs. 100 per annum per kW plus 2 paisa per kVARh.

[(i) Rs. 85,000 (ii) Rs. 170,000.] (*Util. of Elect. Power, Punjab Univ.*)

8. A consumer requires one million units per year and his annual load factor is 50%. The tariff in force is Rs. 120 per kW per annum plus 5 paise per unit. Estimate the saving in his energy charges which would result if he improved his load factor to 100%.

[Rs. 13,698.60] (*Util. of Elect. Power, AMIE*)

9. A factory has a maximum demand of 1000 kW and a load factor of 40%. It buys power from a company at the rate of Rs. 45 per kW plus Rs. 0.025/kWh consumed. Calculate the annual electricity bill of the factory. Also work out the overall cost of one unit of electricity.

[Rs. 1,32,600, 3.78 paise] (*Util of Elect. Power, AMIE*)

10. A power station having a maximum demand of 100MW has a load factor of 30%. It is to be supplied by any of the following schemes :

(*a*) a steam power station in conjunction with a hydro station, the latter supplying 108 kWh/annum with a maximum output of 40 MW.

(*b*) a steam station capable of supplying whole load,

(*c*) a hydro station capable of supplying whole load.

	The following data may be assumed :	steam	hydro
(*i*)	Capital cost/kW installed capacity	Rs. 600	Rs. 1500
(*ii*)	Interest and depreciation on capital cost	12%	10%
(*iii*)	Operating cost/unit	5 p	1 p
(*iv*)	Transmission cost/unit negligible	0.25 p	

Neglect spares. Calculate the overall cost per unit generated in each case.

[(a) 7.5 P/ kWh (b) 7.74 P/kWh (c) 6.95 P/kWh]

11. An electricity undertaking having a maximum load of 100 MW generates 375 million kWh per annum and supplies consumers having an aggregate maximum demand of 165 MW.

The annual expenses including capital charges are : for fuel Rs. 30 lakhs, fixed charges concerning generation Rs. 40 lakhs. Fixed charges connected with transmission and distribution: Rs. 50 lakhs. Assume that 90% of the fuel cost is essential to running charges and with the losses in transmission and distribution as 15% of the kWh generated, deduce a two-part tariff to find the actual cost of supply to consumers. **[Rs. 56.3 per kW Max. demand ; 0.84 P/kWh]**

12. Obtain a tariff for the consumers of supply undertaking which generates 39×10^7 kWh/year and has maximum demand of 130 MW connected to it. The cost is distributed as follows :

Fuel : Rs. 37.5 lakhs : Generation : Rs. 18 lakhs ; Transmission : Rs. 37.5 lakhs ; Distribution : Rs.25.5 lakhs.

Of these items, 90%, 10% 5% and 7% respectively are allocated to running charges, the remainder being fixed charges. The total loss between the station and the consumers is 10% of the energy generated.

Find also the load factor and overall cost per unit.

[Rs. 61 per kW max. demand and 1.1 P/kWh ; 34.2% ; 3.38 P/kWh]

13. A generating station has got maximum demand of 80 kW. Calculate the cost per kWh delivered from the following data :

Capital cost = Rs. 16×10^6
Annual cost of fuel and oil = Rs. 14×10^5
Taxes, wages and salaries = Rs. 9×10^5
The rate of interest and depreciation is 12%; Annual load factor is 60%.

[10.036 paise] (*Utili. of Elect. Power. AMIE*)

14. A S/S supplies power by four feeders to its consumers. Feeder No. 1 supplies six consumers whose individual daily maximum demands are 70 kW, 90 kW, 20 kW, 50 kW, 10 kW and 20 kW, while the maximum demand on feeder is 200 kW. Feeder No. 2 supplies four consumers, whose daily max. demands are 60 kW, 40 kW, 70 kW, and 30 kW, while the maximum demand on feeder No. 2 is 160 kW. Feeders Nos. 3 and 4 have a daily maximum demand of 150 kW and 200 kW respectively, while the maximum demand on the station is 600 kW.

Determine the diversity factor for the consumers of feeder No. 1, feeder No. 2 and for the four feeders. **[1.3, 1.25, 1.18] (*AMIE*)**

15. A generating station has a M.D. of 75 MW and a yearly load factor of 40%. Generating costs inclusive of station capital cost are Rs. 60 per annum per kW demand plus 1 paisa/ kWh transmitted. The annual capital charge for the transmission system is Rs. 1.5 million, the respective diversity factors being 1.2 and 1.25 respectively. The efficiency of transmission system is 90% and that of the distribution system inclusive of substation losses 85%. Find the yearly cost per kW demand and the cost per kWh supplied (*a*) at the substation and (*b*) at the consumer's premises.

[(*a*) Rs. 72.2 per kW M.D. and 1.1. P/kWh (b) Rs. 71.1 per kW M.D. and 1.31 P/kWh]

50.21. Kelvin's Law

We will now consider the application of Kelvin's economy law to power transmission through feeder cables. Since, in feeders, voltage drop is not of vital importance, they can be designed on the basis of their current-carrying capacity and where feasible, of minimum financial losses.

The financial loss occuring in a current-carrying conductor is made up of two parts :

(*i*) interest on the capital cost of the conductor plus allowance for depreciation and

(*ii*) the cost of energy loss due to **(*a*)** ohmic resistance *i.e.*, I^2R loss, **(*b*)** losses in the metallic sheaths of sheathed cables and **(*c*)** in the case of high tension insulated cables, loss in the insulating material used.

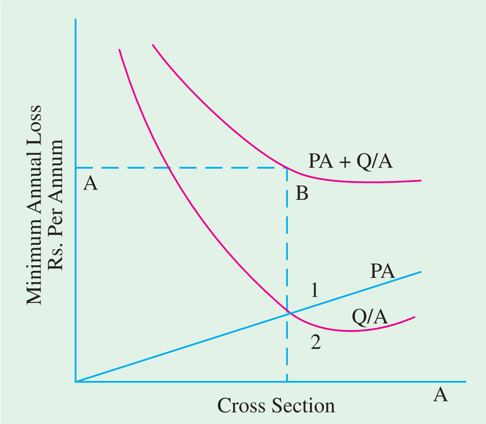


Fig. 50.11

For a given length of the cable, the weight and hence cost of copper required is directly proportional to the cable cross-section. The annual combined interest on capital cost and depreciation is also directly proportional to the cross-section of the cable and can be written as Rs. PA where P = a constant and A = cross-section of the cable.

Now, we will consider the cost of I^2R loss, neglecting at the moment losses other than ohmic. The ohmic resistance of the conductor is proportional to $1/A$. For a given annual load curve, the energy loss is proportional to resistance and so proportional to $1/A$. It can be written as Q/A where Q is another constant. If the load is variable, then current used in calculating I^2R loss will be the r.m.s. value of current reckoned over a period of one year.

$\therefore$ total annual financial loss is $L =$ Rs. $\left(PA + \dfrac{Q}{A}\right)$

Now, it can be proved that this annual financial loss will be minimum when $PA = Q/A$ *i.e.*, ***when annual charge on the capital outlay is equal to the annual value of the energy loss***.* The most economical cross-section which will make the above cost equal is given by :

$$PA = \frac{Q}{A} \text{ or } A^2 = Q/P \therefore A = \sqrt{Q/P}$$

The most economical cross section can be also found graphically as shown in Fig. 50.11. It is seen that graph of PA is a straight line (curve 1) and that of Q/A is a rectangular hyperbola (curve 2). The graph for total annual loss is obtained by combining curves 1 and 2 and is found to exhibit a minimum for that value of 'A' which corresponds to the intersection of the two curves.

Since we have neglected the dielectric and sheath losses while deriving the above condition of minimum financial loss, it is obvious that it applies only to ***bare conductors.***

In practice, Kelvin's law may have to be modified because the most economical size so calculated may not always be the practical one because it may be too small to carry the current.

Example 50.55. *If the cost of an overhead line is Rs. 2000 A (where A is the cross-sectional area in cm^2) and if the interest and depreciation charges on the line are 8%, estimate the most economical current density to use for a transmission requiring full-load current for 60% of the year. The cost of generating electric energy is 5 paise/kWh. The resistance of a conductor one kilometre long and 1 cm^2 cross-section is 0.18 Ω.*

Solution. Take one kilometre of the overhead line and let I be the full-load current, then if R is the resistance of each line, the full-load power loss in the line is

$$= 2I^2R = 2I^2 \times 0.18 \times \frac{1}{A} \text{ watt} = \frac{0.36\, I^2}{A} \times 10^{-3} \text{ kW}$$

Since the line works for 60% of the year *i.e.* for $(0.6 \times 365 \times 24)$ hours, hence total annual loss is $= \dfrac{0.35\, I^2}{A} \times 10^{-3} \times 0.6 \times 365 \times 24$ kWh

$$\therefore \text{ annual cost of this loss} = \text{Rs.} \left(\frac{0.35\, I^2}{A} \times 10^{-3} \times 0.6 \times 365 \times 24 \times 0.05 \right) = 0.0946 \text{ I2/A}$$

The annual value of interest on capital outlay and depreciation = 8% of 2,000 A = Rs. 160 A

For minimum total financial loss : $160\, A = 0.0946\, I^2/A \quad I/A = \sqrt{(160/0.0946)}$ = **41.12 A/cm^2**

Example 50.56. *A 500-V, 2-core feeder cable 4 km long supplies a maximum current of 200 A and the demand is such that the copper loss per annum is such as would be produced by the full-load current flowiing for six months. The resistance of the conductor 1 km long and 1 sq cm. cross-sectional area is 0.17 Ω. The cost of cable including installation is Rs. (120 A + 24) per metre where A is the area of cross-section in sq. cm and interest and depreciation charges are 10%. The cost of energy is 4 paise per kWh. Find the most economical cross-section.*

(Electrical Technology, M.S. Univ. Baroda)

Solution. Let us consider one kilometer length of the feeder cable.**

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* For this expression to have minimum value $\dfrac{dL}{dA} = 0$

$$\frac{d}{dA}(P + Q/A) = 0 \quad \text{or} \quad P - \frac{Q}{A^2} = 0 \quad \therefore \quad P = \frac{Q}{A^2} \quad \text{or} \quad PA = \frac{Q}{A} \quad \text{or} \quad A = \sqrt{Q/P}$$

** Even through we are given 4 km length of the cable, so far as our calculations are concerned, any convenient unit of length can be taken.

Cable cost/metre $= \text{Rs. } (120\,A + 24)$

Cost of 1 km long cable $= \text{Rs. } 120\,A \times 1000 = \text{Rs. } 12 \times 10^4\,A$

Interest and depreciation per annum is $= 10\%$ of $\text{Rs. } 12 \times 10^4\,A = \text{Rs. } 12 \times 10^3\,A$

Resistance of one km long cable is $R = 0.17/A$ ohm. —where A is in cm^2

Cu loss in the cable $= 2I^2R = 2 \times 200^2 \times 0.17/A\ W = 13.6 \times 10^3/A\,W = 13.6/A$ kW

Energy loss over six months $= (13.6/A) \times (8760/2)$ kWh $= 59{,}568/A$ kWh

Cost of this energy loss $= \text{Rs. } 59{,}568 \times 0.04/A = \text{Rs. } 2383/A$

For most economical cross-section ; $12 \times 10^3\ A = 2{,}383/A$, $A = \sqrt{0.1986} =$ **0.446 cm^2**

Example 50.57. *A 2-core, 11-kV cable is to supply 1 MW at 0.8 p.f. lag for 3000 hours in a year. Capital cost of the cable is Rs. (20 + 400a) per metre where a is the cross-sectional area of core in cm^2. Interest and depreciation total 10% and cost per unit of energy is 15 paise. If the length of the cable is 1 km, calculate the most economical cross-section of the conductor. The specific resistance of copper is 1.75×10^{-6} ohm-cm.* **(Power Systems-I, AMIE, Sec. B, 1993)**

Solution. Cost of 1 km length of the cable = Rs. $(20 + 400a) \times 1000 = (20{,}000 + 400 \times 10^3 a)$

If a is the cross-sectional area of each core of the cable, then resistance of 1 km cable length is

$= 1.75 \times 10^{-6} \times 1000 \times 100/a = 0.175/a\ \Omega$.; F.L. current $= 1 \times 10^6/11 \times 10^3 \times 0.8 = 113.6$ A

Power loss in the cable $= 2I^2R = 2 \times 113.6^2 \times (0.175/a) = 4516.7/a$ W

Annual cost of energy loss $= \text{Rs. } (4516.7/a) \times 3000 \times 10^{-3} \times (15/100) = \text{Rs. } 2032.5/a$.

Interest and depreciation per annum $= 0.1 \times 400 \times 10^3 a = 40{,}000\ a$

As per Kelvin's law, the most economical cross-section would be given by

$$40{,}000\ a = 2032.5/a \qquad a = 0.2254 \text{ cm}^2 \text{ and}$$

$$d = \sqrt{0.2254 \times 4/\pi} = \mathbf{0.536\ cm.}$$

50.22. Effect of Cable Insulation

Let us now consider the effect of insulation in the case of an insulated cable. The cost of insulation would now be added to the annual value of interest and depreciation. Since for a given type of cable, a given type of armouring and for a given voltage of transmission, the insulation cost does not vary much with the cross-section of the cable, it is taken care of by merely adding a constant R to the term Rs. PA. Hence, the annual interest and depreciation, and insulation cost is represented by the term Rs. $(PA + R)$ where R is a constant representing insulation cost. The addition of term Rs. R results in merely raising the graph of Rs. PA vertically through a distance representing Rs. R, as shown in Fig. 50.12.

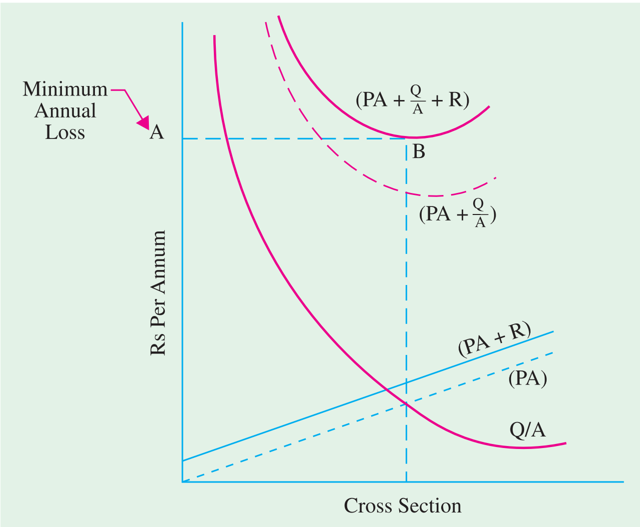

Fig. 50.12

The total annual financial loss of insulated cable is thus represented by Rs. $(PA + R + Q/A)$. It should be noted that curve for Q/A is not disturbed at all. However, the graph for total loss *i.e.* $(PA + Q/A + R)$ is only shifted vertically through a distance equal to Rs.

R without any horizontal displacement of the point of minimum financial loss. In other words, the insulation cost does not affect the value of most economical cross-section. Hence, for an insulated cable, Kelvin's law is : *the most economical cross-section is that which makes the interest on the capital outlay plus depreciation due to the conductor in the cable equal to the annual cost of energy lost.*

Example 50.58. *The cost of a two-core feeder cable including insulation is Rs. (130 A + 24) per metre and the interest and depreciation charges 10% per annum. The cable is two km in length and the cost of energy is 4 paisa per unit. The maximum current in the feeder is 250 amperes and the demand is such that the copper loss is equal to that which would be produced by the full current flowing for six months. If the resistance of a conductor of 1 sq. cm cross-sectional area and one km in length be 0.18 Ω, find the most economical section of the same.*

Solution. Cable cost/m = Rs. $(130\,A + 24)$ where A is the cable cross-section in cm^2.

Cost of 1 km long conductor = Rs. $(130\,A \times 1000)$ = Rs. $(13 \times 10^4\,A)$

Interest and depreciation per annum = $0.1 \times 13 \times 10^4\,A$ = Rs. $(13 \times 10^3\,A)$

Resistance of one km long conductor = 0.18/A ohm

Cu loss in the cable $= 2I^2R = 2 \times 250^2 \times \dfrac{0.18}{A}\ \text{W} = 2 \times 250^2 \times \dfrac{0.18}{A} \times 10^{-3}\ \text{kW}$

Annual value of this energy loss = Rs. $\left(2 \times 250^2 \times \dfrac{0.18}{A} \times 10^{-3}\ \dfrac{4}{100}\right) \times \dfrac{8760}{2}$

= Rs. 3941/A

For most economical cross-section 13×10^3 A = 3,942/A $\therefore$ $A^2 = 3942/13 \times 10^3$

$\therefore$ A = **0.505 cm^2**

Example 50.59. *An 11-kV, 3-core cable is to supply a works with 500-kW at 0.9 p.f. lagging for 2,000 hours p.a. Capital cost of the cable per core when laid is Rs. (10,000 + 32,00 A) per km where A is the cross-sectional area of the core in sq. cm. The resistance per km of conductor of 1 cm^2 cross-section is 0.16 Ω.*

If the energy losses cost 5 paise per unit and the interest and sinking fund is recovered by a charge of 8% p.a., calculate the most economical current density and state the conductor diameter.

(Power Systems-I, AMIE-1994)

Solution. The annual charge on cost of conductors per km is = $0.08 \times 32{,}000\,A$ = Rs. 2,560 A

Current per conductor is = $500{,}000/\sqrt{3} \times 11{,}000 \times 0.9 = 29.2$ A

Losses in all the three cores $= 3I^2R = 3 \times 29.2^2 \times \dfrac{0.16}{A} = \dfrac{409.3}{A} \times 10^{-3}$ kW

Annual cost of this loss = Rs. $\dfrac{409.3 \times 10^{-3}}{A} \times \dfrac{5}{100} \times 200$ = Rs. $\dfrac{40.93}{A}$

Obviously, the values of the two constants per km are P = 2,560 and Q = 40.93

The most economical cross-section is given by $A = \sqrt{Q/P} = \sqrt{(40.93/2{,}560)} = 0.1265$ cm^2

The current density is 29.2/0.1265 = **231 A/cm^2** and the conductor diameter is

$d = \sqrt{4 \times 0.1256/\pi}$ = **0.4 cm.**

Example 50.60. *Discuss limitations of the application of Kelvin's law.*

An industrial load is supplied by a 3-phase cable from a sub-station at a distance of 6 km. The voltage at the load is 11 kV. The daily load cycle for six days in a week for the entire year is as given below :

(i) *700 kW at 0.8 p.f. for 7 hours,* *(ii)* *400 kW at 0.9 p.f. for 3 hours,*

(iii) *88 kW at unity p.f. for 14 hours.*

Compute the most economical cross-section of conductors for a cable whose cost is Rs. (5000 A + 1500) per km (including the cost of laying etc.). The tariff for the energy consumed at the load is Rs. 150 per annum per kVA of M.D. plus 5 paise per unit. Assume the rate of interest and depreciation as 15%. The resistance per km of the conductor is (0.173/A) Ω.

(Electrical Power-I ; Bombay Univ.)

Solution. Capital cost of the cable = 6 × Rs. (5000 A + 1500) = Rs. (30,000 A + 9000).

Annual cost of interest and depreciation = Rs. 0.15 (30,000 A + 9000) = Rs. (4500 A + 1350).

Resistance of each conductor = 6 × 0.173/A = 1038/A ohm

Line currents due to different loads are :

(i) At 700 kW, 0.8 p.f.; $I = 700 \times 10^3/\sqrt{3} \times 11 \times 10^3 \times 0.8 =$ **45.9 A**

(ii) At 400 kW, 0.9 p.f.; $I = 400 \times 10^3/\sqrt{3} \times 11 \times 10^3 \times 0.9 =$ **23.3 A**

(iii) At 88 kW, u.p.f. $I = 88/ \times 11 =$ **4.62 A**

The corresponding energy losses per week in the cable are :

(i) loss $= 3 \times 45.9^2 \times (1.038/A) \times (6 \times 7)/1000 = (276/A)$ kWh

(ii) loss $= 3 \times 23.3^2 \times (1.038/A) \times (6 \times 3)/1000 = (30.4/A)$ kWh

(iii) loss $= 3 \times 4.62^2 \times (1.038/A) \times (6 \times 14) \times 10^{-3} = (5.6/A)$ kWh

Total weekly loss $= (276/A) + (30.4/A) + (5.6/A) = (312/A)$ kWh

Taking 52 weeks in one year, we have

Annual Cu loss $= \text{Rs. } \frac{5}{100} \cdot \frac{312}{A} \times 52 = \text{Rs. } \frac{811}{A}$

Max. voltage drop in each conductor $= 45.9 \times 1.038/A = (47.64/A)$ volt

Max. kVA demand charge due to this drop for three conductors is

$= \text{Rs. } 3 \times (47.64/A) \times 45.9 \times 10^{-3} \times 150 = \text{Rs. } 984/A$

Total annual charges due to cable loss = Rs. (811/A) + Rs. (984/A) = Rs. (1795/A)

For the most economical size of the cable ; 4500 A=1795/A ; A = 0.63 cm^2.

Note. Though not given, it has been assumed that A appearing in Rs. (5000 A + 1500) is in cm^2.

Tutorial Problem No. 50.2

1. The daily load cycle of a 3-phase, 110-kV transmission line is as follows :

(*a*) 6 hours–20 MW, (*b*) 12 hours–5 MW, (*c*) 10 hours – 6 MW. The load p.f. is 0.8 lag for all the three loads. Determine the most economical cross-section if the cost of the line including erection is Rs. (9,000 + 600 A) per km where A is the cross-section of each conductor in cm^2. The rate of interest and depreciation is 10% and energy cost is 6 paise/unit. The line is in use all the year. The resistance per km of each conductor is 0.176/A ohm. **[1.64 cm^2]**

2. Find the most economical cross-section of conductor for a system to transmit 120 A at 250 V all the year round with a cost of 10 paise per kWh for energy wasted. The two-core cable costs Rs. 20 A per metre where A is the cross-section of each core in cm^2. The length of feeder is one km. The interest and depreciation charges total 8% of total cost. One km of Cu wire 1 cm^2 in cross-section area has a resistance of 0.15 W.

[1.52 cm^2]

50.23. Note on Power Factor

Irrespective of the nature of voltage and current, the power factor may be defined as the ratio of true power consumed to the apparent power (product of voltage and current).

$$\therefore \qquad \text{p.f.} = \frac{W}{VA} = \frac{kW}{kVA}$$

If voltage and current are both sinusoidal, then p.f. = kW/kVA = cos ϕ

where ϕ is the phase difference between voltage and current.

The power factor of all a.c. motors (except over-excited synchronous motors and certain types of commutator motors) and transformers is less than unity (lagging). Majority of industrial motors are induction motors which have a reasonable high power factor (0.9 or so) at full-load but very low power factor at light loads and at no-load as low a value as 0.1. Induction motor is, in fact, a simple resistance-inductance circuit. The current taken by such a circuit has two components ***(i)*** active or wattful component Iw which is in phase with the voltage. Its value is $I \cos \phi$ where I is total circuit current. It is this component which represents the useful power in the circuit and ***(ii)*** reactive or wattless or idle component I_μ which lags behind the applied voltage by 90°. Its value is $I \sin \phi$ and its purpose is to produce alternating magnetic flux. Hence, it is also known as wattless or magnetising current.

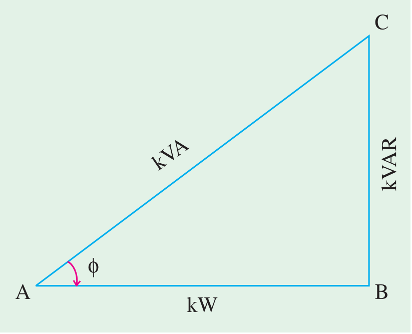

Fig. 50.13

Obviously $\qquad I^2 = I_w^2 + I_\mu^2$

Multiplying both sides by V_2, we get

$$V^2I^2 = V^2I_w^2 + V^2I_\mu^2$$

or $\qquad (VI)^2 = (VI_w)^2 + (VI_\mu)^2$

Now $\qquad VI$ = volt-amperes written as VA ; VI_w = wattage W

$\qquad VI_\mu$ = reactive volt-ampere written as $VA\,R$.

Hence, in any inductive circuit as shown in Fig. 50.13

$$\text{kVA}^2 = \text{kW}^2 + \text{kVA R}^2$$

For example, suppose an R-L circuit draws a current of 100 A from a supply of 250 V at a power factor of 0.8, then total kilo-voltampere of the circuit is 250 × 100/1000 = 25 kVA. The true power is 250 × 100 × 0.8/1000 = 20 kW and reactive voltampere is 250 × 100 × 0.6/1000 = 15 kVAR.

Obviously, $\qquad \text{kVA} = \sqrt{20^2 + 15^2} = 25$

50.24. Disadvantages of Low Power Factor

Suppose a 3-phase balanced system is supplying a load W at a voltage of V and p.f. cos ϕ, then current flowing through the conductor is $I = W/\sqrt{3}\,.\,V\,.\cos\phi$.

A lower power factor, obviously, means a higher current and this affects in the following three ways :

1. Line losses are proportional to I^2 which means proportional to $1/\cos^2\phi$. Thus losses at $\cos\phi = 0.8$ are $1/0.8^2 = 1.57$ times those at unity power factor.
2. Ratings of generators and transformers etc. are proportional to current, hence to $1/\cos\phi$, therefore, larger generators and transformers are required.
3. Low lagging power factor causes a large voltage drop, hence extra regulation equipment is required to keep voltage drop within prescribed limits.

Explanation

Low power factor means higher wattless current which presents a serious problem from supply viewpoint. Consider the following numerical example. Suppose an alternator (single-phase) is to have an output of 1500 kW at 10,000 V.

(*a*) Load at unity power factor $P = VI \times 1 = VI$ watt and $I = \frac{P}{V} = \frac{1,500,000}{10,000} = 150$ A

In this case $\quad$ kVA $=$ kW $= 1500$

(*b*) If the load has a lagging p.f. of 0.6, then

$$I = \frac{P}{V\cos\phi} = \frac{1,500,000}{10,000 \times 0.6} = 250 \text{ A and kVA} = \mathbf{2500}$$

It is obvious that in case **(*b*)**, the current-carrying capacity of alternator winding will have to be (250-150)/150 = 0.67 or 67% greater. Now, the size and hence cost of a given type of alternator is decided by kVA output and not by kW.

In case **(*a*)**, kVA = kW1500. In case **(*b*)** kVA = 1500/0.6 = 2500.

It is seen that with a load p.f. of 0.6, the cost of an alternator to give the same power output and hence the earning capacity, is about 67% greater than when p.f. is unity. Moreover, the switchgear will have to carry 250 A instead of 150 A and so, will be correspondingly larger and more costly. Since transformers are almost always used in the line, they too will be more expensive.

The cables connecting the alternator with the load will also have to carry more current. If current density is the same, then cable cross-section and hence the weight of copper required will be increased.

Therefore, it is seen that a low power factor leads to a high capital cost for the alternator, switchgear, transformers and cables etc. Since the value of power factor is decided by the consuming devices like motors etc., the electric supply undertakings encourage the consumers to make their power factor as high as possible. As an inducement, they offer cheaper tariffs in different ways discussed in Art. 50.25.

The disadvantages of low power factor are summarized below :

In a transmission line (supplying an inductive load) it is only the in-phase component of line current which is active in the transmission of power. When power factor is low, then in-phase (or active) component is small, but reactive component is large hence unnecessarily large current is required to transmit a given amount of power. Large reactive component means large voltage drop and hence greater Cu losses with the result that the regulation is increased and efficiency is decreased.

Lighting and heating loads, being mainly resistive, have unity p.f. but electromagnetic machinery has a low p.f. especially on no-load or light loads.

A low power factor (lagging) system has the following disadvantages as compared to a system carrying the same power but at higher p.f.

1. As current is large and at a large lagging angle, it cause greater losses and requires higher excitation in the generators, thereby reducing the efficiency of generation.

2. Increased Cu losses are incurred in the cables and machinery because of large current. Both these factors raise the running cost of the system.

3. The ratings of the generators, transformers, switchgears and cables etc., have to be increased, which means additional capital charges with increased depreciation and interest.

4. Due to voltage drop in generators, transformers and cables etc. the regulation becomes poor. As supply authorities are usually bound to maintain the voltage at consumer's terminals within prescribed limits, they have to incur additional capital cost of tap-changing gear on transformers to compensate for the voltage drop. Hence, the supply authorities penalise industrial consumers for their low power factor by charging increased tariff for kVA maximum demand in addition to usual kWh charge. Obviously, it is advantageous for the consumer to improve his load p.f. with the help of phase-advancing equipment (*i.e.* synchronous capacitors) or static capacitors.

50.25. Economics of Power Factor

In order to induce the customers to keep their load p.f. as high as possible, tariff rates for a.c. power are such that the overall charge per kWh of the energy consumed depends, in some way or the other, on the load p.f. of the consumer. Some of the ways in which it is done are given below :

(*i*) The total bill for consumption is so adjusted as to make it depend on the deviation of the load p.f. from a standard value,say 0.9. The bill is increased by a constant percentage for each unit p.f. deviation from 0.9, say, for every decrease of 0.01 from 0.9. Similarly, there would be bonus for each increase of unit p.f. value.

(*ii*) The total power bill is adjusted in some way on the total reactive kilo-volt ampere-hour *i.e.* kVARh instead of kWh. Special meters are installed for measuring kVARh.

(*iii*) But the most commonly-used tariff is the two-part tariff (Art. 50.20). The fixed charges instead of being based on kW maximum demand are based on kVA maximum demand so that they become inversely proportional to the power factor (because kVA = kW/cos ϕ)

Explanation

Consider a single-phase, 1500-kW, 10,000-V alternator already discussed in Art. 50.24. Suppose whole of its output is taken up by one consumer and the two-part tariff is Rs. 60 per annum per kVA max. demand plus 2 paise per kWh. Let the load factor be 30%. Then,

Average power throughout the year = $0.3 \times 1500 = 450$ kW

$\therefore$ annual consumption = $450 \times 8{,}760$ kWh

Annual cost at 2 paise per kWh = Rs. $450 \times 8{,}760 \times 0.02$ = Rs. 78,840

For the fixed kVA charge, we have

Annual charge at unit p.f. = Rs. 60×1500 = Rs. 90,000

Annual charge at of 0.6 p.f. = Rs. $60 \times (1500/0.6)$ = Rs. 150,000

Total charges at unity p.f. = Rs. 90,000 + 78,840 = Rs. 168,840

Total charges of 0.6 p.f. = Rs. 150,000 + 78,840 = Rs. 228,840

It is seen that annual saving for unity p.f. is Rs. 60,000. Of course, this may not represent the net total saving when p.f. correction has been done either by phase-advancing equipment or static capacitors because the cost and maintenance of p.f. correcting apparatus has still to be taken into account. The cost will be by way of interest on capital required to install the p.f. improvement apparatus plus depreciation, maintenance expenses etc.

Consider the undergiven case. To illustrate how two-part tariff operates for different cases, suppose that rate of charging power to a large factory is Rs. 90 per annum per kVA maximum demand and 2 paise per kWh consumed. Also, suppose that the factory has 450 kW maximum load demand and an annual consumption of 10,000,000 kWh.

(*i*) If load p.f. were 0.9, then 450 kW max. demand is equal to 450/0.9 = 500 kVA max. demand.

(*ii*) If load p.f. were 0.7, then 450 kW max. demand is equal to 450/0.7 = 643 kVA max. demand.

Cost on kWh consumed will be the same for both power factors, it is only the maximum demand charge which will vary with the power factor.

At 0.7, the annual fixed charge for 643 kVA at Rs. 90 is Rs. 90×643 = Rs. 57,870

At 0.9, the annual fixed charge for 500 kVA = Rs. 90×500 = Rs. 45,000

$\therefore$ extra charge = Rs. 12,870

Now, let us assume that phase advancing equipment required to improve the power factor from 0.7 to 0.9 costs Rs. 50,000. Then, taking combined interest and depreciation at 12%, the annual charge will be Rs. $50{,}000 \times 0.12$ = Rs. 6,000.

Hence, the net annual saving comes to Rs. 12,870 – Rs. 6,000 = Rs. 6,870.

If, suppose, the p.f. correcting plant had cost Rs. 107,250, then interest and depreciation allowance on it would be = Rs. 107,250 × 0.12 = Rs. 12,870. In that case, there would be no point in installing such an apparatus because there would be no net saving.

It can be seen from the above example that higher the desired power factor improvement *i.e.* greater the kVAR reduction, the more costly the p.f. improvement plant and hence greater the charge on interest on capital outlay and depreciation. A point is reached, in practice, when any further improvement in power factor costs more than the saving in the power bill. Hence, it is necessary for the consumer to find out the value of power factor at which his net saving will be maximum. The value can be found if ***(i)*** annual charge per kVA maximum demand and ***(ii)*** the cost per kVA rating of phase advancing equipment are known.

50.26. Economical Limit of Power Factor Correction

Suppose a consumer is charged at Rs. A per kVA maximum demand plus a flat rate per kWh. Further, suppose that he is taking power of P kW at a power factor of $\cos\phi_1$. As shown in Fig. 50.14, his kVA_1 is $P/\cos\phi_1$ and his kVAR_1 is $P\tan\phi_1$. Suppose by installing static capacitors or synchronous capacitors, he improves his power factor to $\cos\phi_2$ (his power consumption P remaining the same). In that case, his kVA_2 is $P/\cos\phi_2$ and kVAR_2 is $P\tan\phi_2$.

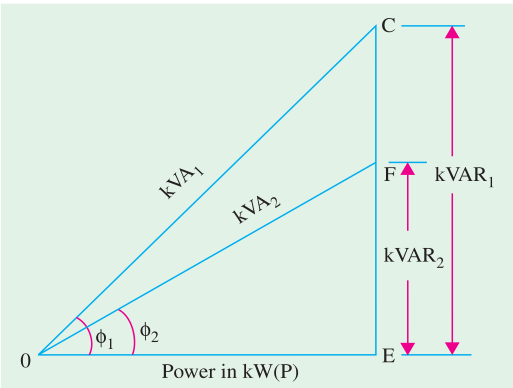

Fig. 50.14

Reduction in his kVA maximum demand is = $(\text{kVA}_1 - \text{kVA}_2) = (P/\cos\phi_1 - P/\cos\phi_2)$. Since charge is Rs. A per kVA maximum demand, his annual saving on this account is

$$= A\,(P/\cos\phi_1 - P/\cos\phi_2).$$

His kVAR is reduced from kVAR_1 to kVAR_2, the difference $(\text{kVAR}_1 - \text{kVAR}_2) = (P\tan\phi_1 - P\tan\phi_2)$ being neutralized by the leading kVAR supplied by the phase advancer.

$\therefore$ Leading kVAR supplied by phase advancer is $= (P\tan\phi_1 - P\tan\phi_2)$.

If the cost per kVAR of advancing plant is Rs. B and the rate of interest and depreciation is p percent per year, then its cost per annum is

$$= \frac{BP}{100}(P\tan\phi_1 - P\tan\phi_2)$$

$$= C\,(P\tan\phi_1 - P\tan\phi_2) \text{ where } C = BP/100$$

$\therefore$ Net annual saving $S = A\,(P/\cos\phi_1 - P/\cos\phi_2) - C(P\tan\phi_1 - P\tan\phi_2)$

This net saving is maximum when $dS/d\phi_2 = 0$

$$\therefore \quad \frac{dS}{d\phi_2} = \frac{d}{d\phi_2}[A\,(P/\cos\phi_1 - P/\cos\phi_2) - C\,(P\tan\phi_1 - P\tan\phi_2)]$$

$$= -AP\sec\phi_2\tan\phi_2 + CP\sec^2\phi_2$$

For maximum saving, $dS/d\phi_2 = 0$

$$\therefore \quad -AP\sec\phi_2\tan\phi_2 + CP\sec^2\phi_2 = 0 \quad \text{or} \quad \sin\phi_2 = C/A = BP/A$$

From this expression, ϕ_2 and hence $\cos\phi_2$ can be found. It is interesting to note that the most economical angle of lag ϕ_2 is independent of the original value ϕ_1.

Obviously, most economical p.f. is

$$\cos\phi_2 = \sqrt{1-\sin^2\phi_2} = \sqrt{1-(C/A)^2} = \sqrt{1-(BP/100\,A)^2}$$

Example 50.61. *A 3-phase, 50-Hz, 3,000-V motor develops 600 h.p. (447.6 kW), the power factor being 0.75 lagging and the efficiency 0.93. A bank of capacitors is connected in delta across the supply terminals and power factor raised to 0.95 lagging. Each of the capacitance units is built of five similar 600-V capacitors. Determine capacitance of each capacitor.* **(London Univ.)**

Solution. Power input $P = 447{,}600/0.93 = 480{,}000$ W

$\cos\phi_1 = 0.75\,;\ \phi_1 = \cos^{-1}(0.75) = 41°24',\ \cos\phi_2 = \cos^{-1}(0.95) = 18°12'.$

$\therefore\quad \tan\phi_1 = \tan 41°24' = 0.8816,\ \tan\phi_2 = \tan 18°12' = 0.3288$

As shown in Fig. 50.14, leading VAR supplied by capacitor bank is

$$= P(\tan\phi_1 - \tan\phi_2) = 480{,}000\,(0.8816 - 0.3288) = 48 \times 5{,}528$$

Leading VAR supplied by each of the three sets $= 48 \times 5{,}528/3 = 16 \times 5{,}528$ ...(*i*)

Phase current of capacitor is $I_{CP} = \dfrac{V}{X_C} = \dfrac{3{,}000}{X_C} = \dfrac{3{,}000}{1/\omega C} = 3{,}000 \times 314\,C$

where C is the total capacitance in each phase.

$\therefore\quad$ VAR of each phase $= VI_{cap} = 3{,}000 \times 3{,}000 \times 314\,C$ ...(*ii*)

Equating (*i*) and (*ii*) we get, $3{,}000 \times 3{,}000 \times 314\,C = 16 \times 5{,}528$ $\quad\therefore\quad C = 31.22\ \mu$F

Since it is the combined capacitance of five equal capacitors joined in series, the capacitance of each $= 5 \times 31.22 =$ **156 μF**.

Example 50.62. *A synchronous motor having a power consumption of 50 kW is connected in parallel with a load of 200 kW having a lagging power factor of 0.8. If the combined load has a p.f. of 0.9, what is the value of leading reactive kVA supplied by the motor and at what p.f. is it working?* **(I.E.E. London)**

Solution. Let ϕ_1 = p.f. angle of motor ; ϕ_2 = p.f. angle of load

ϕ_t = combined p.f. angle ; $\phi_2 = \cos^{-1}(0.8) = 36°52'$

$\tan\phi_2 = \tan 36°\,52' = 0.75\,;\ \phi_t = \cos^{-1}(0.9) = 25°51'$

$\tan\phi_t = \tan 25°51' = 0.4854$

Combined power $P = 200 + 50 = 250$ kW

Total kVAR $= P\tan\phi_t = 250 \times 0.4854 = 121.1$

Load kVAR $= 200 \times \tan\phi_2 = 200 \times 0.75 = 150$

$\therefore$ leading kVAR supplied by synchronous motor = 150 – 121.1 = **28.9**

$\tan\phi_1 = 28.9/50\,;\ \phi_1 = 30.1°,\quad \cos\phi_1 =$ **0.86 (lead)**

Example 50.63. *A generating station supplies power to the following, lighting load 100-kW ; an induction motor 400 h.p. (298.4 kW), power factor 0.8, efficiency, 0.92 ; a rotary converter giving 100 A at 500 V at an efficiency of 0.94. What must be the power factor of the rotary converter in order that the power factor of the supply station may be unity.*

Solution. Since, lighting load has no kVAR (unity p.f. assumed), the lagging kVAR of induction motor are neutralized by the leading kVAR of rotary converter only,

(*i*) Motor input = 298.4/0.92 = 324.4 kW

motor p.f. angle $\phi_m = \cos^{-1}(0.8) = 36°52' \quad\therefore\quad \tan\phi_m = \tan 36°52' = 0.75$

$\therefore$ lagging motor kVAR $= 324.4 \times \tan\phi_m = 324.4 \times 0.75 = 243.3$

$\therefore$ leading kVAR to be supplied by rotary converter = 243.3

Rotary converter intake $= 500 \times 100/0.94 \times 1000 = 53.2$ kW

$$\tan\phi = \frac{\text{kVAR}}{\text{kW}} = \frac{243.3}{53.2} = 4.573$$

$\therefore\quad \phi = 77°40'$ or $\cos\phi =$ **0.214 (leading)**

Example 50.64. *A factory has an average annual demand of 50 kW and an annual load factor of 0.5. The power factor is 0.75 lagging. The tariff is Rs. 100 per kVA maximum demand per annum plus five paise per kWh. If loss-free capacitors costing Rs. 600 per kVAR are to be utilized, find the value of the power factor at which maximum saving will result. The interest and depreciation together amount to ten per cent. Also, determine the saving affected by improving the power factor to this value.* **(Electrical Technology ; M.S. Univ. Baroda)**

Solution. The most economical power factor angle is given by $\sin\phi = C/A$

C = 10% of Rs. 600 = Rs. 60, A = Rs. 100 $\quad\therefore\ \sin\phi = 60/100 = 0.6$

$\phi = \sin^{-1}(0.6) = 36°52'$; New p.f. = $\cos 36°52'$ = **0.8**

The net annual saving due to improving in power factor can be found as follows :

Max. demand = 50/0.5 = 100 kW

At load p.f. of 0.75, maximum demand of 100 kW represents 100/0.75 = 400/3 kVA maximum demand.

At load p.f. of 0.8, 100 kW represent 100/0.8=125 kVA maximum demand.

Max. kVA demand charge

At 0.75 p.f. = Rs. 100 × 400/3 = Rs. 13,333.3

At 0.8 p.f. = Rs. 100 × 125 = Rs. 12,500

Annual saving = Rs. (13,333.3 – 12,500) = **Rs. 838.3**

Example 50.65. *For increasing the kW capacity of a plant working at 0.7 lag p.f. the necessary increase of power can be obtained by raising the p.f. to 0.85 or by installing additional plant. What is the maximum cost per kVA of p.f. correction apparatus to make its use more economical than additional plant at Rs. 500 kVA ?* **(Gen. Protect. and Switchgear, Madras Univ.)**

Solution. Let kVA_1 be the initial capacity of the plant and kVA_2 its increased capacity with extra or additional plant. As seen from Fig. 50.15.

$KVA_1 \cos\phi_2 = kVA_2 \cos\phi_1$

$\therefore\quad kVA_2 = kVA_1 \times \cos\phi_2/\cos\phi_1$

$= kVA_1 \times 0.85/0.7 = (17/14)\ kVA_1$

kVA of the additional plant $= kVA_2 - kVA_1 = (17/14)\ kVA_1 - kVA_1 = (3/14)\ kVA_1$

Capital cost of additional plant = Rs. $(500 \times 3/14)\ kVA_1$ = Rs. $(750/7)\ kVA_1$

Now, $\cos\phi_1 = 0.7,\ \phi_1 = 45°34'$; $\sin\phi_1 = 0.714$

$\cos\phi_2 = 0.85,\ \phi_2 = 31.8°$; $\sin\phi_2 = 0.527$

$\therefore$ kVAR supplied by p.f. correction apparatus

$$= (kVA_2 \sin\phi_1 - kVA1 \sin\phi_s) = \left(\frac{7}{14} \times 0.714\ kVA_1 - kVA_1 \times 0.527\right) = 0.34\ kVA_1$$

If the capital cost of the p.f. correction apparatus be Rs. x per kVAR, the total cost is = $0.34x\ kVA_1$.

If the annual cost of the additional plant is to be the same as that of the p.f. correction apparatus (assuming same annual interest and depreciation), then

$(750/7)\ kVA_1 = 0.34\,x\ kVA_1\ x$ = Rs. 315.2

If p.f. correction apparatus is loss-free, then its kVAR = kVA. Hence, the maximum cost per kVA of p.f. correction apparatus that can be paid is **Rs. 315.2.**

Example 50.66. *A consumer taking a steady load of 160 kW at a p.f. of 0.8 lag is charged at Rs. 80 per annum per kVA of maximum demand plus 5 paise per kWh consumed. Calculate the value to which he should improve the p.f. in order to affect the maximum saving if the leading kVA cost Rs.100 per kVA and interest and depreciation be at 12% per annum.*

Calculate also the saving.

(Electrical Technology ; Gujarat Univ.)

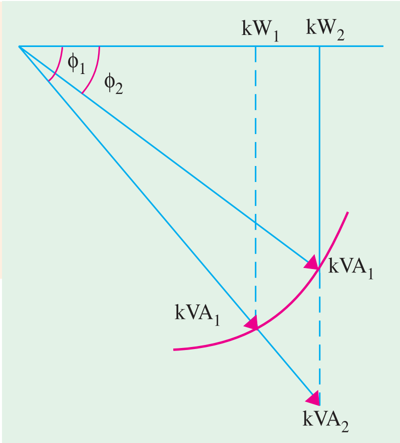

Fig. 50.15

Solution. With reference to Art. 50.26, $C = BP/100 = 12 \times 100/100 = 12$, $A = 80$

Most economical p.f. $\cos\phi_2 = \sqrt{1-(BP/100A)^2}$

$= \sqrt{1-(12/80)^2}$

$=$ **0.9887 (lag)**

Since load remains steady, maximum load is also 160 kW.

At load p.f. of 0.8, maximum load of 160 kW represents 160/0.8 = 200 kVA.

Similarly, at 0.9887 p.f., it represents 160/0.9887 = 162 kVA.

Max. kVA demand charge at 0.8 p.f. = Rs. 80 × 200 = Rs. 16000

Max. kVA demand charge at 0.9887 p.f. = Rs. 80 × 162 = Rs. 12,960

$\therefore$ annual saving = Rs. (16,000 – 12,960) = **Rs. 3,040.**

Example 50.67. *A consumer takes a steady load of 1500 kW at a p.f. of 0.71 lagging and pays Rs. 50 per annum per kVA of maximum demand. Phase advancing plant costs Rs. 80 per kVA. Determine the capacity of the phase advancing plant required for minimum overall annual expenditure. Interest and depreciation total 10%. What will be the value of the new power factor of the supply?*

(Electrical Power-III, Bangalore Univ.)

Solution. Minimum overall annual expenditure corresponds to most economical power factor given by $\cos\phi_2 = \sqrt{1-(BP/100A)^2}$ as shown in Art. 50.26.

Now, $B = 80,\ P = 10,\ A = 50 \quad \cos\phi_2 = \sqrt{1-(80\times 10/100\times 50)^2} =$ **0.987**

Now, $\cos\phi_1 = 0.71,\ \phi_1 = 44.7°$

$\tan\phi_1 = 0.9896$; $\cos\phi_2 = 0.987$. $\phi_2 = 9.2°$, $\tan\phi_2 = 0.162$

kVA supplied by phase advancing plant = $P(\tan\phi_1 - \tan\phi_2)$ = 1500 (0.9896 – 0.162) = **1240.**

Example 50.68. *A factory takes a load of 200 kW at 0.85 p.f. (lagging) for 2,500 hours per annum and buys energy on tariff of Rs. 150 per kVA plus 6 paise per kWh consumed. If the power factor is improved to 0.9 lagging by means of capacitors costing Rs. 525 per kVA and having a power loss of 100 W per kVA, calculate the annual saving affected by their use. Allow 8% per annum for interest and depreciation on the capacitors.*

Solution. Factory load = 200 kW

$\cos\phi_1 = 0.85$ (lagging)

$\therefore \quad \phi_1 = \cos^{-1}(0.85) = 31°48' \quad \tan\phi_1 = 0.62$

$\therefore$ lagging kVAR of factory load = 0.62 × 200 = 124

(or first find total kVA = 200/0.85 and then multiply it by $\sin\phi_1$ to get kVAR)

Suppose, x = capacitor's kVAR (leading), Total kVAR = $124 - x$

Because loss per kVA is 100 W *i.e.*, 1/10 kW per kVA

$\therefore$ capacitor's loss $= x/10$ kW

$\therefore$ Total kW $= 200 + (x/10)$

Now, overall p.f. $= \cos\phi = 0.9 \quad \therefore \quad \phi = 25°51' \quad \therefore \tan 25'51' = 0.4845$

Now $$\tan\phi = \frac{\text{total kVAR}}{\text{total kW}} = \frac{124 - x}{200 + (x/10)} \quad \therefore \quad x = 25.9 \text{ kVAR}$$

Cost per annum before improvement

kVA = 200/0.85 = 235.3 Units consumed per annum = $200 \times 2500 = 5 \times 10^5$ kWh

$\therefore$ annual cost = Rs. $(235.3 \times 150 + 5 \times 105 \times 6/100)$ = Rs. 65,295

Cost per annum after improvement

kVA = 200/0.9 = 222.2, Cost = Rs. 222.2×150 = Rs. 33,330

Cost of energy = Rs. $5 \times 10^5 \times 6/100$ = Rs. 30,000 (as before)

Cost of losses occuring in capacitors = Rs. $\dfrac{25.9 \times 100 \times 2500 \times 6}{1000 \times 100}$ = Rs. 389

Annual interest and depreciation cost on capacitors =Rs. $525 \times 25.9 \times 8/100$ = Rs. 1088

$\therefore$ total annual cost is = Rs. (33,330 + 30,000 + 389 + 1088) = Rs. 64,807

$\therefore$ annual saving = Rs. (65,295 – 64,807) = **Rs. 488.**

Example 50.69. *A 30 h.p. (22.38 kW) induction motor is supplied with energy on a two-part tariff of Rs. 60 per kVA of maximum demand per annum plus 5 paise per unit. Motor (A) has an efficiency of 89% and a power factor of 0.83. Motor (B) with an efficiency of 90% and a p.f. of 0.91 costs Rs. 160 more. With motor (A) the p.f. would be raised to 0.91 (lagging) by installing capacitors at a cost Rs. 50 per kVA.*

If the service required from the motor is equivalent to 2,280 hr. per annum at full load, compare the annual charges in the two cases. Assume interest and depreciation charges to be % per annum for the motor and 8% per annum for the capacitors.

Solution. For Motor A

Full-load motor input $= 22.38 \times 10^3/0.89 = 25.14$ kW ; kVA = 25.14/0.83 = 30.29

Now, when p.f. is raised to 0.91, then kVA demand of motor = 25.14/0.91 = 27.6

Annual cost of energy supplied to motor = $27.6 \times 60 + (25.14 \times 2{,}280 \times 5/100)$

= 1,656 + 2,866 = Rs. 4,522

By using capacitors, the phase angle of load is decreased from ϕ_1 ($\cos\phi_1 = 0.83$) to ϕ_2 ($\cos\phi_2 = 0.91$).

kVAR necessary for this improvement = load kW $(\tan\phi_1 - \tan\phi_2)$

= 25.14 (0.672 – 0.4557) = 5.439

Annual charges on capacitors = Rs. $50 \times 5.439 \times 8/100$ = Rs. 21.8

Total charges per annum including interest and depreciation on motor

= Rs. (4,522 + 21.8) = Rs. 4,544.

For Motor B

F.L. motor input = 22.38/0.9 = 24.86 kW ; kVA = 24.86/0.91 = 27.32

Annual cost of energy supplied to motor = $27.32 \times 60 + (24.86 \times 2280 \times 5/100)$ = Rs. 4,473

If we do not include the interest and depreciation on the motor itself, then *B* is cheaper than *A* by Rs. (4544 – 4473) i.e., by Rs. 71. But *B* cost Rs. 160 more than *A*. Hence, additional annual charges on *B* are % of Rs. 160 *i.e.*, Rs. 20. Hence, B is cheaper than *A* by Rs. (71 – 20) = Rs. 51 per annum.

Example 50.70. *The motor of a 22.5 kW condensate pump has been burnt beyond economical repairs. Two alternatives have been proposed to replace it by*

Motor A. Cost = Rs. 6000 ; η at full-load=90% ; at half-load = 86%.

Motor B. Cost = Rs. 4000 ; η at full-load=85% ; at half-load= 82%.

The life of each motor is 20 years and its salvage value is 10% of the initial cost. The rate of interest is 5% annually. The motor operates at full-load for 25% of the time and at half-load for the remaining period. The annual maintenance cost of motor A is Rs. 420 and that of motor B is Rs. 240. The energy rate is 10 paise per kWh.

Which motor will you recommend ? **(Power Plant Engg., AMIE)**

Solution. Motor A

Annual interest on capital cost = Rs. 6000 × 5/100 = Rs. 300

$$\text{Annual depreciation charges} = \frac{\text{original cost-salvage value}}{\text{life of motor in years}}$$

$$= \text{Rs. } \frac{6000 - (6000 \times 10/100)}{20} = \text{Rs. } 270$$

Annual maintenance cost = Rs. 420

$$\text{Energy input per annum} = \frac{22.5 \times 0.25 \times 8760}{0.9} + \frac{11.25 \times 0.75 \times 8760}{0.86} = 1,40,695 \text{ kWh}$$

Annual energy cost = Rs. 0.1 × 1,40,695 = Rs. 14,069

Total annual cost = Rs. (300 + 270 + 420 + 14,069) = Rs. 15,059

Motor B

Annual interest on capital cost = Rs. 4000 × 5/100 = Rs. 200

$$\text{Annual depreciation charges} = \text{Rs. } \frac{4000 - (4000 \times 10/100)}{20} = \text{Rs. } 180$$

Annual maintenance charges = Rs. 240

$$\text{Energy input per annum} = \frac{22.5 \times 0.25 \times 8760}{0.85} + \frac{11.25 \times 0.75 \times 8760}{0.82} = 1,48,108 \text{ kWh}$$

Annual energy cost = Rs. 0.1 × 1,48,108 = Rs. 14,811

Total annual cost = Rs. (200 + 180 + 240 + 14,811) = Rs. 15,431

Since the annual cost of Motor *B* is Rs. (15431 – 15059) = Rs. 372 more than that of Motor *A*, hence **motor A would be recommended.**

Example 50.71. *An industrial load takes 10^6 kWh a year, the power factor being 0.707 lagging. The maximum demand is 500 kVA. The tariff is Rs. 75 per annum per kVA maximum demand plus 3 paise per unit. Calculate the yearly cost of supply and find the annual saving in cost by installing phase advancing plant costing Rs. 45 per kVA which raises the plant power factor from 0.707 to 0.9 lagging. Allow 10% per annum on the cost of the phase advancing plant to cover all additional costs.*

Solution. Max. demand charge per annum = Rs. 500 × 75 = Rs. 37,500

Annual energy charges = Rs. $10^6 \times 3/100$ = Rs. 30,000

∴ yearly cost of supply = Rs. (37,500 + 30,000) = **Rs. 67,500**

Now, when p.f. is increased from 0.707 to 0.9, then maximum kVA demand is reduced to 500 × 0.707/0.9 = 392.8.

Now, annual cost of the supply = Rs. 75 × 392.8 + Rs. 30,000

= 29,460 + 30,000 = Rs. 59,460

kVAR to be supplied by the phase-advancing plant for improving the p.f. from 0.707 to 0.9 = kW demand ($\tan\phi_1 - \tan\phi_2$) where $\phi_1 = \cos^{-1}(0.707)$ and $\phi_2 = \cos^{-1}(0.9)$.

∴ kVAR = 500 × 0.707 × 0.516 = 182.4

Annual cost by way of interest and depreciation etc. on the phase advancing plant

= Rs. 182.4 × 45 × 0.1 = Rs. 821

The annual saving would be due to reduction in the maximum demand charge only because units consumed remain the same throughout.

$\therefore$ yearly saving = initial charges for kVA – (new charges for kVA + annual cost of phase advancer) = 67,500 – (59,460 + 821) = **Rs. 7,219.**

Example 50.72. *It is necessary to choose a transformer to supply a load which varies over 24 hour period in the manner given below :*

500 kVA for 4 hours, 1000 kVA for 6 hours, 1500 kVA for 12 hours and 2000 kVA for the rest of the period.

Two transformers each rated at 1500 kVA have been quoted. Transformer I has iron loss of 2.7 kW and full-load copper loss of 8.1 kW while transformer II has an iron loss and full-load copper loss of 5.4 kW each.

(i) Calculate the annual cost of supplying losses for each transformer if electrical energy costs 10 paise per kWh.

(ii) Determine which transformer should be chosen if the capital cost of the transformer I is Rs. 1000 more than that of the transformer II and annual charges of interest and depreciation are 10%.

(iii) What difference in capital cost will reverse the decision made in (ii) above ?

(Utilisation of Elect. Power AMIE)

Solution. (i) Transformer No. 1

Iron loss/day $= 2.7 \times 24 = 64.8$ kW

Cu loss/day $= 8.1\,(500/1500)^2 \times 4 + 8.1 \times (1000/1500)^2 \times 6 + 8.1 \times (1500/1500)^2 \times 12 + 8.1 \times (2000/1500)^2 \times 2 = 151.2$ kWh

Annual energy loss $= 365\,(64.8 + 151.2) = 78,840$ kWh

Annual cost of both losses = Rs. $0.1 \times 78,840$ = **Rs. 7884**

Transformer No. 2

Iron loss/day $= 5.4 \times 24 = 129.6$ kWh

Cu loss/day $= 5.4\,(500/1500)^2 \times 4 + 5.4\,(1000/1500)^2 \times 6 + 5.4\,(1500/1500)^2 \times 12 + 5.4\,(2000/1500)^2 \times 2 = 100.8$ kWh

Annual energy loss $= 365\,(129.6 + 100.8) = 84,096$ kWh

Annual cost of both losses = Rs. $0.1 \times 84,096$ = **Rs. 8410**

(ii) Since the cost of transformer No. 1 is Rs. 1000 more than that of transformer No. 2, extra annual charge in case transformer No. 1 is selected = Rs. $1000 \times 10/100$ = Rs. 100

Annual saving in energy cost due to losses = Rs. (8410 – 7884) = Rs. 526

Since the annual saving in the energy cost of supplying losses is much than the extra annual charges due to the higher cost of the transformer No. 1, therefore transformer No. 1 will be selected.

(iii) Transformer No. 2 would be chosen in case annual charges due to extra capital cost of transformer No. 1 exceed the annual saving on the energy cost of supplying losses.

If x is the extra capital cost of transformer No. 1, then transformer No. 2 could be selected if

$x \times 10/100 > 526$ or $x >$ **Rs. 5260**

Example 50.73. *Three-phase 50-Hz power is supplied to a mill, the voltage being stepped down to 460-V before use.*

The monthly power rate is 7.50 per kVA. It is found that the average power factor is 0.745 while the monthly demand is 611 kVA.

To improve power factor, 210 kVA capacitors are installed in which there is negligible power loss. The installed cost of the equipment is Rs. 11,600 and fixed charges are estimated at 15% per year. What is the yearly saving introduced by the capacitors.

Solution. Monthly demand $= 611$ kVA ; p.f. $= 0.745$

$\phi = \cos^{-1}(0.745) = 41°5'$ $\therefore$ $\sin\phi = \sin 41°5' = 0.667$

$\therefore$ kW component $=$ kVA $\cos\phi = 611 \times 0.745 = 455$

kVAR component $=$ kVA $\sin\phi = 611 \times 0.667 = 408$ (lagging)

leading kVAR supplied by capacitor $= 210$

Hence, kVAR after p.f. improvement $= 408 - 210 = 198$

$\therefore$ kVA after p.f. improvement $= \sqrt{455^2 + 198^2} = 491$

Reduction in kVA $= 611 - 491 = 120$

Hence, monthly saving on kVA charge = Rs. 120×7.5 = Rs. 900

Yearly saving on kVA charge = Rs. 12×900 = Rs. 10,800

Capital cost on capacitors = Rs. 11,600

Fixed charge per annum = Rs. $0.15 \times 11{,}600$ = Rs, 1,740

Hence, net saving per annum = Rs. (10,800 – 1,740) = **Rs. 9,060**

Example 50.74. *A supply undertaking is offering the following two tariffs to prospective customers :*

Tariff A : Lighting : 20 paise per unit; domestic power: 5 paise per unit, meter rent: 30 paise per meter per month.

Tariff B : 12 per cent on the rateable value of the customer's premises plus 3 paise per unit for all purposes.

If the annual rateable value of the customer's premises is Rs. 2,500 and his normal consumption for lighting per month is 40 units, determine what amount of domestic power consumption will make both the tariffs equally advantageous.

Solution. Let x = minimum number of power units consumed per month.

Tariff A

Total cost per month = meter rent + lighting charges + power charges

$= (2 \times 30) + (40 \times 20) + 5x = 860 + 5x$ paise

Tariff B

Total cost per month = 12% of 2500 + energy charges for all purposes

$= 0.12 \times 2500 \times 100/12 + 3(40 + x)$

$\therefore$ $860 + 5x = 2500 + 120 + 3x$ $\therefore$ $2x = 1760$ $\therefore$ $x =$ **880 units**

Example 50.75. *Transformers and low-tension motors of a certain size can be purchased at Rs. 12 per kVA of full output and Rs. 24 per kW output respectively. If their respective efficiencies are 98% and 90%, what price per kW output could be paid for high-tension motors of the same size but of average efficiency only 89%? Assume an annual load factor of 30%, the cost of energy per unit as 7 paise and interest and depreciation at the rate of 8% for low-tension motors.*

Solution. Let S = the price of h.t. motors per output kW in rupees.

Low-tension Motors with Transformers

(*a*) Interest and depreciation on the motor input kW = Rs. $\dfrac{24 \times 0.08}{0.9}$ = Rs. 2.13

(*b*) Let ∞ a p.f.or the mers and calculate their standing charges on the basis of their input expressed in kW.

$\therefore$ interest and depreciation on transformers per input kW = Rs. $\dfrac{12 \times 0.9}{0.99} \times 0.08$ = Rs. 0.88

(*c*) Running charges $= \dfrac{(8,760 \times 0.3) \times 7}{(0.98 \times 0.9) \times 100}$ = Rs. 208.6

Hence, total cost of transformers and low-tension motors per input kW

= 2.13 + 0.88 + 208.6 = Rs. 211.6

High Tension Motors

Standing charges/output kW = Rs. $\dfrac{S}{.89} \times .12$ = Rs. $\dfrac{12S}{89}$

Running cost $= \dfrac{(1 \times 8,760 \times 0.3) \times 7}{0.89 \times 100}$ = Rs. 206.7

Total cost of h.t. motors = (12 S/89) + 206.7.

$\therefore$ 12 S/89 + 206.7 = 211.6 $\therefore$ S = **Rs. 36.34 kW output.**

Example 50.76. *An industrial load can be supplied on the following alternative tariffs* **(a)** *high-voltage supply at Rs. 45 per kVA per annum plus 1.5 paise per kWh or* **(b)** *low-voltage supply at Rs. 50 per annum plus 1.8 paise per kWh. Transformers and switchgear etc. for the H.V. supply cost Rs. 35 per kVA, the full-load transformer losses being 2%. The fixed charges on the capital cost of the high-voltage plant are 25% and the installation works at full-load. If there are 50 working weeks in a year, find the number of working hours per week above which the H.V. supply is cheaper.*

Solution. Let x be the number or working hours per week above which H.V. supply is cheaper than the L.V. supply. Also, suppose that the load = 100 kW.

Rating of transformers and switchgear = 100/0.98=10,000/98 kW, because transformer losses are 2%.

Cost of switchgear and transformer = (10,000/98) × 35 = Rs. 3,571.5

Annual fixed charges = 3,571.5 × 0.25 = Rs. 892.9

Annual energy consumption = 100 × x × 50 = 5,000 x kWh

On the H.V., the total kWh which will have to be paid for is 5,000 x/0.98.

(*a*) H.V. Supply

Total annual cost = 45 × kVA + energy charges + charge on H.V. plant

$= 45 \times \dfrac{100}{0.98} + \dfrac{1.5 \times 5,000\, x}{100 \times 0.98}$ + Rs. 892.9

= Rs. 4,592 + Rs. 76.54 x + Rs. 892.9 = **Rs. (5,485 + 76.54 x)**

(*b*) L.V. Supply

Total annual cost = Rs. 50 × kVA + energy charges = 50 × 100 + $\dfrac{1.8 \times 5,000\, x}{100}$

= **Rs. (5,000 + 90 x)**

If the two annual costs are to be equal, then

5,485 + 76.54 x = 5,000 + 90 x

$\therefore$ 13.46 x = 485 or x = **36 hr.**

Hence, above 36 hours per week, the H.V. supply would be cheaper.

Example 50.77. *For a particular drive in a factory requiring 10 h.p. (7.46 kW) motors, following tenders have been received. Which one will you select ?*

	cost	*efficiency*
Motor X	*Rs. 1,150*	*86%*
Motor Y	*Rs. 1,000*	*85%*

Electrical tariff is Rs. 50 per kW + 5 paise per kWh. Assume interest and depreciation as 10% and that the factory works for 10 hours a day for 300 days a year.

(Electrical Engineering, Bombay Univ.)

Solution. (*i*) Motor X

F.L. power input	= 7.46/0.86	= 8.674 kW
Units consumed/year	= 8760 × 8.674	= 75,980 kWh
kW charges	= Rs. 50 × 8.674	= Rs. 433.7
kWh charges	= Rs. 5 × 75,980/100	= Rs. 3,799
Fixed charges	= Rs. 0.1 × 1150	= Rs. 115
Total annual charges	= (3,799 + 433.7 + 115)	= **Rs. 4,347.7**

(*ii*) Motor Y

F.L. power input	= 7.46/0.85	= 8.776 kW
units consumed/year	= 8760 × 8.766	= 76,880 kWh
kW charges	= Rs. 50 × 8.766	= Rs. 438.8
kWh charges	= Rs. 5 × 76,880/100	= Rs. 3,844
Fixed charges	= Rs. 0.1 × 1000	= Rs. 100
Total annual charges	= (438.8 + 3,844 + 100)	= Rs. 4,382.8

Obviously, motor *X* is cheaper by Rs. (4,382.8 × 4,347.7) = **Rs. 35.1**

Example 50.78. *A 200-h p. (149.2 kW) motor is required to operate at full-load for 1500 hr, at half-load for 3000 hr per year and to be shut down for the remainder of the time. Two motors are available.*

Motor A : efficiency at full load = 90% ; at half-load = 88%

Motor B : efficiency at full load = 90% ; at half-load = 89%

The unit of energy is 5 paise/kWh and interest and depreciation may be taken as 12 per cent per year. If motor A cost Rs. 9,000 ; what is the maximum price which could economically be paid for motor B ? **(Electrical Power-II, Bombay Univ.)**

Solution. (*a*) Motor A

F.L. power input = 149.2/0.9 = 165.8 kW

Half-load power input = (149.2/2)/0.88 = 84.7 kW

Total energy consumed per year = (1500 × 165.8) + (3000 × 84.7) = 502,800 kWh

Cost of energy = Rs. 5 × 502,800/100 = Rs. 25,140

Interest and depreciation on motor = Rs. 0.12 × 9000 = Rs. 1,080

Total charge = Rs. 25,140 + Rs. 1,080 = **Rs. 26,220**

(*b*) Motor B

F.L. input = 149.2/0.9 = 165.8 kW : Half-load input = (149.2/2)/0.89 = 83.8 kW

Total energy consumed per year = (1500 × 165.8) + (3000 × 83.8) = 497,100 kWh

Cost of energy consumed = Rs. 5 × 497,100/100 = Rs. 24,855

Let x be the maximum price which could be paid for motor B.

Annual interest and depreciation = Rs. $0.12\,x$

annual charges = Rs. (24,855 + 0.12x)

For the two charges to be equal = $24{,}855 + 0.12x = 26{,}220$; x = Rs. 11,375

Example 50.79. *Two tenders A and B for a 1000-kVA, 0.8 power factor transformer are : A, full-load efficiency = 98.5% and iron loss = 6 kW at rated voltage ; B, 98.8% and iron loss 4 kW but costs Rs. 1,500 more than A. The load cycle is 2000 hours per annum at full-load, 600 hours at half-load and 400 hours at 25 kVA. Annual charges for interest and depreciation are 12.5% of capital cost and energy costs 3 paise per kWh. Which tender is better and what would be the annual saving.*

Solution. Tender A Transformer full-load output = $1000 \times 0.8 = 800$ kW

Efficiency = 98.5% Input = 800/0.985

Total losses = (800/0.985) – 800 = 12.2 kW ; F.L. Cu losses = 12.2 – 6 = 6.2 kW

Total losses per year for a running period of 3000 hr. are—

(*i*) Iron loss = $3{,}000 \times 6 = 18{,}000$ kWh

(*ii*) F.L. Cu losses for 2000 hours = $2{,}000 \times 6.2 = 12{,}400$ kWh

(*iii*) Cu loss at half-load for 600 hours = $\frac{1}{4} \times 6.2 \times 600 = 930$ kWh

(*iv*) Cu loss at 25 kVA load for 400 hours = $\left(\frac{25}{1000}\right)^2 \times 6.2 \times 400 = 1.5$ kWh

Total energy loss per year = 18,000 + 12,400 + 930 + 1.5 = 31,332 kWh

Cost = Rs. $31{,}332 \times 3/100$ = Rs. 939.96

Tender B.

Total loss of transformer = (800/0.988) – 800 = 9.7 kW

Iron loss = 4 kW ; Full-load Cu loss = 9.7 – 4 = 5.7 kW

Total losses for 3,000 hours per year are as follows :

(*i*) Iron loss = $3{,}000 \times 4 = 12{,}000$ kWh

(*ii*) F.L. Cu loss for 2000 hours = $5.7 \times 2000 = 11{,}400$ kWh

(*iii*) Cu loss at half-load for 600 hours = $(5.7/4) \times 600 = 855$ kWh

(*iv*) Cu loss at 25 kVA for 400 hours = $(25/1000)^2 \times 5.7 \times 400 = 1.4$ kWh

Total yearly loss = 12,000 + 11,400 + 855 + 1.4 = 24,256 kWh

Cost = Rs. $24{,}256 \times 3/100$ = Rs. 727.68 ; Extra cost of B = Rs. 1500

Annual interest and depreciation etc. = 12.5% or 1500 = Rs. 187.50

$\therefore$ yearly cost of B = Rs. 727.68 + Rs. 187.50 = Rs. 915.18

Obviously, tender B is cheaper by Rs. (939.96 – 915.18) = **Rs. 24.78 annually.**

Example 50.80. *Transformer A has iron loss of 150 kWh and load loss of 140 kWh daily while the corresponding losses of transformer B are 75 kWh and 235 kWh. If annual charges are 12.5% of the capital costs and energy costs 5 paise per kWh, what should be the difference in the cost of the two transformers so as to make them equally economical ?*

Solution. Transformer A

Yearly loss = $365 \times (150 + 140) = 365 \times 290$ kWh

Transformer B

Yearly loss = $365 \times (75 + 235) = 365 \times 310$ kWh

Difference in yearly loss = 365 (310 – 290) = 7,300 kWh

Value of this loss = Rs. 7,300 × 5/100 = Rs. 365

Hence, transformer B has greater losses per year and so is costlier by Rs. 365. If it is to be equally advantageous, then let it cost Rs. x less than A. Annual interest and depreciation on it is Rs. 0.125 x.

$\therefore$ $0.125x = 365$

or x = Rs. 2,920

Hence, transformer B should cost Rs. 2,920 less than A.

Example 50.81. *Quotations received from three sources for transformers are :*

	Price	*No-load loss*	*Full-load loss*
A	*Rs. 41,000*	*16 kW*	*50 kW*
B	*Rs. 45,000*	*14 kW*	*45 kW*
C	*Rs. 38,000*	*19 kW*	*60 kW*

If the transformers are kept energized for the whole of day (24 hours), but will be on load for 12 hours per day, the remaining period on no-load, the electricity cost being 5 paise per kWh and fixed charges Rs. 125 per kW of loss per annum and if depreciation is 10% of the initial cost, which of the transformers would be most economical to purchase ?

Solution. Transformer A

F.L. loss = 50 kW

No-load losses = 16 kW (this represents iron loss)

$\therefore$ F.L. Cu loss = 50 – 16 = 34 kW

F.L. Cu loss for 12 hours* = 12 × 34 = 408 kWh

Iron losses for 24 hours = 24 × 16 = 384 kWh

Total loss per day = 408 + 384 = 792 kWh

Annual loss = 792 × 365 kWh

Cost of this loss = Rs. 792 × 365 × 5/100 = Rs. 14,454

Annual fixed charges = Rs. 125 × 50 = Rs. 6,250

Annual depreciation = Rs. 0.1 × 41,000 = Rs. 4,100

Total annual charges for transformer

A = Rs. 14,454 + Rs. 6,250 + Rs. 4,100 = Rs. 24,804

Transformer B

F.L. loss = 45 kW ; Iron or no-load loss = 14 kW

F.L. Cu loss = 45 – 14 = 31 kW; Iron loss/day = 24 × 14 = 336 kWh

F.L. Cu losss for 12 hours* = 12 × 31 = 372 kWh

Total loss/day = 708 kWh ; Annual loss = 708 × 365 kWh

Cost of this loss = 708 × 365 × 5/100 = Rs. 12,921

Fixed charges = Rs. 125 × 45 = Rs. 5,625

Annual depreciation = Rs. 0.1 × 45,000 = Rs. 4,500

Total annual charges for transformer B

= Rs. 12,921 + Rs. 5,625 + Rs. 4,500 = Rs. 23,046

Transformer C

F.L. loss = 60 kW ; Iron or N.L. loss = 19 kW

F.L. Cu loss = 60 – 19 = 41 kW

* Cu loss is proportional to $(\text{kVA})^2$

Iron loss for 24 hr. = $24 \times 19 = 456$ kWh
F.L. Cu loss for 12 hr = $12 \times 41 = 492$ kWh
Total losses per day = 948 kWh ; Annual losses = 948×365 kWh
Cost of these losses = Rs. $948 \times 365 \times 5/100$ = Rs. 17,301
Fixed charges = Rs. 125×60 = Rs. 7,500 ;
Annual depreciation = Rs. $0.1 \times 38{,}000$ = Rs. 3,800

Total annual charges on transformer C = Rs. 17,301 + Rs. 7,500 + Rs. 3,800 = Rs. 28,601

Obviously, transformer B is cheaper and should be purchased.

Example. 50.82. *A 37.3 kW induction motor has power factor 0.9 and efficiency 0.9 at full-load, power factor 0.6 and efficiency 0.7 at half-load. At no-load, the current is 25% of the full-load current and power factor 0.1. Capacitors are supplied to make the line power factor 0.8 at half-load. With these capacitors in circuit, find the line power factor at (i) full-load and (ii) no-load.*

(Utilisation of Elect. Power, AMIE)

Solution. Full-load motor input, $P_1 = 37.3/0.9 = 40.86$ kW

Lagging kVAR drawn by the motor at full-load, $\text{kVAR}_1 = P_1 \tan \phi_1 = 40.86 \tan (\cos^{-1} 0.9) = 19.79$.

Half-load motor input, $P_2 = (0.5 \times 37.3)/0.7 = 26.27$ kW

Lagging kVAR drawn by the motor at half-load,

$$\text{kVAR}_2 = P_2 \tan \phi_2 = 26.27 \tan (\cos^{-1} 0.6) = 35.02$$

Full-load current, $I_1 = 37.3 \times 10^3/\sqrt{3}\, V_L \times 0.9 \times 0.9 = 26,\, 212/V_L$

Current at no-load, $I_0 = 0.25\, I_1 = 0.25 \times 26{,}212/V_L = 6553/V_L$

Motor input at no-load, $P_0 = \sqrt{3} V_L I_0 \cos \phi_0 = \sqrt{3} \times 6553 \times V_L \times 0.1/V_L = 1135 \text{ W} = 1.135$ kW

Lagging kVAR drawn by the motor at no-load, $\text{kVAR}_0 = 1.135 \tan (\cos^{-1} 0.1) = 11.293$

Lagging kVAR drawn from the mains at half-load with capacitors,

$$\text{kVAR}_{2C} = 26.27 \tan (\cos^{-1} 0.8) = 19.7$$

kVAR supplied by capacitors, $\text{kVAR}_C = \text{kVAR}_2 - \text{kVAR}_{2C} = 35.02 - 19.7 = 15.32$

kVAR drawn from the main at full-load with capacitors $\text{kVAR}_{1C} = \text{kVAR}_K - \text{kVAR}_C = 19.79 - 15.32 = 4.47$.

(*i*) Line power factor at full-load = $\cos (\tan^{-1} \text{kVAR}_{1C}/P_1) = \cos (\tan^{-1} 4.47/40.86)$ = **0.994 lagging**

(*ii*) kVAR drawn from mains at no-load with capacitors = $11.293 - 15.32 = -4.027$

Line power factor at no-load = $\cos (\tan^{-1} - 4.027/1.135) = \cos (-74.26^\circ)$
= **0.271 leading.**

Tutorial Problem No. 50.3

1. A power plant is working at its maximum kVA capacity with a lagging p.f. of 0.7. It is now required to increase its kW capacity to meet the demand of additional load. This can be done by raising the p.f. to 0.85 by correction apparatus or by installing extra generating plant which costs Rs. 800 per kVA. Find the minimum cost per kVA of p.f. apparatus to make its use more economical than the additional generating plant. **[Rs. 502 per kVAR]**
2. An industrial load of 4 MW is supplied at 11 kV, the p.f. being 0.8 lag. A synchronous motor is required to meet additional load of 1500 h.p. (1,119 kW) and at the same time to raise the resultant p.f. to 0.95 lag. Find the kVA capacity of the motor and the p.f. at which it must operate. The efficiency of the motor is 80%. **[1875 kVA ; 0.7466 (lead)]**

3. A factory takes a load of 300 kW at 0.8 p.f. lag for 4800 hours per annum and buys energy at a tariff of Rs. 100 per kVA plus 3 p/kWh consumed. If p.f. is improved to 0.9 (lag) by means of capacitors costing Rs. 200 per kVA and having a power loss of 50 W per kVA, calculate the annual saving affected by their use. Allow 10% per annum for interest and depreciation of the capacitors.
[Rs. 693 per year]

4. The following two tenders are received in connection with the purchase of 1000-kVA transformer.
Transformer A : full-load efficiency = 98% ; core loss = 8 kW
Transformer B : full-load efficiency = 91.5% ; core loss = 6 kW
The service needed per year is 200 hours as 1000 kVA, 1000 hours at 500 kVA and 1000 hours at 200 kVA. If the transformer A is costing Rs. 25,000, estimate the maximum price that could be paid for transformer *B*. Take interest and depreciation at 10 per cent per annum and cost of energy 5 paise per kWh. Assume working at unity power factor. **[Max. Price for B = Rs. 21,425]**

5. Tenders *A* and *B* for a 1000-kVA, 0.8 p.f. transformer are :
A : full-load η = 98.3% ; core loss = 7 kW at rated voltage
B : full-load η = 98.7% ; core loss = 4 kW but costs Rs. 5000 more than *A*.
The service needed is 1800 hours per annum at 1000 kVA, 600 hours at 600 kVA, 400 hours at 25 kVA and remaining period shut down. Take the annual charges for interest and depreciation at 12.5 per cent of capital cost and energy costs at 8 paise/kWh ; which is the better tender and what will be annual saving ? **[Tender B is cheaper by Rs. 211]**

6. A 440-V, 50-Hz induction motor takes a line current of 45 A at a power factor of 0.8 (lagging). Three D-connected capacitors are installed to improve the power factor to 0.95 (lagging). Calculate the kVA of the capacitor bank and the capacitance of each capacitor.
[11.45 kVA ; 2.7 μ F] *(I.E.E. London)*

7. A generator station supplies power to the following : lighting 100 kW; an induction motor 400 h.p. (298.4 kW) power factor 0.8, efficiency 0.92, a rotary converter giving 100 A at 800 V at an efficiency of 0.94. What must be the p.f. of the rotary converter in order that the p.f. of the supply station may be unity. **[0.214 leading] *(C. & G. London)***

8. A substation which is fed by a single feeder cable supplies the following loads : 1,000 kW at p.f. 0.85 lagging, 1,500 kW at 0.8 lagging, 2,000 kW at 0.75 lagging : 500 kW at 0.9 leading. Find the p.f. of the supply to the substation and the load the feeder cable could carry at unity p.f. with the same cable heating. **[0.84 ; 5,960 kW] *(I.E.E. London)***

9. A 3-phase synchronous motor is connected in parallel with a load of 200 kW at 0.8 p.f. (lagging) and its excitation is adjusted until it raises the total p.f. to 0.9 (lagging). If the mechanical load on the motor, including losses, takes 50 kW, calculate the kVA input to the motor.
[57.8 kVA] *(I.E.E. London)*

10. A 3-phase, 3,000-V, 50-Hz motor develops 375 h.p. (223.8 kW), the p.f. being 0.75 lagging and the efficiency 92%. A bank of capacitors is star-connected in parallel with the motor and the total p.f. raised to 0.9 lagging. Each phase of the capacitor bank is made up of 3 capacitors joined in series. Determine the capacitance of each. **[118 μF] *(London Univ.)***

11. For increasing the kW capacity of a power plant working at 0.7 lagging power factor, the necessary increase in power can be obtained by raising the power factor to 0.9 or by installing additional plant. What is the maximum cost per kVA of power factor correction apparatus to make its use more economical than additional plant at Rs. 800 per kVA?
[Rs. 525] *(Utili. of Elect. Power, AMIE)*

12. A 340-kW, 3300-V, 50-Hz, 3f star-connected induction motor has full-load efficiency of 85% and power factor of 0.8 lagging. It is desired to improve power factor to 0.96 lagging by using a bank of three capacitors. Calculate

(*i*) the kVA rating of the capacitor bank,

(*ii*) the capacitance of each limb of the capacitor bank connected in delta,

(*iii*) the capacitance of each capacitor, if each one of the limbs of the delta-connected capacitor bank is formed by using 6 similar 3300-V capacitors.

[(*i*) 183.33 kVAR (*ii*) 17,86 μF (*iii*) 2.977 μF] (*Util. of Elect. Power, AMIE*)

13. A system is working at its maximum kVA capacity with a lagging power factor of 0.71. An anticipated increase of load could be met by

(*i*) raising the power factor of the system to 0.87 by the installation of phase advancers,

(*ii*) installing extra generating plant, cables etc. to meet the increased power demand.

Estimate the limiting cost per kVA of phase advancing plant which would justify its use if the cost for generating plant is Rs. 60 per kVA. Interest and depreciation charges may be assumed to be 1% in each case. **[Rs. 36.5] (*Util. of Elect. Power, AMIE*)**

14. A consumer requires an induction motor of 36.775 kW. He is offered two motors of the following specifications :

Motor *A* : efficiency 88% and p.f. = 0.9 Motor *B* : efficiency 90% and p.f. = 0.81

The consumer is being charged on a two-part tariff of Rs. 70 per kVA of the maximum demand plus 5 paise per unit. The motor (*B*) power factor is raised to 0.89 by installing capacitors. The motor *B* costs Rs. 150 less than *A*.

The cost of capacitor is Rs.60 per kVAR. Determine which motor is more economical and by how much. Assume rate of interest and depreciation as 10% and working hours of motors as 2400 hours in a year. **[Motor B, Rs. 111.10] (*Power Systems-II, AMIE*)**

15. A 250-V, 7.46 kW motor is to be selected for a workshop from two motors *A* and *B*. The cost of each motor is same but the losses at full load are different as given below :

	Motor A	Motor B
Stray loss	1000 W	900 W
Shunt field loss	250 W	200 W
Armature copper loss	300 W	450 W

The motor has to work on full-load for 8 hours, half-load for 4 hours and quarter load for 4 hours each day. Which motor should be selected ? **[Motor B] (*Util. of Elect. Power AMIE*)**

16. What is load duration curve? *(Anna University, April 2002)*

17. How will you classify loads? *(Anna University, April 2002)*

18. Why should there be diversity? *(Anna University, April 2002)*

19. Define load factor. *(Anna University, April 2002)*

20. What is M.D.? *(Anna University, April 2002)*

21. Distinguish between a base load plant and a peak load plant. *(Anna University, April 2002)*

OBJECTIVE TESTS – 50

1. While calculating the cost of electric power generation, which of the following is NOT considered a fixed cost ?

(*a*) interest on capital investment

(*b*) taxes and insurance

(*c*) most of the salaries and wages

(*d*) repair and maintenance.

2. Maximum demand of an installation is given by its

(*a*) instantaneous maximum demand

(*b*) greatest average power demand

(*c*) average maximum demand over a definite interval of time during a certain period.

(*d*) average power demand during an interval of 1-minute.

3. A diversity factor of 2.5 gives a saving of .... percent in the generating equipment.

(*a*) 60

(*b*) 50

(*c*) 40

(*d*) 25.

4. Mark the WRONG statement.
High load factor of a generating equipment
(*a*) leads to lesser charges per kWh
(*b*) implies lower diversity in demand
(*c*) gives more profit to the owner
(*d*) can be obtained by accepting off-peak loads.

5. In a generating station, fixed, charges at 100% load factor are 6 paise/kWh. With 25% load factor, the charges would become .... paise/kWh.
(*a*) 1.5
(*b*) 10
(*c*) 24
(*d*) 3

6. When considering the economics of power transmission, Kelvin's law is used for finding the
(*a*) cost of energy loss in bare conductors
(*b*) most economical cross-section of the con ductors
(*c*) interest on capital cost of the con- ductor
(*d*) the maximum voltage drop in feeders.

7. A 3-phase balanced system working at 0.9 lagging power factor has a line loss of 3600 kW. If p.f. is reduced to 0.6, the line loss would become .... kW.
(*a*) 8100
(*b*) 1600
(*c*) 5400
(*d*) 2400.

8. Mark the WRONG statement.
While considering power factor improve- ment, the most economical angle of lag depends on the
(*a*) cost/kVA rating of phase advancer
(*b*) rate of interest on capital outlay
(*c*) rate of depreciation
(*d*) value of original lagging p.f. angle.

9. Load factor of a power station is defined as
(*a*) maximum demand / average load
(*b*) average load × maximum demand
(*c*) average load / maximum demand
(*d*) (average load × maximum demand)$^{1/2}$

10. Load factor of a power station is generally
(*a*) equal to unity
(*b*) less than unity
(*c*) more than unity
(*d*) equal to zero

11. Diversity factor is always
(*a*) equal to unity
(*b*) less than unity
(*c*) more than unity
(*e*) more than twenty

12. Load factor for heavy industries may be taken as
(*a*) 10 to 20%
(*b*) 25 to 40%
(*c*) 50 to 70%
(*d*) 70 to 80%

13. The load factor of domestic load is usually
(*a*) 10 to 15%
(*b*) 30 to 40%
(*c*) 50 to 60%
(*d*) 60 to 70%

14. Annual depreciation cost is calculated by
(*a*) sinking fund method
(*b*) straight line method
(*c*) both (*a*) and (*b*)
(*d*) none of the above

15. Depreciation charges are high in case of
(*a*) thermal plant
(*b*) diesel plant
(*c*) hydroelectric plant

16. Demand factor is defined as
(*a*) average load/maximum load
(*b*) maximum demand/connected load
(*c*) connected load/maximum demand
(*d*) average load × maximum load

17. High load factor indicates
(*a*) cost of generation per unit power is increased
(*b*) total plant capacity is utilised for most of the time
(*c*) total plant capacity is not properly utilised for most of the time
(*d*) none of the above

18. A load curve indicates
(*a*) average power used during the period
(*b*) average kWh (kW) energy consumption during the period
(*c*) either of the above
(*d*) none of the above

19. Approximate estimation of power demand can be made by
(*a*) load survey method
(*b*) statistical methods
(*c*) mathematical method
(*d*) economic parameters
(*e*) all of the above

20. Annual depreciation as per straight line method, is calculated by
(*a*) the capital cost divided by number of year of life
(*b*) the capital cost minus the salvage value, is divided by the number of years of life
(*c*) increasing a uniform sum of money per annum at stipulated rate of interest
(*d*) none of the above

21. A consumer has to pay lesser fixed charges in
(*a*) flat rate tariff
(*b*) two part tariff
(*c*) maximum demand tariff
(*d*) any of the above

22. In two part tariff, variation in load factor will affect
(*a*) fixed charges
(*b*) operating or running charges
(*c*) both (*a*) and (*b*)
(*d*) either (*a*) & (*b*)

23. In Hopkinson demand rate or two part tariff the demand rate for fixed charges are
(*a*) dependent upon the energy consumed
(*b*) dependent upon the maximum demand of the consumer
(*c*) both (*a*) and (*b*)
(*d*) neither (*a*) and (*b*)

24. Which plant can never have 100 percent load factor?
(*a*) Peak load plant
(*b*) Base load plant
(*c*) Nuclear power plant
(*d*) Hydro electric plant

25. The area under a load curve gives
(*a*) average demand
(*b*) energy consumed
(*c*) maximum demand
(*d*) none of the above

26. Different generating stations use following prime movers
(*a*) diesel engine
(*b*) hydraulic turbine
(*c*) gas turbine
(*d*) steam turbine
(*e*) any of the above

27. Diversity factor has direct effect on the
(*a*) fixed cost of unit generated
(*b*) running cost of unit generated
(*c*) both (*a*) and (*b*)
(*d*) neither (*a*) nor (*b*)

28. Following power plant has instant starting
(*a*) nuclear power plant
(*b*) hydro power plant
(*c*) both (*a*) and (*b*)
(*d*) none of the above

29. Which of the following generating station has minimum running cost?
(*a*) Nuclear
(*b*) Hydro
(*c*) Thermal
(*d*) Diesel

30. Power plant having maximum demand more than the installed rated capacity will have utilisation factor
(*a*) equal to unity
(*b*) less than unity
(*c*) more than unity
(*d*) none of the above

31. Load curve is useful in deciding the
(*a*) operating schedule of generating units
(*b*) sizes of generating units
(*c*) total installed capacity of the plant
(*d*) all of the above

32. Load curve of a power plant has always
(*a*) zero slope
(*b*) positive slope
(*c*) negative slope
(*d*) any combination of (*a*), (*b*) and (*c*)

33. Annual operating expenditure of a power plant consists of
(*a*) fixed charges
(*b*) semi-fixed charges
(*c*) running charges
(*d*) all of the above

34. Maximum demand on a power plant is
(*a*) the greatest of all "short time interval averaged" demand during a period
(*b*) instantaneous maximum value of kVA supplied during a period
(*c*) both (*a*) or (*b*)
(*d*) none of the above

35. Annual instalment towards depreciation reduces as rate of interest increases with
(*a*) sinking fund depreciation
(*b*) straight line depreciation
(*c*) reducing balances depreciation
(*d*) none of the above

36. Annual depreciation of the plant is proportional to the earning capacity of the plant vide

(*a*) sinking fund depreaciation
(*b*) straight line depreciation
(*c*) reducing balances depreciation
(*d*) none of the above

37. For high value of diversity factor, a power station of given installed capacity will be in a position to supply
(*a*) less number of consumers
(b) more number of consumers
(*c*) neither (*a*) nor (*b*)
(*d*) either (*a*) or (*b*)

38. Salvage value of the plant is always
(*a*) positive
(*b*) negative
(*c*) zero
(*d*) any of the above

39. Load curve helps in deciding
(a) total installed capacity of the plant
(b) size of the generating units
(c) operating schedule of generating units
(d) all of the above

40. .... can generate power at unpredictable or uncontrolled times.
(*a*) Solar power plant
(*b*) Tidal power plant
(*c*) Wind power plant
(*d*) Any of the above

41. Direct conversion of heat into electric power is possible through
(*a*) fuel cell
(*b*) batteries
(*c*) thermionic converter
(*d*) all of the above

42. A low utilization factor for a plant indicates that
(*a*) plant is used for stand by purpose only
(*b*) plant is under maintenance
(*c*) plant is used for base load only
(*d*) plant is used for peak load as well as base load

43. Which of the following is not a source of power?
(*a*) Thermocouple
(*b*) Photovoltaic cell
(*c*) Solar cell
(*d*) Photoelectric cell

44. Which of the following should be used for extinguishing electrical fires?
(*a*) Water
(*b*) Carbon tetrachloride fire extinguisher
(*c*) Foam type fire extinguisher
(*d*) CO_2 fire extinguisher

45. Low power factor is usually not due to
(*a*) arc lamps
(*b*) induction motors
(*c*) fluorescent tubes
(*d*) incandescent lamp

46. Ships are generally powered by
(*a*) unclear power plants
(*b*) hydraulic turbines
(*c*) diesel engines
(*d*) steam accumulators
(*e*) none of the above

47. Direct conversion of heat into electrical energy is possible through
(*a*) fuel cells
(*b*) solar cells
(*c*) MHD generators
(*d*) none of the above

48. Which of the following place is not associated with nuclear power plants in India?
(*a*) Narora
(*b*) Tarapur
(*c*) Kota
(*d*) Benglore

49. During load shedding
(*a*) system power factor is changed
(*b*) some loads are switched off
(*c*) system voltage is reduced
(*d*) system frequency is reduced

50. Efficiency is the secondary consideration in which of the following plants?
(*a*) Base load plants
(*b*) Peak load plants
(*c*) Both (*a*) and (*b*)
(*d*) none of the above

51. Air will not be the working substance in which of the following?
(*a*) Closed cycle gas turbine
(*b*) Open cycle gas turbine
(*c*) Diesel engine
(*d*) Petrol engine

52. A nuclear power plant is invariably used as a
(*a*) peak load plant
(*b*) base load plant
(*c*) stand-by plant
(*d*) espinning reserve plant
(*e*) any of the above

53. ....power plant is expected to have the longest life.

(a) Steam
(b) Diesel
(c) Hydroelectric
(d) Any of the above

54. .... power plant cannot have single unit of 100 MW.
(a) Hydroelectric
(b) Nuclear
(c) Steam
(d) Diseal
(e) Any of the above

55. Which of the following, in a thermal power plant, is not a fixed cost?
(a) Fuel cost
(b) Interest on capital
(c) Depreciation
(d) Insurance charges

56. .... will offer the least load.
(a) Vacuum cleaner
(b) Television
(c) Hair dryer
(d) Electric shaver

57. In .... fuel transportation cost is least.
(a) nuclear power plants
(b) diesel generating plants
(c) steam power stations

58. Which of the following equipment provides fluctuating load?
(a) Exhaust fan
(b) Lathe machine
(c) Welding transformer
(d) All of the above

59. The increased load during summer months is due to
(a) increased business activity
(b) increased water supply
(c) increased use of fans and air conditioners
(d) none of the above

60. .... is the reserved generating capacity available for service under emergency conditions which is not kept in operation but in working order.
(a) Hot reserve
(b) Cold reserve
(c) Spinning reserve
(d) Firm power

61. Generating capacity onnected to the bus bars and ready to take load when switched on is known as ....
(a) firm power
(b) cold reserve
(c) hot reserve
(d) spinning reserve

62. .... offers the highest electric load.
(a) Television set
(b) Toaster
(c) Vacuum cleaner
(d) Washing machine

63. .... industry has the least power consumption per tonne of product.
(a) Soap
(b) Sugar
(c) Vegetable oil
(d) Caustic soda

64. With reference to a power station which of the following is not a fixed cost?
(a) Fuel cost
(b) Interest on capital
(c) Insurance changes
(d) Depreciation

65. .... is invariably used as base load plant.
(a) Diesel engine plant
(b) Nuclear power plant
(c) Gas turbine plant
(d) Pumped storage plant

66. In a power plant if the maximum demand on the plant is equal to the plant capacity, then
(a) plant reserve capacity will be zero
(b) diversity factor will be unity
(c) load factor will be unity
(d) load factor will be nearly 60%

67. In case of .... fuel transportation is the major problem.
(a) diesel power plants
(b) nuclear power plants
(c) hydro-electric power plants
(d) thermal power plants

68. Which of the following power plants need the least period for installation?
(a) Thermal power plant
(b) Diesel power plant
(c) Nuclear power plant
(d) Hydro-electric power plant

69. For which of the following power plants highly skilled engineers are required for running the plants?
(a) Nuclear power plants
(b) Gas turbine power plants
(c) Solar power plants
(d) Hydro-electric power plants

70. In which of the following power plants the maintenance cost is usually high?
(*a*) Nuclear power plant
(*b*) Hydro-electric power plants
(*c*) Thermal power plants
(*d*) Diesel engine power plants

71. .... is inveriably used for peak load
(*a*) Nuclear power lant
(*b*) Steam turbine plant
(*c*) Pumped storage plant
(*d*) None of the above

72. Which of the following is not an operating cost?
(*a*) Maintenance cost
(*b*) Fuel cost
(*c*) Salaries of high officials
(*d*) Salaries of operating stall

73. Which of the following is the essential requirement of peak load plant?
(*a*) It should run at high speed
(*b*) It should produce high voltage
(*c*) It should be small in size
(*d*) It should be capable of starting quickly

74. Large capacity generators are invariably
(*a*) water cooled
(*b*) natural air cooled
(*c*) forced air cooled
(*d*) hydrogen cooled

75. By the use of which of the following power factor can be improved?
(*a*) Phase advancers
(*b*) Synchronous compensators
(*c*) Static capacitors
(*d*) Any of the above

76. An induction motor has relatively high power factor at
(*a*) rated r.p.m.
(*b*) no load
(*c*) 20 percent load
(*d*) near full load
(*e*) none of the above

77. Which of the following is the disadvantage due to low power factor?
(*a*) Poor voltage regulation
(*b*) Increased transmission losses
(*c*) High cost of equipment for a given load
(*d*) All of the above

78. In a distribution system, in order to improve power factor, the synchronous capacitors are installed
(*a*) at the receiving end
(*b*) at the sending end
(*c*) either (*a*) or (*b*)
(*d*) none of the above

79. Satic capacitors are rated in terms of
(*a*) kW
(*b*) kWh
(*c*) kVAR
(*d*) none of the above

80. Base load plants usually have .... capital cost, ... operating cost and .... load factor.
(*a*) high, high, high
(*b*) high, low, high
(*c*) low, low, low
(*d*) low, high, low

81. Which of the following is the disadvantage of a synchronous condenser?
(*a*) High maintenance cost
(*b*) Continuous losses in motor
(*c*) Noise
(*d*) All of the above

82. For a consumer the most economical power factor is generally
(*a*) 0.5 lagging
(*b*) 0.5 leading
(*c*) 0.95 lagging
(*d*) 0.95 leading

83. A synchronous condenser is virtually which of the following?
(*a*) Induction motor
(*b*) Underexcited synchronous motor
(*c*) Over excited synchronous motor
(*d*) D.C. generator
(*e*) None of the above

84. For a power plant which of the following constitutes running cost?
(*a*) Cost of wages
(*b*) Cost of fuel
(*c*) Cost of lubricants
(*d*) All of the above

85. In an interconnected system, the diversity factor of the whole system
(*a*) remains unchanged
(*b*) decreases
(*c*) increases
(*d*) none of the above

86. Generators for peak load plants are usually designed for maximum efficiency at
(*a*) 25 to 50 percent full load
(*b*) 50 to 75 percent full load
(*c*) full load
(*d*) 25 percent overload

87. .... will be least affected due to charge in supply voltage frequency.
(*a*) Electric clock
(*b*) Mixer grinder
(*c*) Ceiling fan
(*d*) Room heater

88. For the same maximum demand, if load factor is decreased, the cost generation will
(*a*) remain unchanged
(*b*) decrease
(*c*) increase

89. The connected load of a domestic consumer is around
(*a*) 5 kW
(*b*) 40 kW
(*c*) 80 kW
(*d*) 120 kW

90. Which of the following is not necessarily an advantage of interconnecting various power stations?
(*a*) Improved frequency of power supplied
(*b*) Reduction in total installed capacity
(*c*) Increased reliability
(*d*) Economy in operation of plants

91. A power transformer is usually rated in
(*a*) kW
(*b*) kVAR
(*c*) kWh
(*d*) kVA

92. .... public sector undertaking is associated with erection and sometimes running of thermal power plants
(*a*) NTPC
(*b*) SAIL
(*c*) BEL
(*d*) BHEL

93. Most efficient plants are normally used as
(*a*) peak load plants
(*b*) base load plants
(*c*) either (*a*) or (*b*)
(*d*) none of the above

94. For a diesel generating station the useful life is expected to be around
(*a*) 15 to 20 years
(*b*) 20 to 50years
(*c*) 50 to 75 years
(*d*) 75 to 100 years

95. Which of the following is not a method for estimating depreciation charges?
(*a*) Sinking fund method
(*b*) Straight line method
(*c*) Diminishin value method
(*d*) Halsey's 50–50 formula

96. The expected useful life of an hydroelectric power station is around
(*a*) 15 years
(*b*) 30 years
(*c*) 60 years
(*d*) 100 years

97. In a load curve the highest point represents
(*a*) peak demand
(*b*) average demand
(*c*) diversified demand
(*d*) none of the above

98. Which of the following source of power is least reliable?
(*a*) Solar energy
(*b*) Geothermal power
(*c*) Wind power
(*d*) MHD

99. In Indial production and distribution of electrical energy is confined to
(*a*) private sector
(*b*) public sector
(*c*) government sectors
(*d*) joint sector
(*e*) none of the above

100. A pilot exciter is provided on generators for which of the following reasons?
(*a*) To excite the poles of main exciter
(*b*) To provide requisite starting torque to main exciter

(*c*) To provide requisit starting torque to generator
(*d*) None of the above

101. The primary reason for low power factor is supply system is due to installation of
(*a*) induction motors
(*b*) synchronous motors
(*c*) single phase motors
(*d*) d.c. motors

102. an over excited synchronous motor on no-load is known as
(*a*) syachronous condenser
(*b*) generator
(*c*) induction motor
(*d*) alternator

103. Which of the following is an advantage of static capacitor for power factor improvement?
(*a*) Little maintenance cost
(*b*) Ease in installation
(*c*) Low losses
(*d*) All of the above

104. For any type of consumer the ideal tariff is
(*a*) two part tariff
(*b*) three part tariff
(*c*) block rate tariff
(*d*) any of the above

105. The efficiency of a plant is of least concern when it is selected as
(*a*) peak load plant
(*b*) casual run plant
(*c*) either (*a*) or (*b*)
(*d*) base load plant

106. Power generation cost reduces as
(*a*) diversity factor increases and load factor decreases
(*b*) diversity factor decreases and load factor increases
(*c*) both diversity factor as well as load factor decrease
(*d*) both diversity factor as well as load factor increase

107. The depreciation charges in diminishing value method are
(*a*) light in early years
(*b*) heavy in early years
(*c*) heavy in later years
(*d*) same in all years

108. The area under daily load curve divided by 24 hours gives
(*a*) average load
(*b*) least load
(*c*) peak demand
(*d*) total kWh generated

109. Maximum demand tariff is generally not applied to domestic consumers because
(a) they consume less power
(*b*) their load factor is low
(*c*) their maximum demand is low
(*d*) none of the above

110. A 130 MW generator is usually .... cooled
(*a*) air
(*b*) oxygen
(*c*) nitrogen
(*d*) hydrogen

111. For cooling of large size generators hydrogen is used because
(*a*) it is light
(*b*) it offers reduced fire risk
(*c*) it has high thermal conductivity
(*d*) all of the above

112. Major share of power produced in India is through
(*a*) diesel power plants
(*b*) hydroelectric power plants
(*c*) thermal power plants
(*d*) nuclear power plants

113. Which of the following may not be the effect of low plant operating power factor?
(*a*) Improved illumination from lighting
(*b*) Reduced voltage level
(*c*) Over loaded transformers
(*d*) Overloaded cables

114. Which of the following plants is almost inevitably used as base load plant?
(*a*) Diesel engine plant
(*b*) Gas turbine plant
(*c*) Nuclear power plant
(*d*) Pumped storage plant

115. Which of the following component, in a steam power plant, needs maximum maintenance attention?
(*a*) Steam turbine
(*b*) Condenser
(*c*) Water treatment plant
(*d*) Bioler

116. For the same cylinder dimensions and speed, which of the following engine will produce least power?
(*a*) Supercharged engine
(*b*) Diesel engine

(c) Petrol engine
(d) All of the above engines will equal power

117. The least share of power is provided in India, by which of the following power plants?
(a) Diesel power plants
(b) Thermal power plants
(c) Hydro-electric power plants
(d) Nuclear power plants

118. Submarines for under water movement, are powered by which of the following?
(a) Steam accumulators
(b) Air motors
(c) Diesel engines
(d) Batteries

119. An alternator coupled to a .... runs at slow speed, as compared to as compared to others.
(a) diesel engine
(b) hydraulic turbine
(c) steam turbine
(d) gas turbine

120. The effect of electric shock on human body depends on which of the following
(a) current
(b) voltage
(c) duration of cantact
(d) all of the above

121. Which lightening stroke is most dangerous?
(a) Direct stroke on line conductor
(b) Indirect stroke on conductor
(c) Direct stroke on tower top
(d) Direct stroke on ground wire

122. Which of the following devices may be used to provide protection against lightening over voltages?
(a) Horn gaps
(b) Rod gaps
(c) Surge absorbers
(d) All of the above

123. When the demand of consumers is not met by a power plant, it will resort to which of the following?
(a) Load shedding
(b) Power factor improvement at the generators
(c) Penalising high load consumers by increasing the charges for electricity
(d) Efficient plant operation.

124. Load shedding is possible through which of the following?
(a) Switching of the loads
(b) Frequency reduction
(c) Voltage reducteion
(d) Any of the above

125. In power plants insurance cover is provided for which of the following?
(a) Unskilled workers only
(b) Skilled workers only
(c) Equipment only
(d) All of the above

126. A company can raise funds through
(a) fixed deposits
(b) shares
(c) bonds
(d) any of the above

127. Which of the following are not repayable after a stipulated period?
(a) Shares
(b) Fixed deposits
(c) Cash certificates
(d) Bonds

128. The knowledge of diversity factor helps in determining
(a) plant capacity
(b) average load
(c) peak load
(d) kWh generated
(e) none of the above

129. Load shedding is done to
(a) improve power factor
(b) run the equipment efficiently
(c) repair the machine
(d) reduce peak demand

130. when a plant resorts to load shedding it can be concluted that
(a) peak demand is more than the installed capacity
(b) daily load factor is unity
(c) diversity factor is zero
(d) plant is under repairs

131. Which of the following is the disadvantage of static capacitor for power factor improvement?
(a) Easily damaged by high voltage
(b) Cannot be repaired
(c) Short service life
(d) All of the above

132. If the tariff for electrical energy charages provides incentive by way of reduced charges for higher consumption, then it can be concluede that
(a) Load factor is unity

(*b*) power is generated through hydroelectric plant
(*c*) plant has sufficient reserve capacity
(*d*) station has more than two generators

133. Anything having some heat value can be used as fuel in case of
(*a*) open cycle gas turbines
(*b*) closed cycle gas turbines
(*c*) petrol engines
(*d*) diesel engines

ANSWERS

1. (*d*)	**2.** (*c*)	**3.** (*a*)	**4.** (*b*)	**5.** (*c*)	**6.** (*b*)	**7.** (*a*)
8. (*d*)	**9.** (*c*)	**10.** (*b*)	**11.** (*c*)	**12.** (*d*)	**13.** (*a*)	**14.** (*c*)
15. (*a*)	**16.** (b)	**17.** (*b*)	**18.** (*b*)	**19.** (*e*)	**20.** (*b*)	**21.** (*c*)
22. (*b*)	**23.** (*b*)	**24.** (*a*)	**25.** (*b*)	**26.** (*e*)	**27.** (*a*)	**28.** (*d*)
29. (*b*)	**30.** (*c*)	**31.** (*d*)	**32.** (*d*)	**33.** (*d*)	**34.** (*a*)	**35.** (*a*)
36. (*c*)	**37.** (*b*)	**38.** (*d*)	**39.** (*d*)	**40.** (*d*)	**41.** (*c*)	**42.** (*a*)
43. (*a*)	**44.** (*b*)	**45.** (*d*)	**46.** (*c*)	**47.** (*c*)	**48.** (*d*)	**49.** (*b*)
50. (*b*)	**51.** (*a*)	**52.** (*b*)	**53.** (*c*)	**54.** (*d*)	**55.** (*a*)	**56.** (*d*)
57. (*a*)	**58.** (*c*)	**59.** (*c*)	**60.** (*b*)	**61.** (*d*)	**62.** (*b*)	**63.** (*c*)
64. (*a*)	**65.** (*b*)	**66.** (*a*)	**67.** (*d*)	**68.** (*b*)	**69.** (*a*)	**70.** (*c*)
71. (*c*)	**72.** (*c*)	**73.** (*d*)	**74.** (*d*)	**75.** (*d*)	**76.** (*d*)	**77.** (*d*)
78. (*a*)	**79.** (*c*)	**80.** (*b*)	**81.** (*d*)	**82.** (*c*)	**83.** (*c*)	**84.** (*d*)
85. (*c*)	**86.** (*b*)	**87.** (*f*)	**88.** (*c*)	**89.** (*a*)	**90.** (*a*)	**91.** (*d*)
92. (*a*)	**93.** (*b*)	**94.** (*a*)	**95.** (*d*)	**96.** (*d*)	**97.** (*a*)	**98.** (*c*)
99. (*b*)	**100.** (*a*)	**101.** (*a*)	**102.** (*a*)	**103.** (*d*)	**104.** (*b*)	**105.** (*c*)
106. (*d*)	**107.** (*b*)	**108.** (*a*)	**109.** (*c*)	**110.** (*d*)	**111.** (*d*)	**112.** (*c*)
113. (*a*)	**114.** (*c*)	**115.** (*d*)	**116.** (*d*)	**117.** (*a*)	**118.** (*d*)	**119.** (*b*)
120. (*d*)	**121.** (*a*)	**122.** (*d*)	**123.** (*a*)	**124.** (*d*)	**125.** (*c*)	**126.** (*d*)
127. (*d*)	**128.** (a)	**129.** (*d*)	**130.** (*a*)	**131.** (*d*)	**132.** (*c*)	**133.** (*b*)